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Innovation Under Ambiguity and Risk

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Abstract

We view innovation investments as real options and explore the implications of risk (volatility) as well as a newly defined outcome independent measure of ambiguity (Knightian uncertainty) for innovation decisions. The empirical analysis uses stock returns to compute an implementable measure of ambiguity. We also control for risk and other determinants of innovation. We find a consistently significant negative effect of ambiguity on R&D, patents, and citations, as predicted. The effect of risk on R&D is positive and significant, but the corresponding effect on patents and citations is negative and significant. Ambiguity matters more for high-tech firms, consistent with intuition.

I. Introduction

A large and growing body of literature investigates the determinants of innovation decisions, including industry competition (Aghion, Bloom, Blundell, Griffith, and Howitt (2005)), institutional ownership (Aghion, Van Reenen, and Zingales (2013)), organizational structure (Lerner, Sorensen, and Strömberg (2011), Seru (2014), and Bernstein (2015)), management quality (Chemmanur, Kong, Krishnan, and Yu (2019)), and anti-takeover provisions (Chemmanur and Tian (2018)). Special attention has been paid to risk as a determinant of innovative activity. *Risk* (the uncertainty of outcomes) assumes a unique known distribution of future outcomes. In reality, however, it may be very difficult (and, perhaps,

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impossible) to predict a distribution of future outcomes for a new innovative product such as a new drug (Krieger et al. (2017)). Therefore, *ambiguity* (the uncertainty of probabilities) seems a natural determinant of future prospects. We utilize a new concept of ambiguity, which is theoretically underpinned and empirically testable, to investigate how each type of uncertainty (risk and ambiguity) affects innovative decisions, and which type of uncertainty is more salient for innovating firms.

Early studies on risk and investments (e.g., Hartman (1972), Abel (1983)) suggest that, since the marginal value product of capital is a convex function of the risk faced by the firm, greater risk raises the marginal valuation of each additional unit of capital invested, thereby increasing investment. This view was translated into a real options framework (Brennan and Schwartz (1985), Childs, Ott, and Triantis (1998)). Building on this concept, Schwartz (2004) and Kraft, Schwartz, and Weiss (2018) view research and development (R&D) and patent decisions as real options. The perceived payoff of the option is the difference between the revenue generated by a new product and the eventual investment required to put the technology into production (the exercise price). If the technology is abandoned, the payoff is 0. The standard real options approach thus implies a positive effect of risk on R&D.

Another strand of literature, analyzing risk (or, more generally, uncertainty) and investments in a dynamic, multiperiod setup, concludes that uncertainty may decrease corporate investments due to the irreversibility effect: delaying an investment decision allows the firm to wait for new and possibly better information (Dixit and Pindyck (1994) and the references therein). Specifically, the opportunity cost associated with irreversible investments increases in uncertainty, and high uncertainty may increase the value of delaying irreversible investments, thereby reducing immediate investments (McDonald and Siegel (1986), Ingersoll and Ross (1992)). In other words, higher uncertainty may lead to a drop in investment because of irreversibility (a "delay" effect), while also making firms less responsive to any given shock or policy (a "caution" effect; Bloom (2007), (2014)).[1] Thus, different studies suggest opposite effects of uncertainty on investments in general and on R&D investments in particular.

Bloom (2014) emphasizes the importance of considering a wider set of uncertainty measures: "e.g., there is little data on (…) the nature of uncertainty (risk vs. Knightian)." Consistent with this observation, a new strand of literature shows that option values are significantly affected not only by risk, but also by ambiguity, or Knightian uncertainty (Izhakian and Yermack (2017), Augustin and Izhakian (2020)). Intuitively, ambiguity captures the variation in outcomes' frequencies (probabilities) but ignores the magnitudes of outcomes (returns). In contrast, risk captures the variation in the magnitudes of outcomes for a given distribution, ignoring the variation in probabilities. Following this literature, we introduce ambiguity, alongside risk, into the real-option approach to valuing innovation

[1] Bloom (2007) attributes investment irreversibility to adjustment costs, which operate differently for investments in capital stock versus investments in knowledge stock. The former typically incurs *stock* adjustment costs, while the latter incurs *flow* adjustment costs from changing the flow of new knowledge from R&D.

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investments. Conceptually, an investment in innovation (R&D or patents) is analogous to creating an option to invest in production at a later stage. We also consider the effects of ambiguity and risk on the *timing* of innovation investments: the decision maker (DM, a manager or an investor) decides how much to immediately invest in innovation and how much to divert into future investments.

Many experimental studies show that DMs tend to be ambiguity averse (Ellsberg (1961), Halevy (2007)). In particular, DMs prefer alternatives involving clear probabilities (risk, the *known unknowns*) over alternatives involving vague probabilities (ambiguity, the *unknown unknowns*). Ambiguity aversion has been shown to be economically relevant and to persist in experimental market settings and among business owners and managers.[2] The effect of ambiguity on investment decisions is very different from the effect of risk. An ambiguity-averse DM overweights the likelihoods of bad outcomes and underweights the likelihoods of good outcomes (Tversky and Kahneman (1992), Izhakian (2017)).[3] Therefore, as ambiguity increases, DMs will tend to value options lower. An increase in ambiguity will discourage creating real options (investing in innovation).

While both ambiguity and risk may affect any investment decision, we expect ambiguity to be more important for new innovative investments rather than, say, for investments in renovations or expansions of existing product lines. For example, it would be difficult to view an investment in refurbishing an office building as creating a real option and the probabilities associated with this project as ambiguous. However, an investment in a new lab may create a real option to license a new drug commercially, and thus such a setting is closer to our conceptual view and hypotheses. The accounting treatment of these two types of investments differs as well. Whereas renovations appear under capital expenditures (CAPEX), investments in developing a new technology appear under R&D in firms' accounting statements. R&D is treated as expenses, whereas CAPEX investments are depreciated. Therefore, in our main tests, we measure innovation investments using R&D expenses and measure later-stage innovation using patents and citations.

We find that ambiguity reduces innovation investments, both when measured by R&D and when measured by patents. We also find that high-tech firms are more sensitive to ambiguity, although the level of ambiguity they face is lower than that of "low-tech" firms. Consistent with the intuition in Herron and Izhakian (2017), (2019), non-high-tech firms typically do not have many internal growth opportunities, and the opportunities that exist are likely to be inorganic and not in the firm's core business. Thus, they may be characterized by ambiguous prospects (e.g., entering new markets). High-tech firms tend to have internal organic growth opportunities in their core business (e.g., expansion of existing activities) whose characteristics are similar to the firm's assets in place and therefore are less ambiguous. However, an increase in ambiguity may affect the entire business of high-tech firms; therefore, such firms may be more sensitive to changes in ambiguity.

[2]See, e.g., Mangelsdorff and Weber (1994), Viscusi and Chesson (1999), Abdellaoui, Vossmann, and Weber (2005), Du and Budescu (2005), Maffioletti and Santoni (2005), and Wakker, Timmermans, and Machielse (2007).

[3]Behavior consistent with this way of thinking was also found in several experimental studies (e.g., Wu and Gonzalez (1999), Abdellaoui and Kemel (2013), and Crockett, Izhakian, and Jamison (2019)).

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We find a significant positive effect of risk on R&D investments, which is consistent with the standard option perspective. However, we also document a significant negative effect of risk on patents and citations, consistent with a multi-period setting, suggesting that it may be optimal to delay investments until more information is acquired and uncertainty decreases (McDonald and Siegel (1986), Dixit and Pindyck (1994), and Bloom (2007)). The different effects of risk on R&D investments and on patent investments may also be attributed to the different nature of these two types of innovation activities. For example, the costs of delaying patent filings may be lower than the costs of delaying R&D investments.

We add a new dimension to studies on the relation between innovation investments and various types of uncertainty. Related empirical studies include Bernstein, McQuade, and Townsend (2021), who suggest that macroeconomic risk, measured by negative housing shocks, reduces employees' interest in risky and exploratory projects. Krieger et al. (2017) investigate the tradeoff between conservative and riskier investments in drug development. Chemmanur et al. (2019) find that higher quality management invests more in R&D, and creates more exploratory patents with more citations, which they interpret as riskier strategies. Lyandres and Palazzo (2016) suggest that returns to R&D investments depend on market competition. Kumar and Li (2018) document a positive association between idiosyncratic volatility and the response rate of subsequent innovation-related investments.[4]

Finally, our article is also related to studies investigating the effect of ambiguity on corporate finance decisions, including capital structure decisions to fund corporate investment (Malenko and Tsoy (2020), Izhakian, Yermack, and Zender (2022b)), mergers and acquisitions and payout policy. If, due to high ambiguity, managers do not find attractive internal investments, they will choose to "bid instead of build," that is, attempt to acquire existing (less ambiguous) technology (Herron and Izhakian (2019)). If neither attractive internal investment opportunities nor attractive external acquisition opportunities are available due to their high ambiguity, the firm will tend to increase payout through dividends or share repurchases (Herron and Izhakian (2017)).

II. Theoretical Background and Hypotheses Development

We introduce the concept of ambiguity into the innovation investment literature. To this end, we build on earlier work that views innovation investments as real options, and we also incorporate insights from the literature on the value of waiting to invest.

A. The Ambiguity Concept

Ambiguity, or Knightian uncertainty, provides the basis for a rich literature in decision theory. This literature has taken a variety of approaches to model decision-

[4]In Kumar and Li (2018), the response rate of innovation-related investments is defined as the *absolute* percentage change in innovation investments. This finding is interpreted in light of a feedback model in which idiosyncratic volatility proxies for investors' private information regarding the prospects of the firm's innovation projects.

making under ambiguity (e.g., Gilboa and Schmeidler (1989), Schmeidler (1989), and Klibanoff, Marinacci, and Mukerji (2005)). A common concept in these models is that, in the presence of ambiguity, ambiguity-averse DMs act *as if* they overweight the probabilities of the unfavorable outcomes and underweight the probabilities of the favorable outcomes, which, in our case, lowers the perceived expected payoff of innovation investments. Expected utility with uncertain probabilities (EUUP; Izhakian (2017)) is a new theory that models ambiguity as independent of outcomes. It distinguishes the concepts of risk and ambiguity by specifying distinct preferences for both. Importantly, the EUUP framework allows us to measure ambiguity empirically, independently of risk and of the attitudes toward ambiguity and risk. In this framework, the degree of ambiguity can be measured by the volatility of probabilities—just as the degree of risk has been measured by the volatility of outcomes (Izhakian (2020)). This measure accounts for the variations of all moments of the outcome distribution and can be utilized in empirical investigations.[5]

Outcome independence is new and central to this notion of ambiguity. To illustrate this idea, consider an innovation investment whose payoff is determined by a flip of an unbalanced coin for which the DM does not know the odds of heads or tails. The payoff of the innovation investment is $1,000 in the case of heads and $0 in the case of tails. Suppose that prior to flipping the coin, the payoff in the case of heads changes to $2,000. Since this change in payoffs provides no new information about the probabilities involved, and they stay the same, ambiguity stays the same as well, and so do the perceived ("certainty equivalent") probabilities. In other words, formally, ambiguity is *outcome independent* up to a state-space partition since it applies exclusively to probabilities. However, it is clear that risk does increase in this example, since it is outcome dependent and the outcomes have changed.

B. The Real Options View

Innovation investments (R&D and patents) can be viewed as real options. To clarify the concepts involved, consider a one-period, real options setup where the firm faces a one-time decision whether to invest in innovation or not.[6] For example, the firm has to decide whether to invest in developing a new drug or a new technology. At the beginning of the period, the firm will make an investment in innovation (pay the option "premium") only if the value of the option created is greater than or equal to the premium, given the "exercise price" (i.e., the eventual outlay for production) and the other parameters in question.[7]

It is well-known that the value of a (real) option increases in risk. The effect of ambiguity is very different. Since ambiguity-averse DMs act *as if* they overweight

[5]The EUUP measure of ambiguity is employed in several empirical studies using equity market data (e.g., Izhakian and Yermack (2017), Brenner and Izhakian (2018), Augustin and Izhakian (2020), Izhakian et al. (2022b), and Ben-Rephael et al. (2022a)) and bond data (e.g., Izhakian, Lewis, and Zender (2022a)).

[6]Analogous to a financial call option, this view corresponds to a European call option on a stock.

[7]We should note that there is no ready market for firm-specific innovation options (except, to some extent, the takeover market).

the probabilities of bad outcomes (out-of-the-money states) and underweight the probabilities of good outcomes (in-the-money states), ambiguity reduces the perceived value of an option. In particular, the higher the ambiguity, the lower the perceived probability of a positive payoff, which the DM uses to compute the expected payoff of the (real) option.[8] As a result, an increase in ambiguity decreases the value of the (real) option, leading to lower investment in innovation.

C. A Binomial Example

To illustrate the effect of ambiguity and risk on the value of an innovation investment (seen as a real option), consider a one-period project to produce a new drug that requires an eventual outlay of $F = \$100$. Suppose that the project's payoff may be either $H = \$120$ or $L = \$80$. The firm can create this option to invest in production by investing in R&D, and then it will decide whether or not to incur the additional cost of production when the state of the world materializes. In the case of the high payoff, the option pays the difference between the outlay in production and the drug value (i.e., $H - F = \$120 - \$100 = \$20$). If the low case materializes, the firm will shelve the drug.

For simplicity, assume that the risk-free rate is 0 and the DM is risk neutral.[9] When the probabilities of both the bad and the good outcomes are known to be 50% (no ambiguity is present), the variance of the probabilities is 0. Therefore, the value of the option is $C = 0.5 \times (\$120 - \$100) = \$10$. If the variance of the payoff (i.e., the risk) of production increases, such that the outcomes in the good and bad states are, respectively, \$130 or \$70 (i.e., a mean-preserving spread in outcomes), then the value of the option increases to $C = 0.5 \times (\$130 - \$100) = \$15$. Thus, an increase in risk is associated with a higher value of the option.[10]

To examine the impact of ambiguity, assume instead that the future payoffs remain the same, \$120 or \$80, but the probabilities of these future payoffs are uncertain and can be either $(0.4,0.6)$ or $(0.6,0.4)$. The DM, who does not have any information regarding the likelihood of these probability distributions, acts *as if* she assigns an equal weight to each probability distribution (the principle of insufficient reason). Thus, the expected probability of the favorable state is $E[\varphi(H)] = 0.5 \times 0.4 + 0.5 \times 0.6 = 0.5$ and its variance is $\mathrm{var}[\varphi(H)] = 0.5 \times (0.4 - 0.5)^2 + 0.5 \times (0.6 - 0.5)^2 = 0.01$. The same values apply to the unfavorable state. This implies that the degree of ambiguity (the expected variance of the probabilities) is $\mho^2 = 0.5 \times 0.01 + 0.5 \times 0.01 = 0.01$.

[8]For a similar reason, employees tend to exercise their options early when the expected ambiguity increases (Izhakian and Yermack (2017)), credit default swap (CDS) spreads decrease in ambiguity (Augustin and Izhakian (2020)), individuals pay less attention to stocks (Izhakian, Levi, Shalev, and Zur (2020)), and stock options value and trading activities decrease in ambiguity (Ben-Rephael et al. (2022b)).

[9]The EUUP framework allows for different combinations of risk attitudes and ambiguity attitudes. Typically, DMs are both risk averse and ambiguity averse (e.g., Abdellaoui et al. (2005), Du and Budescu (2005)). However, to focus on the effect of ambiguity, the current example is a simplification to a risk-neutral but ambiguity-averse DM. The extension to a risk-averse DM is fairly straightforward.

[10]This holds true also for a risk-averse DM and for continuous-type models (e.g., Black and Scholes (1973)).

In the EUUP framework, an ambiguity-averse DM forms perceived probabilities by certainty-equivalent probabilities and uses them to assess her expected utility. The perceived probability of the favorable payoff is $E[\varphi(H)] \times (1 + \frac{\Upsilon''}{\Upsilon'}\text{var}[\varphi(H)])$, where $\varphi(\cdot)$ is the probability mass function, Υ is the function that describes preferences for ambiguity, and $-\frac{\Upsilon''}{\Upsilon'}$ is the coefficient of absolute ambiguity aversion.[11] Assume first an ambiguity-neutral DM. This DM's preference for ambiguity is characterized by a linear function Υ, implying that the perceived probabilities are equal to the expected probabilities. Accordingly, the value of the option remains the same and is equal to $C = 0.5 \times (\$120 - \$100) = \$10$. Now assume instead an ambiguity-averse DM with a constant absolute ambiguity aversion $-\frac{\Upsilon''}{\Upsilon'} = \eta = 2$. Due to aversion to ambiguity, this DM does not form perceived probabilities through a linear compounding of probabilities, but aggregates probabilities in a nonlinear way as described above. As a result, the value of the option becomes $C = 0.5 \times (1 - 2 \times 0.01) \times (\$120 - \$100) = \$9.8$. For a DM with a higher aversion to ambiguity, the value of the option drops even further. For example, if the coefficient of absolute ambiguity aversion is $\eta = 4$, the value of the option is $C = 0.5 \times (1 - 4 \times 0.01) \times (\$120 - \$100) = \$9.6$. Thus, an increase in the aversion to ambiguity *decreases* the option value.

Assume now that the ambiguity of the drug production payoff increases. If future payoffs are distributed either $(0.3,0.7)$ or $(0.7,0.3)$ with equal likelihoods (a mean-preserving spread in probabilities), then the expected probability of the favorable (and the unfavorable) state remains unchanged: $E[\varphi(H)] = 0.5 \times 0.3 + 0.5 \times \; 0.7 = 0.5$, but the variance of its probabilities increases to $\text{var}[\varphi(H)] = 0.5 \times (0.3 - 0.5)^2 + 0.5 \times (0.7 - 0.5)^2 = 0.04$, implying a degree of ambiguity of $\mho^2[X] = 0.04$. Assuming a coefficient of ambiguity aversion $\eta = 2$, the value of the option then drops to $C = 0.5 \times (1 - 2 \times 0.04) \times (\$120 - \$100) = \$9.2$. That is, as ambiguity increases, the value of the option decreases.

D. Main Hypotheses

We now state our hypotheses based on the conceptual view illustrated in the example above.

Hypothesis 1. Innovation investment decreases in ambiguity.

The effect of aversion to ambiguity is similar. Higher ambiguity aversion reduces the perceived expected payoff of the investment. However, empirically, we cannot measure ambiguity aversion, so we focus on the effects of ambiguity itself on option values. In relative terms, ambiguity should naturally matter more for young and small firms and for firms that do not yet have a proven record of successful innovations, as there is less of a history of how various market conditions affect the set of probabilities such firms may face.

In contrast, even in the presence of ambiguity, an increase in risk increases the value of the option, because of the convex nature of the option payoff. This effect,

[11]This expression for the perceived probability of positive payoffs is derived formally in Izhakian (2020).

which is consistent with the standard options literature, implies that an increase in risk leads to higher investment in innovation.

Hypothesis 2. Innovation investment increases in risk.

Hypothesis 2 coincides with Schwartz (2004) and Kraft et al. (2018) and with earlier corporate investments literature (e.g., Hartman (1972), Abel (1983)). However, when the firm has the possibility of delaying an irreversible investment (see Section II.E), a competing prediction (Hypothesis 4a below) suggests an opposite effect of risk on innovation investment.

E. The Timing of Innovation Investments

A comprehensive analysis of innovation investments must also consider the *timing* of innovation. In real life, a firm can choose to invest in innovation (R&D or patents) immediately or to *delay* innovation investments and wait for more information.[12] Waiting for additional information about the prospect of the innovation has value, and the value of this option-to-wait is affected by both ambiguity and risk.

The literature on investment under uncertainty suggests that, when risk increases, immediate investment in an *irreversible* investment opportunity with positive net present value (NPV) in not necessarily optimal (e.g., McDonald and Siegel (1986), Dixit and Pindyck (1994)). The reason is that the conventional NPV investment rule ignores the opportunity cost of making a commitment now and giving up the option of waiting for new information. Instead, the optimal investment rule for an irreversible investment with stochastic cost F_t and stochastic present value V_t is that the investment should be undertaken only if its gross return $\frac{V_t}{F_t}$ is at least as large as a critical value C_t^* that exceeds 1, implying that $V_t \geq V_t^* > F_t$ for a critical value V_t^* (McDonald and Siegel (1986)).[13] The investment decision schedule $\{C_t^*\}_{t=1,2,\ldots}$ is chosen so as to maximize the time 0 expected present value of the payoff $V_t - F_t$. Since V_t and F_t are stochastic, the time when $\frac{V_t}{F_t} \geq C_t^*$ can be viewed as the optimal stopping time; that is, the optimal time to exercise the option (the optimal innovation time). The value of the option-to-wait is the difference between the present value of investing at all possible times in the future and the value of investing today, which are mutually exclusive alternatives. McDonald and Siegel (1986) show that both the value of the investment option V_t and the level of gross return $\frac{V_t}{F_t}$ at which investment should occur are increasing in the volatilities ("risk" or σ) of the investment cost and the investment value. This is consistent with the basic intuition that risk increases the option value, as well as with the notion that higher risk raises the value of the delay option, as developed further by Dixit and Pindyck (1994) and

[12]Analogous to a financial call option, this view corresponds to an American call option on a stock.

[13]F_t can be nonstochastic. However, in the context of innovation investments, it is reasonable to assume that R&D investment is flexible. That is, there is no ex ante fixed exercise price, which implies that F_t is stochastic. We thank an anonymous referee for pointing out this issue.

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the real options literature. The empirical implication of the latter notion is that as volatility increases, immediate investment will decline.[14]

When allowing for flexibility regarding the timing of innovation investments, ambiguity may also affect innovation through the delay channel. By waiting, clarifying news may arrive, or new conditions may be created that reduce ambiguity (Ben-Rephael, Cookson, and Izhakian (2022a), (2022b)); therefore, higher ambiguity increases the value of the option-to-wait. Note that the quality of information, as well as the fundamental nature of the investment, determine ambiguity. Although better information does not always imply lower ambiguity, better information may provide a clearer picture of an ambiguous investment (Ellsberg (1961), Epstein and Schneider (2007)).[15]

The delay effect implies that, all else equal, the value of the option-to-wait increases in both ambiguity and risk, leading to the following hypotheses.

Hypothesis 3a. The propensity to delay innovation investment increases in ambiguity. Therefore, immediate innovation investment decreases in ambiguity.

Hypothesis 4a. The propensity to delay innovation investment increases in risk. Therefore, immediate innovation investment decreases in risk.

This negative effect of risk on immediate innovation, due to the delay effect, contrasts with the positive effect of risk on innovation in the absence of an option to delay (Hypothesis 2). However, even when the firm has the option to delay innovation investment, risk may still have a positive effect on immediate innovation investment, for several reasons. First, firms undertake investments when they reach their optimal stopping boundary, where the investment present value increases in risk (McDonald and Siegel (1986)). Empirically, if we observe only the innovation investments undertaken when the firms are at their respective stopping boundaries, then, in the cross section of firms, we should find that the R&D investment increases in risk.[16]

Second, higher (systematic) risk increases the cost of capital, which, holding the drift of V fixed, increases the opportunity cost of delaying investment (McDonald and Siegel (1986), p. 717). The difference between the cost of capital

[14]In the same vein, Ingersoll and Ross (1992) show that, even if the project payoff and cost are both known with certainty, there is value in waiting when the future interest rate (cost of capital) is uncertain.

[15]DMs may have incentives to acquire costly information to reduce ambiguity. However, this new information may either reduce or increase ambiguity. For example, this new information may either rule out prior probability distributions as impossible or indicate that priors considered impossible are indeed possible. Ellsberg ((1961), p. 659) states that "Ambiguity may be high (and the confidence in any particular estimate of probabilities low) even where there is ample quantity of information, when there are questions of reliability and relevance of information, and particularly where there is conflicting opinion and evidence." Consistent with this intuition, Bachmann, Carstensen, Lautenbacher, and Schneider ((2020), Figure 3) show that managers' Knightian survey responses are due to unusual growth expectations, environment changes, and caution more often than missing information. Even for well-understood investments, information collection and analysis may reduce ambiguity, but cannot eliminate it. For example, Izhakian and Yermack (2017) show that managers of public firms often exercise their stock options early when they expect their firms to face high ambiguity.

[16]We thank an anonymous referee for pointing out this empirical implication.

(the required return on investing in the project immediately) and the drift of V (the expected percentage change in the project's value) represents the portion of the required return on the project that is forgone by delaying the investment (McDonald and Siegel (1986), Dixit and Pindyck (1994)).[17] In our setup, this difference represents the cash flows that the firm could earn by investing in innovation early instead of delaying innovation. Higher risk can thus lead to an increase in the ratio of forgone cash flows to project present value, representing an opportunity cost for delaying innovation investment, and leading to higher immediate innovation investment.

Third, another cost of delay is the risk of competitor firm entry (Dixit and Pindyck (1994)). This risk motivates the firm to invest quickly in order to preempt investments by existing or potential competitors (Tirole (1988), Chapter 8). Fourth, investing itself may reveal information about the cost of the project. When facing *technical uncertainty* (uncertainty about the cost that will ultimately be required to complete the project), the firm may find it optimal to invest early, since early investment can help resolve part of this uncertainty (Dixit and Pindyck (1994), pp. 47 and 346).[18] Similarly to risk, the last three arguments hold true also for ambiguity. Overall, both ambiguity and risk can therefore increase the cost of delay, which may offset their positive effect on the option-to-wait, thus increasing immediate innovation investment.

Hypothesis 3b. The propensity to delay innovation investment decreases in ambiguity. Therefore, immediate innovation investment increases in ambiguity.

Hypothesis 4b. The propensity to delay innovation investment decreases in risk. Therefore, immediate innovation investment increases in risk.

Hypothesis 3b competes with Hypothesis 3a which is in line with Hypothesis 1. Hypothesis 4b is in line with Hypothesis 2 and they both compete with Hypothesis 3a. Since different innovation investments (e.g., R&D and patenting) are of a different nature, ambiguity and risk may have different implications for R&D and patents.

III. Data

The primary data sources for our analysis are: the intraday trade and quote (TAQ) data for estimating the degrees of ambiguity and risk; the patent database of Kogan et al. (2017), extended to 2019, for historical information on patents; and Compustat for accounting data. We combine the original Kogan et al. (2017) data (May 2016 version) with the May 2020 update. In addition, we use analysts' earnings forecasts data from IBES to calculate analysts' forecasts dispersion; patent citation data from PatentsView; institutional ownership data from the Thompson

[17]Analogous to a financial call option on a stock, this difference corresponds to the dividend yield on the stock: a positive dividend yield is an opportunity cost of keeping the call option alive instead of exercising it.

[18]Miao and Wang (2011) present a model in which ambiguity is resolved once an investment is made, and, thus, in their model, ambiguity encourages investments.

Reuters 13F database; and the Bushee (1998) classification of institutional owners. All balance sheet and income statement variables are deflated using the GDP deflator from St Louis Fed (2009=100).

A. Sample Construction

Our sample starts in 1993, the first year of TAQ data. To construct the sample, we start with all firm-quarter observations with strictly positive sales and assets in the Compustat Fundamentals Quarterly files for fiscal years 1993–2016, excluding utilities (SIC codes 4900–4999), financials (SIC codes 6000–6999), public service, international affairs firms, and non-operating establishments (SIC codes 9000–9999).[19] We also restrict the sample to firms listed on the NYSE/AMEX or Nasdaq (EXCHCD = 1, 2, and 3). We organize the data by *calendar quarter–year*,[20] and augment them with the history of patenting activity for Compustat firms using the Kogan et al. (2017) patent data set, combined with PatentsView.[21]

Next, we attempt to identify firm reorganizations that are not accompanied by a change in the Compustat firm identifier (*gvkey*). Following Bloom, Schankerman, and Van Reenen (2013), whenever we observe extremely large changes (greater than 200% or lower than −67%) in annual sales, employment, or assets, we treat the firm as a new entity and assign it a new identifier (*new gvkey*), even if the Compustat *gvkey* remains the same.[22]

As in most other studies on patents, our measure of the patenting process is patent applications. However, patent applications are observed only conditional on the patent being eventually granted. Since our patent data (Kogan et al. (2017), extended) ends in 2019, we are missing patents applied for in recent years, which have not yet been granted. To address the truncation bias (Dass, Nanda, and Xiao (2017)), following Dong, Hirshleifer, and Teoh (2021), we drop the last three years of the patent data, ending the patent sample in 2016. Moreover, patent citations are also subject to truncation, because we only observe citations made by patents already granted. To further reduce the truncation bias, we correct for technology-year effects using the fixed-effects approach (Hall, Jaffe, and Trajtenberg (2001)); specifically, we scale the citation count for each patent by the average number of citations received by all patents in the same International Patent Classification (IPC) technology class and filed in the same year.

We report empirical findings for three different samples: firms with at least one quarter of positive R&D expenditures (*R&D Sample*); firms with at least one patent application (*Patent Sample*); and firms with at least one citation (*Citation Sample*),

[19] We use the historical SIC code from CRSP to identify industries; when the historical SIC code is missing in CRSP, we use the historical SIC code from Compustat. When both are missing, we use the SIC code of the largest business or operating segment from the Compustat Segment Files.

[20] For example, the first quarter of 2000 includes all firm-quarters with the fiscal quarter ending in February, March, or April 2000.

[21] The PatentsView data set starts in 1976. Since we combine the Kogan, Papanikolaou, Seru, and Stoffman (2017) data and PatentsView data to calculate our patent variables, we use the patent history for each firm starting in 1976.

[22] This approach is more general than including a full set of *gvkey* fixed effects, because it allows the fixed effect to change over time, when the firm undergoes major changes.

conditional on non-missing data for all variables of interest during the sample period (1993–2016). This approach is common in the innovation literature, given that many firms have neither any patents nor positive R&D. For all samples, we require firms to have available data for at least four quarters for all variables of interest.[23] In addition, for the *Patent Sample* and the *Citation Sample*, we require firms to have at least four years (16 quarters) of patent data before the first quarter in the sample (the pre-sample period). For firms that enter Compustat after 1993, we use the first four years of data as the pre-sample period, and we include the following years in the sample.[24]

As explained in Section III.D and in Section A of the Supplementary Material, we estimate ambiguity and risk using intraday stock return data. In order to mitigate microstructure effects, we use five-minute stock returns, and we also apply the Scholes and Williams (1977) correction for nonsynchronous trading. To further eliminate potential microstructure effects, in some analyses, we exclude penny stocks, very small firms, and very young firms. Penny stocks are stocks with a price of less than $5 at the end of the previous quarter. Very small firms are firms with a market capitalization of less than $10 million at the end of the previous quarter. Very young firms are firms with less than five years in Compustat.

There are 66,733 firm-quarters for 2,746 different firms in the *R&D Sample*, 63,949 firm-quarters for 2,118 different firms in the *Patent Sample*, and 62,653 firm-quarters for 2,055 different firms in the *Citation Sample.* The *R&D* and *Patent* samples do not overlap completely: only 49,820 firm-quarters are in both samples, while 16,913 firm-quarters are only in the *R&D Sample*, and 14,129 firm-quarters are only in the *Patent Sample.*[25]

B. Estimating Innovation

Our hypotheses apply to "investment projects." Naturally, ambiguity and risk matter more for the innovative activity of the firm, rather than for other activities such as routine maintenance. Therefore, our main measure of innovation is R&D. We also use patents and citations, which create options later in the innovation process. Citations proxy for the prominence of patents and can be viewed as a rough estimate of investment in high-quality patents. However, we expect the findings to be less precise for this proxy, since citations are an ex post measure.

Our measures of innovation intensity are as follows. RD_ASSETS_{t+1} is R&D expenses in quarter $t+1$ (Compustat variable *xrdq*, replaced with 0 when missing), scaled by total assets at the beginning of quarter $t+1$. It is possible that the firm adjusts its R&D with a lag. Thus, to reflect a potential delayed response of R&D to ambiguity and risk, we also analyze the R&D intensity 1 year ahead, $\text{RD\_ASSETS}_{t+1:t+4}$, defined as total R&D expenditures in the four quarters

[23]Specifically, we require nonmissing ASSETS, SALES, *Q*, *K_L*, CASH_FLOW, LEVERAGE, AMBIGUITY, RISK, ANALYST_DISPERSION, ILLIQUIDITY, DIVIDENDS, and SIC code.

[24]See the discussion of the Blundell et al. (1999) pre-sample mean scaling fixed effect estimator in Section IV.

[25]The fact that 22% (14,129 out of 63,949) of firm-quarters in the *Patent Sample* do not have positive R&D expenditures during the sample period is consistent with Koh and Reeb (2015), who find that a significant number of firms with missing R&D in Compustat actually file and receive patents.

$t+1:t+4$, scaled by total assets at the beginning of quarter $t+1$. For robustness, we use two alternative measures of investment in innovation: RD_CAPEX_ASSETS$_{t+1}$, the sum of R&D and CAPEX, scaled by total assets at the beginning of the quarter (Kumar and Li (2016)); and RD_ADJ_ASSETS$_{t+1}$, R&D scaled by total assets at the beginning of the quarter, where total assets are adjusted to include capitalized R&D (Chan, Josef, and Sougiannis (2001), Chambers, Jennings, and Thompson (2002), and Lev, Sarath, and Sougiannis (2005)).

We consider patents and citations up to three years (12 quarters) ahead. PATENTS$_{t+1}$ is the number of patents applied for during quarter $t+1$, conditional on being granted by 2019. To reduce the bias caused by the application-grant lag, following Dong et al. (2021), we end the sample in 2016, dropping all patents filed after Dec. 31, 2016. Finally, following many innovation studies, including recent contributions (e.g., Dong et al. (2021)), we use citations counts i.e., citations-weighted patents, Trajtenberg (1990), Hall et al. (2005)) as a proxy for the quality of the firm's patents. CITATIONS$_{t+1}$ is the number of citations received by the patents that the firm filed during quarter $t+1$, excluding self-cites, and corrected for citation truncation using the fixed-effects approach of Hall et al. (2001). Namely, the raw number of citations received by each patent, excluding self-cites, is scaled by the average number of citations received by all patents in the same IPC technology class filed in the same year.

C. Estimation Methodology

Using stock return data, we estimate the ambiguity and risk of a firm's equity as a proxy for its potential project ambiguity and risk. Since stock prices are forward-looking, they represent the market's best estimate for a firm's future prospects and reflect the risk and ambiguity of future payoffs (Bloom, Davis, Foster, Lucking, Ohlmacher, and Saporta-Eckstein (2017)). Moreover, R&D activities are typically of the same nature as the firm's existing projects.

Many studies examine the effect of risk, measured by stock return volatility, on investment. For example, Leahy and Whited (1996) and Bloom, Bond, and Reenen (2007) study the effect of (outcome) uncertainty, as measured by stock return variance, on firm investment in fixed capital assets. Bloom et al. (2017) provide survey evidence that validates the use of stock market volatility as a proxy for managers' forward-looking assessment of uncertainty. They find a strong and robust positive relation between the manager's subjective uncertainty over the future growth rate (measured by the log standard deviation in the plant manager's growth rate forecasts) and the log standard deviation of the daily stock returns of the plant's parent firm over the prior year. We extend this current practice in the investment literature (estimating risk as the variance or the volatility of stock returns) to ambiguity (the variance of return probabilities that we also estimate from stock return data).

Bloom et al.'s (2017) findings mitigate the concern that the ambiguity and risk faced by the managers who make investment decisions may be significantly different from those observed by investors as reflected in stock market returns. Other direct evidence about the relation between the ambiguity (and risk) the managers face and the ambiguity (and risk) investors face is provided by Izhakian and

Yermack (2017). They find that managers tend to expedite exercising their vested options when expected ambiguity, measured by stock returns, increases. Also, managers tend to delay exercising their vested options when expected risk, measured using stock returns, increases. These findings show that managers' perceptions of ambiguity and risk are in line with those of the investors, as reflected in stock returns.

D. Estimating Ambiguity and Risk

To measure ambiguity, we follow recent implementations of the EUUP framework (Izhakian and Yermack (2017), Augustin and Izhakian (2020), and Izhakian et al. (2022b)). For each firm, we measure ambiguity as the monthly volatility of its return probabilities.[26] To this end, we use five-minute intraday stock returns from the TAQ database, restricting the observations to common stocks (SHRCD = 10 and 11 in CRSP). Each day is thus considered a different manifestation of the distribution of stock returns. In addition, we use the book value of total debt and the market value of equity estimated at every five-minute interval to unlever the intraday returns from which we measure ambiguity. The precise methodology for computing ambiguity is detailed in Section A of the Supplementary Material. In our empirical analysis, we denote the quarterly mean of monthly firm ambiguity estimates ($\mho^2[r_i]$ in Section A of the Supplementary Material) by AMBIGUITY$_{i,t}$, where i denotes a firm and t a quarter.[27]

For consistency, we estimate risk using the same (unlevered) five-minute returns that we use to compute ambiguity. For each firm on each day, we estimate the variance of five-minute intraday returns, applying the Scholes and Williams (1977) correction for nonsynchronous trading. Each month, we estimate risk as the mean of the daily variance estimates. In our analysis, as with ambiguity, we use the quarterly mean of monthly firm risk estimates, which is denoted RISK$_{i,t}$.

E. Control Variables

We control for variables that are known in the literature to be related to innovation. Our firm-level controls include: log sales ($\ln(\text{SALES})$); Tobin's Q (Q); log of 1 plus the ratio of physical capital per employee ($\ln(K\_L)$); cash-flow (CASH_FLOW); leverage (LEVERAGE); log of 1 plus firm age ($\ln(\text{AGE})$); log of 1 plus R&D capital ($\ln(\text{RD\_CAPITAL})$); a dummy for Nasdaq listing (NASDAQ), and an indicator variable for missing R&D expenditures in Compustat (MISSING_RD). To account for other dimensions of uncertainty, besides

[26]Since stock market data are required to calculate risk and ambiguity, we cannot explore the differences between innovation strategies for private and publicly listed firms, as is done in Gao, Hsu, and Li (2018).

[27]Empirical studies sometimes use the dispersion of analysts' forecasts as a proxy for ambiguity (Anderson, Ghysels, and Juergens (2009)). Conceptually, analysts tend to disagree on outcomes rather than probabilities, and disagreement among individuals is not the same as ambiguity (Garlappi, Giammarino, and Lazrak (2017)). Furthermore, as opposed to the measure we use in our study, the dispersion of analysts' forecasts is outcome dependent and, therefore, risk dependent. However, we do control for the dispersion of analysts' forecasts in all our regressions, so the effect of ambiguity that we document is distinct from that of analyst forecast dispersion.

ambiguity and risk, we control for the dispersion of analysts' earnings forecasts (ANALYST_DISPERSION). In addition, for some subsample analyses, we employ a measure of the firm's knowledge capital (KNOWLEDGE_CAPITAL), proxied by the citations stock (the total stock of citations received by the patents filed by the firm).[28] All variables are detailed in Table IA.I in Section B of the Supplementary Material.

To eliminate the effect of outliers, we drop firm-quarters with AMBIGUITY, RISK, or CASH_FLOW below the 1st percentile and above the 99th percentile over the entire sample period. We also drop firm-quarters with RD_ASSETS, RD_CAPEX_ASSETS, RD_ADJ_ASSETS, K_L, and ANALYST_DISPERSION above the 99th sample percentile. Following Aghion et al. (2013), we winsorize Q by setting it equal to 0.10 for values below 0.10 and to 20 for values above 20.

F. Summary Statistics

Table IA.II in Section C of the Supplementary Material presents descriptive statistics for the *R&D Sample* (Panels A and B), and the *Patent Sample* (Panel C). In the *R&D (Patent) Sample*, the median firm has sales of $166.312 ($302.620) million per quarter. The median firm age, approximated by the number of quarters the firm is listed in Compustat, is 59 quarters, or 14.75 years (70 quarters, or 17.5 years) in the *R&D (Patent) Sample.* Overall, these differences suggest that R&D investment and patenting may take place at different stages in the firms' life cycle.[29]

In the *R&D Sample*, the median (mean) RD_ASSETS is 1.4% (2%) per quarter, and 5.1% (7.8%) per year.[30] We also note that 23.3% of the firm-quarters in the *R&D Sample* have missing R&D in the Compustat Fundamentals Quarterly file. Conditional on filing at least one patent across all sample quarters (*Patent Sample*, Panel C of Table IA.II in Section C of the Supplementary Material), the median (mean) firm files 3 (32.233) patents per year and receives 2.388 (31.750) citations for the patents filed in a given year.[31] Thus, the distribution of the number of patents and citations is heavily skewed, as previously documented in the literature. Untabulated analysis reveals that in the subsample of patent-intensive firms (firms in the top tercile according to the average number of patents filed during the

[28]Naturally, there are industry-specific patterns that we cannot control for. For example, in the pharmaceutical industry, drug novelty can be measured based upon the novelty of molecular structure (Krieger, Li, and Papanikolaou (2017)). Similar to most empirical finance studies, applying the theory to project-level investments is not feasible due to a lack of project-specific data. However, this concern is mitigated by the fact that, in our empirical analysis, the strongest findings are obtained for high-tech firms, whose equity often reflects a specific single project; in other words, our results are strongest in the subsample where firm-level ambiguity and risk most closely capture project-level ambiguity and risk.

[29]Part of the difference in firm age between the *R&D Sample* and the *Patent Sample* is due to the fact that, for firms that enter Compustat (broadly speaking, IPO firms) during the sample period, we use the first four years of data to construct pre-sample means of the dependent count variables (PATENTS and CITATIONS), effectively removing these years from the actual sample.

[30]For variables calculated over the four quarters $t+1:t+4$, we require the firm to be in the sample in all four quarters. For this reason, the means for the annual variables are not necessarily exactly four times larger than for the corresponding quarterly variables.

[31]As discussed in Section III.A, the citation count for each patent is scaled by the average number of citations received by all patents in the same technology field filed in the same year.

sample period), the median (mean) firm files 26 (77.209) patents and receives 26.119 (75.156) citations per year.

The median (mean) AMBIGUITY in the *R&D Sample* (Panel A of Table IA.II in Section C of the Supplementary Material) is 0.0143 (0.0207). This implies that the median expected standard deviation of probabilities is $\sqrt{0.0143} = 11.96\%$. The median RISK of 0.0009 per day corresponds to an annualized stock return volatility of approximately $\sqrt{250 \times 0.0009} = 47.43\%$ (or, equivalently, $\sqrt{20 \times 0.0009} = 13.42\%$ per month). The medians for AMBIGUITY and RISK are quite similar in the *R&D Sample* (Panel A) and the *Patent Sample* (Panel C).

We then split the sample into two subsamples: high-tech and non-high-tech industries. Following Brown, Fazzari, and Petersen (2009), we classify the following seven three-digit SIC code industries as high-tech industries: drugs (SIC 283), office and computing equipment (SIC 357), communications equipment (SIC 366), electronic components (SIC 367), scientific instruments (SIC 382), medical instruments (SIC 384), and software (SIC 737). The high-tech/non-high-tech industry classification splits the *R&D Sample* approximately in half: 34,122 firm-quarters (1,635 distinct firms) in the high-tech sample, and 32,273 firm-quarters (1,202 distinct firms) in the non-high-tech sample. Descriptive statistics for the high-tech and non-high-tech subsamples are presented in Panel B of Table IA.II in Section C of the Supplementary Material. As expected, the R&D intensity is larger in high-tech industries. High-tech firms are, in general, smaller, younger, have less leverage, less tangible capital, and more intangible capital than non-high-tech firms.[32]

High-tech firms also appear to have higher risk and lower ambiguity than non-high-tech firms. As discussed, non-high-tech firms typically do not have many internal growth opportunities, and the opportunities that exist are likely to be inorganic and not in the firm's core business and may be typified by ambiguous prospects (e.g., entering new markets). High-tech firms tend to have internal organic growth opportunities in their core business (e.g., expansion of existing activities) whose characteristics are similar to the firm's assets in place and, therefore, less ambiguous. These characterizations correspond to the documented regularities in the life-cycle models of firms, most recently studied in Hoberg and Maksimovic (2022). Discussing the four stages in a firm's life, from heavy investments in R&D to CAPEX and then to acquisitions, the article suggests (Hoberg and Maksimovic (2022), p. 4250): "The natural ordering [of investment sensitivities to q over the life cycle] indicates a progression from organic investment to inorganic investment to, finally, disinvestment and extension strategies."

Table IA.III in Section C of the Supplementary Material reports within-firm correlations for key variables for the *R&D Sample* (Panel A) and the *Patent Sample* (Panel B). Our main interest is the within-firm relation between ambiguity and innovation. Therefore, our regressions include firm fixed effects, and we report within-firm correlations (i.e., Pearson correlations for variables demeaned by

[32]The last two columns of Panel B report that the differences in means and medians between high-tech and non-high-tech firms are highly significant. Untabulated analysis shows that the patterns documented in Panel B of Table IA.II in Section C of the Supplementary Material for the *R&D Sample* also obtain in the *Patent Sample* and in the *Citation Sample.*

firm).[33] The correlations are very similar in the *R&D Sample* and the *Patent Sample*. The within-firm correlation between AMBIGUITY and RISK is 0.1% in the *R&D Sample* and −8.7% in the *Patent Sample*, supporting the idea that ambiguity and risk are different dimensions of uncertainty. This very low correlation also addresses the multi-collinearity concern and motivates the joint presence of ambiguity and risk in our regressions. Table IA.III in Section C of the Supplementary Material also shows that AMBIGUITY is positively correlated with $\ln(\text{SALES})$ and AGE, which is consistent with more complex operations in large firms. In the same vein, Cohen and Lou (2012) show that complicated firms are larger and more difficult to analyze than simple firms, even for similar information flows.

IV. Empirical Methodology

We employ two main empirical models to test the hypotheses presented in Section II. First, to analyze the effect of ambiguity on R&D decisions, we estimate the following OLS model:

$$\text{(1)}\quad \text{RD\_ASSETS}_{i,t+1} = \alpha + \beta_1 \text{AMBIGUITY}_{i,t} + \beta_2 \text{RISK}_{i,t} + \Gamma' X_{i,t} + \mu_i + \nu_t + \varepsilon_{i,t},$$

where i denotes the firm; t denotes the quarter; $X_{i,t}$ is a vector of firm- and time-varying control variables as described above; μ_i are the firm fixed effects; and ν_t are the quarter–year fixed effects. Quarter–year fixed effects absorb any time effects that are constant across all firms, including seasonality effects. Standard errors are clustered by firm.

For patents and citations, we use both Poisson and Negative Binomial regressions to estimate the following count model:

$$\text{(2)}\quad \mathbb{E}[\text{OUT}_{i,k,t+n} | X_{i,t}, \chi_i, \xi_k, \nu_t] = \exp\Big(\alpha + \beta_1 \text{AMBIGUITY}_{i,t} + \beta_2 \text{RISK}_{i,t} + \Gamma' X_{i,t} + \chi_i + \xi_k + \nu_t\Big),$$

where $\mathbb{E}[\cdot]$ stands for expected value; $\text{OUT}_{i,k,t+n}$ is either $\text{PATENTS}_{i,k,t+n}$ or $\text{CITATIONS}_{i,k,t+n}$ for firm i, in industry k, and quarter $t+n$ ($n = 1 \dots 12$); $X_{i,t}$ is a vector of firm- and time-varying control variables; χ_i denotes the Blundell, Griffith, and Van Reenen (1999) pre-sample firm fixed effects; ξ_k denotes the industry fixed effects;[34] and ν_t are the quarter–year fixed effects. We also estimate equation (2) for the total number of patents or citations for each of the next three years: $\text{OUT}_{i,k,t+1:t+4}$, $\text{OUT}_{i,k,t+5:t+8}$, and $\text{OUT}_{i,k,t+9:t+12}$. Standard errors are clustered by firm.

In the count models for PATENTS and CITATIONS, we follow the recent innovation literature (e.g., Aghion et al. (2013), Bloom et al. (2013)), and control

[33] The correlation coefficients for the raw data, without demeaning by firm, are similar to the within-firm correlation coefficients reported in Table IA.III in Section C of the Supplementary Material.

[34] Our sample includes both NYSE/AMEX and Nasdaq firms. We use three-digit SIC code fixed effects because, according to CRSP documentation, until Mar. 2000, the Nasdaq stock exchange reports the first three digits for Nasdaq firms, and CRSP adds a fourth digit of 0 (https://www.crsp.org/products/documentation/data-definitions-s).

for unobserved, time-invariant, firm-level heterogeneity using the pre-sample mean scaling fixed effect estimator of Blundell et al. (1999). This approach exploits the history of patent data for each firm and uses the log of pre-sample averages of the count dependent variable (PREPATENTS and PRECITATIONS) as a proxy for unobserved heterogeneity.[35]

V. Empirical Findings: R&D Investment

Using the methodology described in Section IV, we now turn to test our hypotheses.

A. Preliminary Findings

To test Hypothesis 1 concerning the effect of ambiguity on innovation investment, we start by plotting the mean R&D investment one quarter ahead (RD_ASSETS_{t+1} in Graph A of Figure 1) and one year ahead ($\text{RD\_ASSETS}_{t+1:t+4}$ in Graph C of Figure 1) for portfolios formed by dependent sorts each quarter into risk quintiles and then ambiguity quintiles, balanced by size (market capitalization).[36] This analysis provides a visual representation of the effect of ambiguity on R&D, controlling for both firm size and risk. Graphs A and C of Figure 1 show that, in line with Hypothesis 1, R&D investment decreases in ambiguity, both one quarter and one year ahead. Moreover, the vertical bars, representing 95% confidence intervals, indicate that the effect of ambiguity is statistically significant within each risk quintile.

Graphs B and D of Figure 1 repeat the analysis for portfolios formed by dependent sorts each quarter into ambiguity quintiles and then into risk quintiles, balanced by size (market capitalization).[37] This analysis provides a visual representation of the effect of risk on R&D, controlling for both firm size and ambiguity. Graphs B and D show that, in line with Hypothesis 2, R&D investment increases in risk both one quarter ahead (Graph B) and one year ahead (Graph D), and the effect is statistically significant in each of the top three quintiles of ambiguity.[38]

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[35]In addition, the count models for PATENTS and CITATIONS also include an indicator variable for whether the firm had any patents (citations) in the pre-sample period. Recall that we require firms to have at least four years of pre-sample data (16 quarters) in order to calculate pre-sample averages of the dependent variables. For firms that enter Compustat after 1993 (the first year of patent data in our regression sample), we use the first 16 quarters of data to estimate pre-sample averages and include the following quarters in the sample. Bloom et al. (2013) use a similar approach, requiring four years of pre-sample data in their 1981–2001 data set.

[36]To control for the effect of size on R&D investment, we perform these dependent sorts on risk and then ambiguity within market capitalization quintiles. Specifically, each quarter, the sample is sorted into market capitalization quintiles, then each market capitalization quintile is sorted into risk quintiles, and finally, each market capitalization-risk portfolio is sorted into ambiguity quintiles.

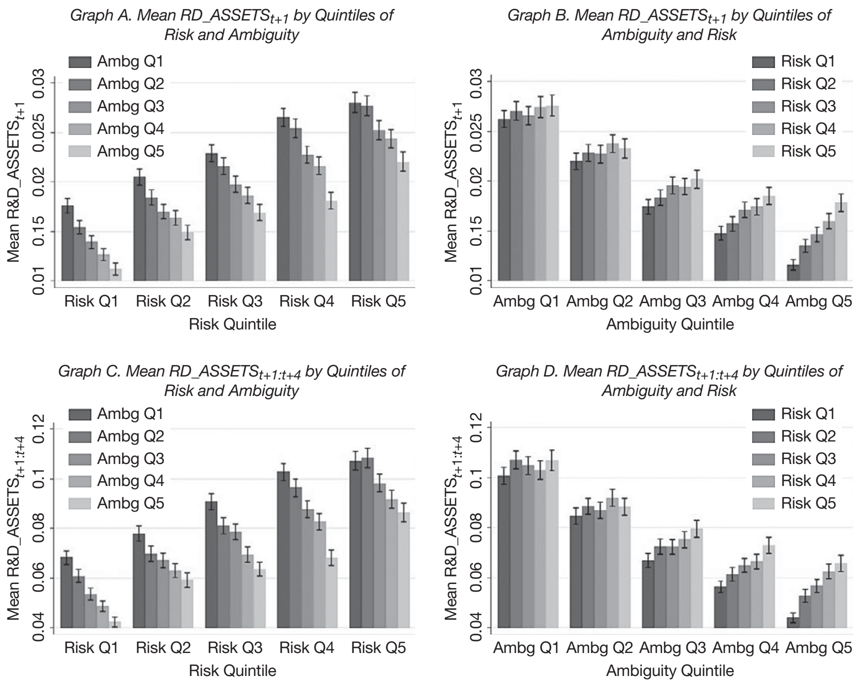
[37]Specifically, each quarter, the data are first sorted into market capitalization quintiles; then each market capitalization quintile is sorted into ambiguity quintiles; and, finally, each market capitalization-ambiguity portfolio is sorted into risk quintiles.

[38]Figure IA.1 in Section C of the Supplementary Material plots histograms similar to those in Figure 1, excluding penny stocks, very small firms, and very young firms. The patterns in Figure IA.1 in Section C of the Supplementary Material are very similar to those in Figure 1.

FIGURE 1

Mean R&D Investment for Dependent Sorts on Risk and Ambiguity

Figure 1 plots mean R&D investments by portfolios formed each quarter within dependent sorts of risk then ambiguity. The sample period is 1993–2016. The sample consists of all firms with at least four quarters of data for all variables of interest and at least one quarter of positive R&D expenditures in Compustat during the sample period (*R&D Sample*). In Graphs A and C, risk quintiles are formed each quarter within market capitalization quintiles to generate size-balanced portfolios; ambiguity quintiles are then formed within each of these market capitalization-risk portfolios. In Graphs B and D, ambiguity quintiles are formed each quarter within market capitalization quintiles; risk quintiles are then formed within each of these market capitalization-ambiguity portfolios. Graphs A and B plot the mean RD_ASSETS one quarter ahead (RD_ASSETS_{t+1}) and Graphs C and D plot the mean RD_ASSETS one year ahead ($RD\_ASSETS_{t+1:t+4}$). Vertical bars indicate 95% confidence intervals.



B. Main Findings

The results of the OLS regressions of R&D investment as a function of AMBIGUITY, RISK, and other explanatory variables are reported in Table 1. The regressions are estimated for all firms in the *R&D Sample.* Panel A shows that ambiguity has a significant negative effect, while risk has a significant positive effect on R&D, both one quarter ahead (column 1) and one year ahead (column 2). These findings support Hypotheses 1 and 2, respectively.

The effect of ambiguity is driven mainly by high-tech firms. The coefficient estimates on AMBIGUITY in that subsample are larger and significant at the 1% level for both one-quarter and one-year-ahead R&D (columns 3 and 4 of Table 1). For non-high-tech firms, the effect of ambiguity is insignificant for one-quarter-ahead R&D (column 5), but it is significant for one-year-ahead R&D (column 6). The finding that the effects of risk and ambiguity are driven by high-tech firms supports the real options view. High-tech firms are more likely to have options for product development in the first place. As risk increases and ambiguity decreases,

TABLE 1

Determinants of R&D Investment

Table 1 presents OLS coefficient estimates for R&D investment. The dependent variable is RD_ASSETS_{t+1}. The sample period is 1993–2016. In Panel A, the sample consists of all firms with at least four quarters of data for all variables of interest and at least one quarter of positive R&D expenditures in Compustat during the sample period (*R&D Sample*). In Panel B, the sample consists of all firms in the *R&D Sample*, excluding penny stocks, very small firms, and very young firms. All regressions in Panel B include the following control variables: $\ln(SALES)_t$, Q_t, $\ln(K\_L)_t$, $CASH\_FLOW_t$, $LEVERAGE_t$, $\ln(AGE)_{t+1}$, $\ln(RD\_CAPITAL)_t$, $NASDAQ_t$, and $MISSING\_RD_{t+1}$. In columns 1, 3, and 5, $MISSING\_RD_{t+1}$ is an indicator variable equal to 1 if the firm has missing R&D expenditures in Compustat in quarter $t+1$. In columns 2, 4, and 6, $MISSING\_RD_{t+1}$ is the number of quarters with missing R&D in Compustat in the period $t+1:t+4$. All regressions include firm (*new gvkey*) fixed effects and quarter–year fixed effects. Sample construction is detailed in Section III.A. Variable definitions are in Table IA.I in Section B of the Supplementary Material. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	All Firms		High-Tech		Non-High-Tech	
	1 Quarter	1 Year	1 Quarter	1 Year	1 Quarter	1 Year
	$t+1$	$t+1:t+4$	$t+1$	$t+1:t+4$	$t+1$	$t+1:t+4$
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. R&D Sample						
$AMBIGUITY_t$	−0.019***	−0.092***	−0.054***	−0.191***	0.001	−0.044***
	(0.005)	(0.019)	(0.011)	(0.042)	(0.005)	(0.015)
$RISK_t$	0.196***	0.914***	0.317***	1.263***	0.049*	0.327***
	(0.032)	(0.126)	(0.049)	(0.200)	(0.027)	(0.103)
$ANALYST\_DISPERSION_t$	0.036***	0.083***	0.033***	0.123***	0.029***	0.018
	(0.008)	(0.027)	(0.012)	(0.038)	(0.010)	(0.036)
$\ln(SALES)_t$	−0.002***	−0.011***	−0.003***	−0.016***	−0.001**	−0.006***
	(0.000)	(0.002)	(0.001)	(0.003)	(0.001)	(0.002)
Q_t	0.001***	0.006***	0.001***	0.006***	0.001***	0.005***
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)
$\ln(K\_L)_t$	−0.002***	−0.008***	−0.002***	−0.010***	−0.001	−0.003*
	(0.000)	(0.002)	(0.001)	(0.003)	(0.000)	(0.002)
$CASH\_FLOW_t$	−0.024***	−0.024*	−0.021***	−0.006	−0.020***	−0.036
	(0.003)	(0.014)	(0.005)	(0.017)	(0.006)	(0.025)
$LEVERAGE_t$	−0.005***	−0.023***	−0.006***	−0.031***	−0.003**	−0.013**
	(0.001)	(0.005)	(0.002)	(0.007)	(0.001)	(0.005)
$\ln(AGE)_t$	0.002***	0.002	0.002	0.001	0.002**	0.004
	(0.001)	(0.003)	(0.001)	(0.005)	(0.001)	(0.002)
$\ln(RD\_CAPITAL)_t$	0.001***	0.004***	0.003***	0.011***	0.000	0.003***
	(0.000)	(0.001)	(0.001)	(0.002)	(0.000)	(0.001)
$NASDAQ_t$	−0.002**	−0.009***	−0.001	−0.009*	−0.002**	−0.006**
	(0.001)	(0.003)	(0.001)	(0.005)	(0.001)	(0.002)
$MISSING\_RD_{t+1}$	−0.016***	−0.006***	−0.024***	−0.009***	−0.014***	−0.004***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)
CONSTANT	0.027***	0.144***	0.029***	0.186***	0.020***	0.068***
	(0.005)	(0.014)	(0.008)	(0.020)	(0.004)	(0.015)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter–year FE	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	66,733	60,586	34,122	30,276	32,273	29,593
No. of firms	2,746	2,472	1,635	1,434	1,202	1,092
Adj. R^2	0.819	0.875	0.773	0.821	0.808	0.893
Panel B. R&D Sample – Excluding Penny Stocks, Very Small Firms, and Very Young Firms						
B1. Excluding Stocks with Price <5 and Market Cap <10 m						
$AMBIGUITY_t$	−0.013**	−0.075***	−0.040***	−0.150***	0.002	−0.038***
	(0.005)	(0.018)	(0.011)	(0.041)	(0.005)	(0.014)
$RISK_t$	0.181***	0.917***	0.283***	1.169***	0.053*	0.411***
	(0.037)	(0.148)	(0.059)	(0.244)	(0.031)	(0.121)
$ANALYST\_DISPERSION_t$	0.055***	0.139***	0.061***	0.190***	0.033***	0.054**
	(0.009)	(0.032)	(0.019)	(0.063)	(0.010)	(0.025)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter–year FE	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	60,392	55,210	29,886	26,714	30,219	27,889
No. of firms	2,471	2,244	1,440	1,272	1,111	1,023
Adj. R^2	0.819	0.881	0.773	0.827	0.799	0.897

(continued on next page)

TABLE 1 (continued)

Determinants of R&D Investment

	All Firms		High-Tech		Non-High-Tech	
	1 Quarter	1 Year	1 Quarter	1 Year	1 Quarter	1 Year
	$t+1$	$t+1:t+4$	$t+1$	$t+1:t+4$	$t+1$	$t+1:t+4$
	(1)	(2)	(3)	(4)	(5)	(6)
B2. Excluding Stocks with Price <5, Market Cap <10 m, and Age <5 Years						
AMBIGUITY_t	−0.012**	−0.072***	−0.038***	−0.142***	0.001	−0.044***
	(0.005)	(0.018)	(0.011)	(0.043)	(0.005)	(0.016)
RISK_t	0.172***	0.809***	0.336***	1.190***	0.021	0.301**
	(0.045)	(0.176)	(0.075)	(0.311)	(0.042)	(0.141)
$\text{ANALYST\_DISPERSION}_t$	0.050***	0.120***	0.066***	0.191***	0.025***	0.041
	(0.010)	(0.033)	(0.022)	(0.068)	(0.009)	(0.025)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter–year FE	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	51,218	47,159	24,413	22,065	26,604	24,652
No. of firms	1,917	1,769	1,074	973	911	848
Adj. R^2	0.816	0.885	0.772	0.835	0.785	0.889

the value of every dollar invested in R&D increases, motivating high-tech firms to invest more.

The coefficient estimates of all control variables in Table 1 have the expected signs. R&D is higher in small firms, in firms with high-growth opportunities (Q), low tangibility (ln(K_L)), low cash flows, and low leverage. The effect of age is positive and significant, but only for non-high-tech firms one quarter ahead, and it becomes insignificant when we exclude very young firms (Panel B). Finally, since ANALYST_DISPERSION is an outcome- and risk-dependent measure, it is expected to have an effect similar to that of RISK. Indeed, ANALYST_DISPERSION has a robust positive and significant effect on R&D.

In terms of economic significance, the coefficient estimates in Table 1 imply that a one-standard-deviation increase in AMBIGUITY across all firms (0.02) decreases the R&D intensity one quarter ahead (RD_ASSETS_{t+1}) by $0.019 \times 0.02 = 0.00038$, which represents approximately 1.7% of the empirical standard deviation of the dependent variable (0.022). In the high-tech subsample, the economic effect is larger: a one-standard-deviation increase in AMBIGUITY decreases the R&D intensity one quarter ahead by 4.2% of the empirical standard deviation in that subsample. Similarly, a one-standard-deviation increase in RISK increases the R&D intensity by approximately 5.3% of the empirical standard deviation for all firms (column 1), and by approximately 8.3% of the empirical standard deviation for high-tech firms (column 3). Thus, the economic effect of ambiguity is of the same order of magnitude as that of risk. Furthermore, ambiguity matters more to high-tech, high-growth firms, which are naturally more sensitive to changes in the prospects of their investments.

In the high-tech subsample, the economic effect of AMBIGUITY is similar to that of LEVERAGE and CASH_FLOW. A one-standard-deviation increase in LEVERAGE (CASH_FLOW) decreases R&D intensity one quarter ahead by 4.5% (4.3%) of the empirical standard deviation for high-tech firms. The economic effect of LEVERAGE and CASH_FLOW on R&D intensity one quarter ahead is similar in the high-tech and non-high-tech subsamples, whereas the economic effect of AMBIGUITY is stronger in the high-tech subsample.[39]

Panel B of Table 1 shows that the effect of ambiguity is robust to eliminating penny stocks, very small firms, and very young firms. This suggests that the findings are unlikely to be driven by microstructure effects.

C. Subsamples by Firm Characteristics

Table 2 reports the findings for splits of the high-tech firms into terciles defined according to the *average* age, leverage, size, and knowledge capital during the sample period.[40] Panel A of Table 2 shows that ambiguity is significant for middle-aged and old firms, but insignificant for young firms, which implies that the effect of ambiguity on R&D is not driven by firms that exit the sample early in their life cycle. The effect of risk on R&D is stronger for young and middle-aged firms, and the negative interaction between risk and old firms is also significant (Panel A1).

When we split the sample by leverage (Panel A of Table 2 and Panel A of Table IA.V in Section C of the Supplementary Material), the interaction between ambiguity and the high-leverage indicator is positive and significant in most specifications. The finding that the effect of ambiguity is stronger for low-leverage firms, together with the finding that R&D, in general, is higher in low-leverage firms, is consistent with the association in the literature between low leverage and high growth.

Panel B of Table 2 shows that the effect of ambiguity is concentrated in the subsamples of small and medium-sized firms.[41] The effect of risk is significant in all size terciles when the sample is split by average firm size, but it is significant only for small firms when the sample is split by firm size at the end of the quarter (Panel B of Table IA.V in Section C of the Supplementary Material).

Innovation is often a sequential process. Initial investments in fundamental R&D are subject to higher and potentially different types of uncertainty than later-stage, follow-on innovations. Moreover, the notion of *unknown unknowns* (Knight (1921)) encompasses not only the uncertainty about probabilities, but also the uncertainty about the set of possible outcomes (payoffs) that may be generated by the R&D efforts. In our view, this applies especially to fundamental R&D, as well as to firms that do not (yet) have a well-established culture of innovation. In

[39]Among the other explanatory variables, ln(SALES) has the strongest effect on R&D. A one-standard-deviation increase in ln(SALES) decreases the R&D intensity one quarter ahead by 16.6% of the empirical standard deviation for all firms, by 21.7% of the empirical standard deviation for high-tech firms, and by 10.3% of the empirical standard deviation for non-high-tech firms. This size effect is common in all corporate finance regressions. As expected, Tobin's Q is also an important determinant of R&D. A one-standard-deviation increase in Q increases the R&D intensity one quarter ahead by 8% of the empirical standard deviation for all firms, by 8.9% of the empirical standard deviation for high-tech firms, and by 7.5% of the empirical standard deviation for non-high-tech firms.

[40]For robustness, Table IA.V in Section C of the Supplementary Material reports the findings for splits of the high-tech firms into terciles defined according to the age, leverage, size, and knowledge capital in the *previous quarter.* The subsample findings based on characteristics measured at the end of the previous quarter are similar to those based on average firm characteristics during the sample period.

[41]When we split the sample by firm size at the end of the previous quarter, the effect of ambiguity on R&D one quarter ahead (RD_ASSETS_{t+1}) is significant only in the bottom size tercile (Panel B1 of Table IA.V in Section C of the Supplementary Material).

TABLE 2

Subsample Analysis of R&D Investment in High-Tech Firms

Table 2 presents OLS coefficient estimates for R&D investment. The dependent variable is RD_ASSETS$_{t+1}$ in Panels A1, B1, and C (column 1), and RD_ASSETS$_{t+1:t+4}$ in Panels A2, B2, and C (column 2). The sample period is 1993–2016. The sample is the same as in Panel B2 of Table 1 (the *R&D Sample*, and excluding penny stocks, very small firms and very young firms, and further restricted to high-tech firms). In Panel A (B), the sample is split into terciles according to the *average* age and leverage (sales and knowledge capital) over the sample period, with terciles defined over all firms in the sample. HIGH is an indicator variable equal to 1 if the split variable is in the top tercile (i.e., for old firms, high-leverage firms, large firms, and high-knowledge-capital firms, respectively), and 0 otherwise. In Panel C, LARGE_SIZE$_t$ is an indicator variable equal to 1 for firm-quarters in the top tercile of size (sales), measured at the end of quarter t (SALES$_t$), and 0 otherwise; SM&MED_SIZE$_t$ is an indicator variable equal to 1 for firm-quarters in the medium and bottom tercile of SALES$_t$, and 0 otherwise; HIGH_KNOWLEDGE$_t$ is an indicator variable equal to 1 for firm-quarters in the top tercile of knowledge capital, measured at the end of quarter t (KNOWLEDGE_CAPITAL$_t$), and 0 otherwise; LOW&MED_KNOWLEDGE$_t$ is an indicator variable equal to 1 for firm-quarters in the medium and bottom tercile of KNOWLEDGE_CAPITAL$_t$, and 0 otherwise. All regressions include the following control variables: ln(SALES)$_t$, Q$_t$, ln(K_L)$_t$, CASH_FLOW$_t$, LEVERAGE$_t$, ln(AGE)$_{t+1}$, ln(RD_CAPITAL)$_t$, NASDAQ$_t$ and MISSING_RD$_{t+1}$. In Panels A1, B1, and C (column 1), MISSING_RD$_{t+1}$ is an indicator variable equal to 1 if the firm has missing R&D expenditures in Compustat in quarter $t+1$, and 0 otherwise. In Panels A2, B2, and C (column 2), MISSING_RD$_{t+1}$ is the number of quarters with missing R&D in Compustat in the period $t+1:t+4$. All regressions include firm (*new gvkey*) fixed effects and quarter–year fixed effects. Sample construction is detailed in Section III.A. Variable definitions are in Table IA.I in Section B of the Supplementary Material. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A. Subsamples by Average Age and Average Leverage

	AGE					LEVERAGE				
	Young	Middle	Old	All Firms		Low	Medium	High	All Firms	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A1. RD_ASSETS *1 Quarter Ahead (Quarter t+1)*										
AMBIGUITY$_t$	−0.038	−0.054***	−0.028*	−0.042***	−0.041**	−0.052**	−0.039**	−0.039***	−0.057***	−0.054***
	(0.026)	(0.018)	(0.016)	(0.016)	(0.016)	(0.026)	(0.019)	(0.013)	(0.017)	(0.017)
RISK$_t$	0.282**	0.553***	0.186*	0.337***	0.478***	0.259**	0.381***	0.299***	0.338***	0.406***
	(0.123)	(0.132)	(0.107)	(0.075)	(0.092)	(0.113)	(0.118)	(0.104)	(0.075)	(0.088)
AMBIGUITY$_t$ × HIGH				0.008	0.006				0.039**	0.032*
				(0.020)	(0.019)				(0.019)	(0.019)
RISK$_t$ × HIGH					−0.283**					−0.309**
					(0.137)					(0.122)
ANALYST_ DISPERSION$_t$	0.045*	0.080***	0.068	0.066***	0.066***	0.067**	0.124***	0.031	0.066***	0.066***
	(0.027)	(0.029)	(0.045)	(0.022)	(0.022)	(0.029)	(0.031)	(0.030)	(0.022)	(0.022)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quarter–year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	4,179	9,579	10,655	24,413	24,413	6,022	9,793	8,598	24,413	24,413
No. of firms	358	358	358	1,074	1,074	358	358	358	1,074	1,074
Adj. R^2	0.818	0.758	0.746	0.772	0.772	0.790	0.740	0.790	0.772	0.772
A2. RD_ASSETS *Four Quarters Ahead (Quarters t+1:t+4)*										
RISK$_t$	1.130**	1.738***	0.734*	1.191***	1.571***	1.326**	0.913**	1.313***	1.191***	1.423***
	(0.496)	(0.606)	(0.397)	(0.310)	(0.438)	(0.630)	(0.443)	(0.422)	(0.311)	(0.366)
RISK$_t$ × HIGH					−0.756					−0.937*
					(0.558)					(0.510)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quarter–year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	3,911	8,581	9,573	22,065	22,065	5,728	8,588	7,749	22,065	22,065
No. of firms	325	324	324	973	973	325	324	324	973	973
Adj. R^2	0.883	0.802	0.833	0.835	0.836	0.828	0.810	0.857	0.836	0.836

Panel B. Subsamples by Average Size and Average Knowledge Capital

	SIZE					KNOWLEDGE_CAPITAL				
	Small	Medium	Large	All Firms		Low	Medium	High	All Firms	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
B1. RD_ASSETS *1 Quarter Ahead (Quarter t+1)*										
AMBIGUITY$_t$	−0.067***	−0.096***	−0.014	−0.071***	−0.073***	−0.055***	−0.059**	−0.029*	−0.061***	−0.065***
	(0.025)	(0.024)	(0.014)	(0.020)	(0.020)	(0.020)	(0.023)	(0.016)	(0.017)	(0.017)
RISK$_t$	0.443***	0.484***	0.572***	0.343***	0.317***	0.247**	0.359**	0.742***	0.340***	0.292***
	(0.099)	(0.109)	(0.212)	(0.075)	(0.077)	(0.098)	(0.139)	(0.200)	(0.074)	(0.076)

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TABLE 2 (continued)

Subsample Analysis of R&D Investment in High-Tech Firms

Panel B. Subsamples by Average Size and Average Knowledge Capital (continued)

	SIZE					KNOWLEDGE_CAPITAL				
	Small	Medium	Large	All Firms		Low	Medium	High	All Firms	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
AMBIGUITY$_t$ × HIGH				0.042*	0.051**				0.040*	0.052***
				(0.023)	(0.023)				(0.021)	(0.020)
RISK$_t$ × HIGH					0.338					0.441**
					(0.210)					(0.205)
ANALYST_DISPERSION$_t$	0.079**	0.078**	0.047	0.066***	0.065***	0.066**	0.065	0.060**	0.066***	0.065***
	(0.034)	(0.031)	(0.030)	(0.022)	(0.022)	(0.028)	(0.047)	(0.024)	(0.022)	(0.022)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quarter–year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	4,387	7,956	12,070	24,413	24,413	5,872	7,662	10,879	24,413	24,413
No. of firms	358	358	358	1,074	1,074	358	358	358	1,074	1,074
Adj. R^2	0.812	0.741	0.736	0.772	0.772	0.748	0.762	0.792	0.772	0.772
B2. RD_ASSETS Four Quarters Ahead (Quarters $t+1:t+4$)										
AMBIGUITY$_t$	−0.249**	−0.341***	−0.055	−0.247***	−0.247***	−0.153*	−0.247***	−0.101	−0.207***	−0.222***
	(0.106)	(0.091)	(0.051)	(0.080)	(0.080)	(0.080)	(0.083)	(0.062)	(0.065)	(0.064)
RISK$_t$	1.573***	1.697***	1.083	1.208***	1.200***	0.937**	1.549**	2.582***	1.198***	1.036***
	(0.397)	(0.491)	(0.778)	(0.312)	(0.321)	(0.382)	(0.633)	(0.758)	(0.310)	(0.316)
AMBIGUITY$_t$ × HIGH				0.130	0.133				0.111	0.153*
				(0.090)	(0.089)				(0.081)	(0.080)
RISK$_t$ × HIGH					0.117					1.452*
					(0.740)					(0.768)
ANALYST_DISPERSION$_t$	0.220	0.171	0.173**	0.190***	0.189***	0.125	0.159	0.205**	0.191***	0.187***
	(0.137)	(0.137)	(0.084)	(0.069)	(0.069)	(0.125)	(0.139)	(0.097)	(0.068)	(0.068)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quarter–year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	4,065	7,348	10,652	22,065	22,065	5,443	6,920	9,702	22,065	22,065
No. of firms	325	324	324	973	973	325	324	324	973	973
Adj. R^2	0.850	0.801	0.833	0.835	0.835	0.845	0.836	0.833	0.835	0.836

Panel C. Interactions of AMBIGUITY *and* RISK *with* SIZE *and* KNOWLEDGE_CAPITAL

	RD_ASSETS$_{t+1}$	RD_ASSETS$_{t+1:t+4}$
	(1)	(2)
(1) AMBIGUITY$_t$ × SM&MED_SIZE$_t$ × LOW&MED_KNOWLEDGE$_t$	−0.076***	−0.261***
	(0.017)	(0.065)
(2) AMBIGUITY$_t$ × LARGE_SALES$_t$ × LOW&MED_KNOWLEDGE$_t$	−0.040*	−0.147*
	(0.022)	(0.076)
(3) AMBIGUITY$_t$ × SM&MED_SIZE$_t$ × HIGH_KNOWLEDGE$_t$	0.037	0.169
	(0.028)	(0.106)
(4) AMBIGUITY$_t$ × LARGE_SALES$_t$ × HIGH_KNOWLEDGE$_t$	−0.020	−0.123**
	(0.016)	(0.059)
(5) RISK$_t$ × SM&MED_SIZE$_t$ × LOW&MED_KNOWLEDGE$_t$	0.326***	1.185***
	(0.073)	(0.307)
(6) RISK$_t$ × LARGE_SALES$_t$ × LOW&MED_KNOWLEDGE$_t$	0.511	0.605
	(0.423)	(1.214)
(7) RISK$_t$ × SM&MED_SIZE$_t$ × HIGH_KNOWLEDGE$_t$	1.433***	4.544***
	(0.428)	(1.102)
(8) RISK$_t$ × LARGE_SALES$_t$ × HIGH_KNOWLEDGE$_t$	0.374	−0.333
	(0.666)	(2.453)
LARGE_SALES$_t$ × HIGH_KNOWLEDGE$_t$	0.003*	0.010
	(0.002)	(0.006)
LARGE_SALES$_t$	−0.001	0.000
	(0.001)	(0.004)
HIGH_KNOWLEDGE$_t$	−0.004***	−0.015***
	(0.001)	(0.005)

(continued on next page)

TABLE 2 (continued)

Subsample Analysis of R&D Investment in High-Tech Firms

Panel C. Interactions of AMBIGUITY *and* RISK *with* SIZE *and* KNOWLEDGE_CAPITAL *(continued)*

	RD_ASSETS_{t+1}	$RD\_ASSETS_{t+1:t+4}$
	(1)	(2)
$ANALYST\_DISPERSION_t$	0.064***	0.190***
	(0.022)	(0.068)
Controls	Yes	Yes
Firm FE	Yes	Yes
Quarter–year FE	Yes	Yes
No. of obs.	24,410	22,065
No. of firms	1,074	973
Adj. R^2	0.773	0.836
P-values for Tests of Differences in the Coefficients		
(1)–(2)	0.163	0.223
(1)–(3)	0.000	0.000
(1)–(4)	0.007	0.082
(7)–(5)	0.010	0.003
(7)–(6)	0.130	0.016
(7)–(8)	0.260	0.052

these cases, the uncertainty about the set of possible payoffs (or events or support of the payoff distribution) is likely to amplify the effect of the uncertainty of probabilities.[42]

To test this conjecture, we split the sample by firm knowledge capital (KNOWLEDGE_CAPITAL), proxied by the firm stock of patent citations; that is, the total number of citations received by the patents filed by the firm (Table IA.I in Section B of the Supplementary Material provides the full definition). A firm that has filed few patents or whose patents are not highly cited, does not have a proven record of innovation, and therefore is likely to face higher uncertainty about the set of possible innovation payoffs. On the other hand, a firm that has a successful track record of innovation is more likely to conduct follow-on R&D geared toward improvements (innovations) that build upon the knowledge capital already developed by the firm (past patents). For these reasons, we expect the effect of ambiguity to be stronger in low-knowledge-capital firms. At the same time, the positive effect of risk on R&D due to the convex nature of the real option payoff (Hypothesis 2) may be weaker when the set of payoffs is subject to considerable uncertainty. Therefore, we expect the effect of risk on R&D to be stronger for high-knowledge-capital firms, which are more likely to conduct follow-on R&D with known potential payoffs.

The findings reported in Panels B and C of Table 2 support these conjectures. First, in Panel B, the coefficient estimate on AMBIGUITY is significant for low and medium KNOWLEDGE_CAPITAL firms, and it is insignificant, or only marginally significant, for high KNOWLEDGE_CAPITAL firms. Consistent with the subsample findings (columns 6–8), the interaction of AMBIGUITY and an indicator for firms in the top tercile of KNOWLEDGE_CAPITAL is positive and significant, indicating that the negative effect of ambiguity is mitigated in high-knowledge-capital firms. In contrast, the positive and significant interaction between RISK and the high KNOWLEDGE_CAPITAL dummy indicates that the effect of risk is stronger for high-knowledge-capital firms, as expected.

[42]We thank an anonymous referee for pointing out this idea.

In Panel B of Table 2, the effect of ambiguity in subsamples of knowledge capital mirrors the effect of ambiguity in the size subsamples. To distinguish the knowledge-capital effect from the size effect, we re-estimate the R&D regressions, including interactions of AMBIGUITY and RISK with SIZE and KNOWLEDGE_CAPITAL indicators (with all variables measured at the end of the previous quarter). The findings reported in Panel C of Table 2 show that the effect of ambiguity is strongest in magnitude and statistical significance for small- and medium-sized firms with low or medium knowledge capital (row 1). In addition, the effect of ambiguity is significant (but only at the 10% level) for large firms that do not have high knowledge capital (row 2), but is insignificant for small- and medium-sized high knowledge capital firms (row 3). These findings suggest that, for ambiguity, the knowledge-capital effect dominates the size effect.[43]

Risk is significant only for small- and medium-sized firms (rows 5 and 7 of Table 2). The tests of differences in coefficients reported at the bottom of Panel C further show that, conditional on small or medium firm size, the effect of risk is stronger for high-knowledge-capital firms (row 7) than for low- and medium-knowledge-capital firms (row 5). On the other hand, conditional on high knowledge capital, the effect of risk is significant only for small- and medium-sized firms.

Overall, as expected, ambiguity and risk play different roles at different stages in the innovation process, especially for small firms. The effect of ambiguity is strongest in small-, low-knowledge-capital firms (row 1 in Panel C of Table 2), whereas the effect of risk is strongest in small, high-knowledge-capital firms (row 7 in Panel C).

D. Further Robustness Tests

Table IA.VI in Section C of the Supplementary Material reports several robustness tests for the OLS analysis in Table 1. Panel A of Table IA.VI in Section C of the Supplementary Material shows that our findings are robust to controlling for dedicated, transient and quasi-indexer institutional ownership (Bushee (1998), Aghion et al. (2013)).[44] The institutional ownership variables themselves are insignificant.[45] Panels B and C of Table IA.VI in Section C of the Supplementary Material further show that the R&D results are also robust to controlling for the Amihud (2002) ILLIQUIDITY measure and for DIVIDENDS.

[43]The negative and significant coefficient estimate on $\text{AMBIGUITY}_t \times \text{LARGE\_SIZE}_t \times \text{HIGH\_KNOWLEDGE}_t$ on R&D one year ahead (column 2, row 4 in Panel C of Table 2) is puzzling in light of the findings documented in Panels A and B. But this coefficient estimate is smaller and significantly different (at the 10% level) than the coefficient estimate on $\text{AMBIGUITY}_t \times \text{SM\&MED\_SIZE}_t \times \text{LOW\&MED\_KNOWLEDGE}_t$ (row 1).

[44]Bushee (1998) finds that total institutional ownership decreases the probability that firms cut R&D in order to reverse an earnings decline, whereas ownership by transient institutional investors has the opposite effect, which suggests that transient institutional ownership encourages myopic investment behavior. On the other hand, Aghion et al. (2013) find that institutional ownership increases innovation, as measured by citation-weighted patents.

[45]An untabulated analysis shows that our findings are also robust to controlling for total institutional ownership instead of controlling for dedicated, transient and quasi-indexer institutional ownership separately, and that total institutional ownership is itself insignificant.

A significant share of capital outlays of R&D-active firms may reflect investments in innovative capacity, such as the construction of a research facility or purchasing patents (Kumar and Li (2016)). Hence, R&D expenditures might actually understate the actual investment in innovation, part of which might be included in capital expenditures.[46] Panel D of Table IA.VI in Section C of the Supplementary Material shows that our findings are robust to measuring innovation investment by the sum of R&D and CAPEX scaled by assets (RD_CAPEX_ASSETS).[47]

High-tech firms have both smaller size (measured by either assets or sales) and larger stocks of capitalized R&D expenditures than firms in traditional industries (Table IA.II in Section C of the Supplementary Material). We follow Chan et al. (2001), Lev et al. (2005), and Chambers et al. (2002), and adjust the book value of total assets to include capitalized R&D (RD_CAPITAL). Panel E of Table IA.VI in Section C of the Supplementary Material shows that our findings are robust to measuring innovation investment by R&D scaled by adjusted assets (RD_ADJ_ASSETS), where adjusted assets include the book value on the balance sheet plus capitalized R&D.[48]

A final concern regarding the findings reported in Table 1 is potential uncontrolled endogeneity; in particular, dynamic reverse causation can arise if AMBIGUITY and RISK are driven or at least significantly influenced by the firm's (past and/or current) R&D investments and if, in addition, these investments are persistent over time.[49] Following Wintoki, Linck, and Netter (2012) and Hoechle, Schmidt, Walter, and Yermack (2012), we control for dynamic endogeneity, unobservable heterogeneity, and simultaneity by estimating a dynamic panel system GMM estimator as proposed by Arellano and Bover (1995) and Blundell and Bond (1998). Specifically, the GMM model is estimated for the high-tech firms in the *R&D Sample.* Overall, the findings reported in Table IA.VII in Section C of the Supplementary Material are in line with the OLS findings reported in Table 1, implying that our results are not driven by dynamic endogeneity.

[46]At the same time, not all investments in innovative capacity would be included in capital expenditures. For example, the purchase of inventories would be reflected as an increase in total assets, but is not included in capital expenditures (Kumar and Li (2016)).

[47]We also regress CAPEX, both one quarter ahead and one year ahead, scaled by assets at the beginning of quarter $t+1$ (CAPEX_ASSETS$_{t+1}$ and CAPEX_ASSETS$_{t+1:t+4}$) on AMBIGUITY, RISK, and the same control variables as for the R&D regressions. We do not find a significant effect of AMBIGUITY or RISK on CAPEX for either high-tech or non-high-tech firms.

[48]These regressions include the same control variables as in Panel A of Table 1, with one difference: as we adjust total assets to include capitalized R&D in the denominator for the dependent variable, we apply the same adjustment to the denominator of Tobin's Q. This adjusted Q is similar to the total Q of Peters and Taylor (2017). An untabulated analysis shows that our findings are also robust to excluding observations with AMBIGUITY and RISK estimated during recession quarters (2001:Q2–2001:Q4 and 2008:Q1–2009:Q2).

[49]We thank an anonymous referee for pointing out this issue. Table IA.IV in Section C of the Supplementary Material reports autocorrelation coefficients for R&D (Panel A) and for patents and citations (Panel B). For RD_ASSETS, the first-order autocorrelation is 0.854 in the pooled sample, and the mean (median) first-order *within-firm* autocorrelation is $0.311(0.439)$. The Woolridge test for first-order serial correlation for panel data rejects the null hypothesis of no autocorrelation, both for all firms and in the high-tech subsample. In Panel B, the mean and median *within-firm* autocorrelation for both patents and citations is always less than 0.2 in absolute value, except for patent- (and citation-) intensive firms, which by construction are likely to have high patent and citation counts for most periods.

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In summary, our findings for R&D investment are consistent with the standard real options view, supporting Hypotheses 1 and 2. Namely, our findings show that investments in R&D decrease with firm ambiguity and increase with firm risk. This is particularly true for high-tech firms.

E. Invest Versus Delay

Allowing for *timing* flexibility regarding the firm's innovation investments leads to a tradeoff between immediate investments and delayed investments (Section II.E). Our hypotheses suggest that firms compare the benefits and costs of investing now versus the benefits and costs of delaying innovation investments and waiting for new information. To empirically explore this tradeoff, we examine the propensity to "Invest" (in innovation) versus to "Delay" (innovation investments) using multinomial logit regressions, and focusing on significant R&D increases at various points in time, as a proxy for innovation investment.

In the spirit of Eberhart et al. (2004), we define a significant R&D increase as an increase of more than 1% in RD_ASSETS. Accordingly, the indicator variable RD_INCREASE is equal to 1 if the firm experiences an increase in RD_ASSETS greater than 1% relative to the same quarter of the previous year, and 0 otherwise.[50] We proxy for the decision to exercise an option-to-invest in innovation, based on information available at the end of quarter t, by a significant R&D increase in quarter $t+1$ (considered to be an *immediate* increase in R&D); and we proxy for the decision to delay innovation investment by the decision to increase R&D in quarter $t+2$, without previously increasing R&D in quarter $t+1$.

Depending on whether we observe a significant R&D increase in quarter $t+1$ and/or in quarter $t+2$, we define three possible outcomes (categories): the first category (0, *No Increase*) consists of observations where the firm does not experience a significant R&D increase in quarter $t+1$, nor in quarter $t+2$; the second category (1, *Delayed Increase*) corresponds to the case where the firm experiences a significant R&D increase in quarter $t+2$, but not in quarter $t+1$; and the third category (2, *Immediate Increase*) corresponds to the case where the firm experiences a significant R&D increase in quarter $t+1$ (whether or not it also experiences a significant R&D increase in quarter $t+2$).[51] We use this categorical variable, RD_INCREASE_CATEGORY, as the dependent variable in the multinomial logit regression model.

Table 3 reports the multinomial logit results for the *R&D Sample* (Panel A) and for the *R&D Sample* restricted to high-tech firms (Panel B). The findings are very

[50]Eberhart, Maxwell, and Siddique (2004) use a 5% cutoff and multiple criteria to identify R&D increases. The reason we use a less stringent definition, compared with Eberhart et al. (2004), is related to the low incidence of significant R&D increases in our sample. Using the 1% threshold as discussed in the text, 5.43% of the observations with nonmissing RD_INCREASE in the *R&D Sample* are classified as firm-quarters with significant R&D increases. Using a 2% threshold, only 2.08% of the observations are classified as significant R&D increases.

[51]For robustness, in untabulated analysis, we define an *Immediate Increase* to be a significant R&D increase in quarter $t+1$ that is not followed by another significant R&D increase in quarter $t+2$; and we add a fourth outcome category, corresponding to the case where the firm experiences a significant R&D increase in both quarters, $t+1$ and $t+2$. The findings are similar to those reported in Table 3.

TABLE 3

Multinomial Logit Analysis of R&D Increases

Table 3 presents coefficient estimates from multinomial logit regressions of significant R&D increases. A significant R&D increase is defined as a quarterly increase in RD_ASSETS greater than 1% relative to the same quarter of the previous year. The dependent variable in the multinomial logit model, RD_INCREASE_CATEGORY$_{t+1}$ takes one of three possible values: 0 (*No Increase*) if the firm does not experience a significant R&D increase in quarter $t+1$, nor in quarter $t+2$; 1 (*Delayed Increase*) if the firm experiences a significant R&D increase in quarter $t+2$, but not in quarter $t+1$; 2 (*Immediate Increase*) if the firm experiences a significant R&D increase in quarter $t+1$ (whether or not it also experiences a significant R&D increase in quarter $t+2$). The sample period is 1993–2016. In Panel A, the sample consists of all firms with at least four quarters of data for all variables of interest and at least one quarter of positive R&D expenditures in Compustat during the sample period (*R&D Sample*). In Panel B, the sample consists of all high-tech firms (three-digit SIC codes 283, 357, 366, 367, 382, 384, or 737) with at least four quarters of data for all variables of interest and at least one quarter of positive R&D expenditures in Compustat during the sample period (*R&D Sample*, restricted to high-tech firms). All regressions include the following control variables: ln(SALES)$_t$, Q$_t$, ln(K_L)$_t$, CASH_FLOW$_t$, LEVERAGE$_t$, ln(AGE)$_{t+1}$, ln(RD_CAPITAL)$_t$, NASDAQ$_t$, and MISSING_RD$_{t+1}$, as well as quarter fixed effects and year fixed effects. Sample construction is detailed in Section III.A. Variable definitions are in Table IA.I in Section B of the Supplementary Material. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	All Firms			Firms with at Least One R&D Increase			Firms with at Least One R&D Increase Excluding Penny Stocks and Very Small Firms		
Base Category:	No Increase		Delayed Increase	No Increase		Delayed Increase	No Increase		Delayed Increase
Outcome Category:	Immediate Increase	Delayed Increase	Immediate Increase	Immediate Increase	Delayed Increase	Immediate Increase	Immediate Increase	Delayed Increase	Immediate Increase
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A. R&D Sample – All Firms									
AMBIGUITY$_t$	−18.248***	−12.174***	−6.074*	−12.444***	−6.596**	−5.848*	−6.490**	−3.636	−2.854
	(3.482)	(3.712)	(3.281)	(3.218)	(3.276)	(3.148)	(3.276)	(3.377)	(3.459)
RISK$_t$	34.116***	13.774	20.342**	30.546***	15.732	14.814	35.864***	24.342*	11.522
	(9.166)	(11.153)	(10.164)	(9.767)	(10.878)	(9.757)	(13.685)	(14.394)	(14.947)
ANALYST_ DISPERSION$_t$	5.330**	4.950*	0.380	6.126**	5.870**	0.256	15.378***	12.856**	2.522
	(2.249)	(2.685)	(2.940)	(2.414)	(2.727)	(2.841)	(5.106)	(5.305)	(5.637)
Controls		Yes			Yes			Yes	
Quarter FE		Yes			Yes			Yes	
Year FE		Yes			Yes			Yes	
No. of obs.		38,107			16,426			13,491	
No. of firms		2,200			887			727	
Log likelihood		−9,529.77			−8,268.51			−6,350.07	
Panel B. R&D Sample – High-Tech Firms									
AMBIGUITY$_t$	−21.472***	−12.570***	−8.902**	−15.751***	−7.078*	−8.672**	−9.190**	−3.554	−5.636
	(4.247)	(4.675)	(4.036)	(3.929)	(4.072)	(3.867)	(4.030)	(4.211)	(4.296)
RISK$_t$	43.412***	13.685	29.728**	45.680***	21.516*	24.163**	49.222***	29.460*	19.762
	(10.795)	(13.019)	(12.023)	(11.183)	(12.415)	(11.279)	(16.429)	(15.855)	(16.688)
ANALYST_ DISPERSION$_t$	4.578*	2.200	2.377	6.734**	4.536	2.198	12.186**	12.035**	0.151
	(2.664)	(3.235)	(3.473)	(2.819)	(3.166)	(3.285)	(5.752)	(5.850)	(6.260)
Controls		Yes			Yes			Yes	
Quarter FE		Yes			Yes			Yes	
Year FE		Yes			Yes			Yes	
No. of obs.		18,828			11,376			9,129	
No. of firms		1,258			656			531	
Log likelihood		−7,069.47			−6,278.90			−4,785.92	

similar in the two panels and, as expected, are usually stronger (in terms of both statistical significance and magnitude) for the subsample of high-tech firms (Panel B).

In Table 3, the omitted outcome is *No Increase* in all columns except for columns 3, 6, and 9, where the omitted outcome is *Delayed Increase.* The negative and significant coefficient estimates of AMBIGUITY$_t$ in columns 1 and 2 indicate that ambiguity has a negative effect on the propensity to invest in R&D either in quarter $t+1$ or in quarter $t+2$, both relative to not increasing R&D at all. Moreover, the negative and significant coefficient estimate on AMBIGUITY in column 3 indicates that an increase in ambiguity is associated with an increase in the propensity to increase R&D in quarter $t+2$, instead of increasing R&D in quarter $t+1$. Overall, these findings support Hypothesis 3a: Ambiguity increases the propensity to postpone R&D investment. The positive and significant coefficient estimate on RISK indicates that an increase in risk is associated with an increase in the propensity to increase R&D in quarter $t+1$; i.e., a decrease in the propensity to delay R&D investments, supporting Hypothesis 4b.

For robustness, columns 4–6 of Table 3 repeat our analysis for all firms with at least one R&D increase, and columns 7–9 further exclude penny stocks and very small firms. The findings are very similar. However, when imposing these filters, the coefficient estimates on AMBIGUITY are smaller, and the effect of ambiguity on delaying investment to quarter $t+2$ becomes insignificant. A possible reason is that the investment decisions of large firms are less sensitive to ambiguity, as they typically have more diversified R&D activities.

The validity of the multinomial logit model relies on the assumption of independence from irrelevant alternatives (IIA).[52] For robustness, we also estimate an ordered logit model (with random effects), where the dependent variable takes the same values as in the multinomial logit model: 0 for *No Increase*; 1 for *Delayed Increase*; and 2 for *Immediate Increase.* The coefficient estimates for the ordered logit model, reported in Table IA.VIII in Section C of the Supplementary Material, show a negative and significant effect of ambiguity and a positive and significant effect of risk, in line with the multinomial logit model. Supporting Hypothesis 3a, these findings indicate that an increase in ambiguity is associated with an increase in the probability of delaying R&D investment relative to immediate investment, and, more generally, with an increase in the probability of delaying the R&D investment decision and potentially not investing at all. That is, an increase in the value of the option-to-wait.[53]

VI. Empirical Findings: Patents and Citations

We now turn to examine the effect of ambiguity and risk on patents and citations.

A. Preliminary Findings

Similar to the analysis of R&D investments, presented in Section V.A, we start by examining the patenting activity for portfolios of firms formed each quarter by

[52]The IIA assumption requires that the ratio of the probabilities for two alternative categories does not depend on what other alternatives are available. In our setting, this would imply that the ratio of the probabilities of delaying investment to quarter $t+2$ versus not investing at all in quarters $t+1$ and $t+2$ does not depend on whether the firm is able to invest in quarter $t+1$.

[53]For robustness, in untabulated analysis, we define a significant R&D increase as an increase of more than 2% in R&D investment (instead of 1%). The findings for the multinomial logit model are qualitatively similar to those reported in Table 3 using the 1% threshold, but weaker in terms of statistical significance. The findings for the ordered logit model are essentially the same as those reported in Table IA.VIII in Section C of the Supplementary Material regarding the coefficient estimates on AMBIGUITY and their statistical significance.

dependent sorts on risk and then ambiguity (Graphs A and of C Figure 2) and by dependent sorts on ambiguity then risk (Graphs B and D of Figure 2), both balanced by size (market capitalization).[54] Overall, Figure 2 suggests a negative effect of ambiguity on patents, in line with Hypotheses 1 and 3a (similar to the R&D findings), and likewise a negative effect of risk on patents, in line with Hypothesis 4a (whereas the R&D findings for risk lend support to Hypotheses 2 and 4b).[55]

B. Main Findings

Table 4 reports the findings of Poisson and Negative Binomial regressions for PATENTS and CITATIONS for all firms in the *Patent Sample* (Panel A) and the *Citation Sample* (Panel B), excluding penny stocks, very small firms and very young firms. All regressions include 3-digit SIC code fixed effects, Blundell et al. (1999) pre-sample firm fixed effects and quarter–year fixed effects.[56]

Overall, Table 4 shows a negative effect of both AMBIGUITY and RISK on patents and citations. The negative effect of RISK on patents and citations is significant both in the Poisson regressions and the Negative Binomial regressions, and AMBIGUITY is always negative and significant in the Poisson regressions for patents.[57]

The negative effect of ambiguity on patenting activity is in line with the R&D results, supporting Hypothesis 1. Risk is significant and negative throughout, suggesting that firms may delay, and hence decrease, patenting in the face of increased risk (McDonald and Siegel (1986), Dixit and Pindyck (1994), and Bloom (2007), (2014)), thus supporting Hypothesis 4a. Whereas the positive effect of risk on R&D is in line with the standard real options view (Hypothesis 2), and also with the interpretation that the cost of delaying R&D investment offsets the benefits of waiting for new information (Hypothesis 4b). The different effects of risk on R&D and on patents are consistent with differential costs of delay. Patent filing is much easier to delay than delaying R&D outlays; therefore, the delay motive (Dixit and Pindyck (1994)) may apply more to patents than to R&D. Another reason for the different effects of risk on R&D and on patents may be related to the fact that R&D investment is measured in dollars; hence, it is a better proxy for the "premium" paid for creating a real option-to-invest. The number of patents, which is our empirical proxy for investment in patents, is a rougher proxy for the firm's innovation

[54]We find very similar patterns for the mean number of patents and for the mean number of citations, but, for brevity, in Figure 2, we plot only the mean number of patents, one quarter ahead (Graphs A and B) and one year ahead (Graphs C and D).

[55]Figure IA.2 in Section C of the Supplementary Material plots the same histograms as in Figure 2, after excluding penny stocks, very small firms, and very young firms. The plots are very similar to those in Figure 2.

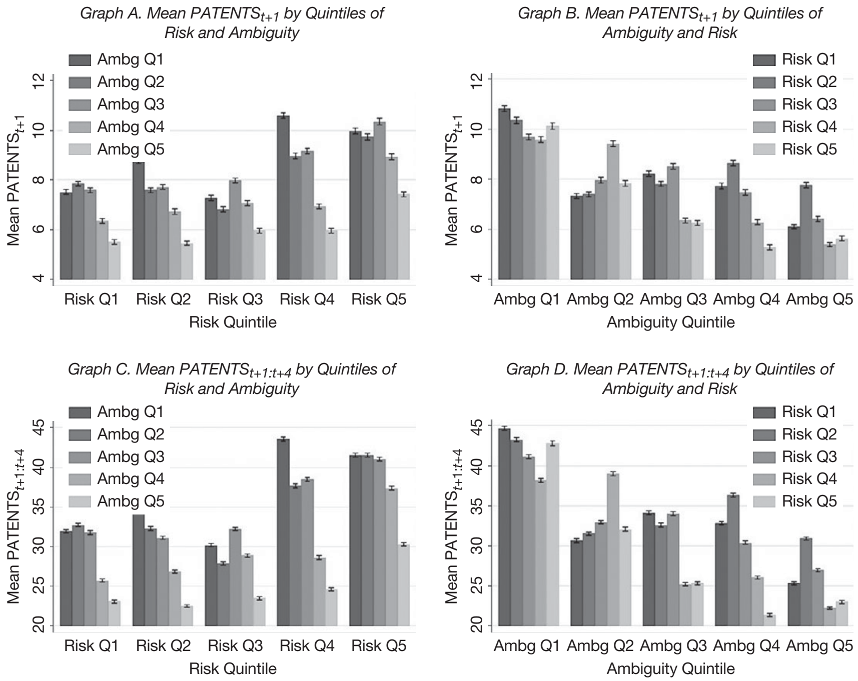
[56]We estimate the regressions separately for each quarter $t+1,\ldots,t+12$, but for brevity report findings only for quarter $t+1$, as well for the combined quarters $t+1:t+4$ (year 1), $t+5:t+8$ (year 2), and $t+9:t+12$ (year three). Importantly, the findings for each of years one–three are not driven by individual quarters within that year.

[57]The advantage of the Poisson model is that it only requires "that the conditional mean be correctly specified—the data need not be Poisson distributed" (Cameron and Triverdi (2015), p. 234). On the other hand, the Negative Binomial model is more general than the Poisson model in that it allows for overdispersion, while the Poisson model assumes equidispersion (the variance–mean equality).

FIGURE 2

Mean Patent Counts for Dependent Sorts on Risk and Ambiguity

Figure 2 plots mean patent counts by portfolios formed each quarter within dependent sorts of risk then ambiguity. The sample period is 1993–2016. The sample consists of all firms with at least four quarters of data for all variables of interest, four years in the pre-sample period, and at least one patent application filed during the sample period (*Patent Sample*). In Graphs A and C, risk quintiles are formed each quarter within market capitalization quintiles to generate size-balanced portfolios; ambiguity quintiles are then formed within each of these market capitalization (risk portfolios). In Graphs B and D, ambiguity quintiles are formed each quarter within market capitalization quintiles; risk quintiles are then formed within each of these market capitalization (ambiguity portfolios). Graphs A and B plot the mean number of patents one quarter ahead (PATENTS$_{t+1}$), and Graphs C and D plot the mean number of patents one year ahead (PATENTS$_{t+1:t+4}$). Vertical bars indicate 95% confidence intervals, where the confidence intervals are calculated assuming the Poisson distribution.



investment, because patents can also be viewed as an intermediate output of the innovation process.

The bottom part of each panel in Table 4 reports marginal effects, calculated as differences between the predicted number of counts (patents or citations) at the 90th and the 10th percentiles of AMBIGUITY and RISK, keeping all the other explanatory variables at their sample means. Column 4 in Panels A and B reveals that, in the Poisson model, the marginal effect of increasing AMBIGUITY (RISK) from the 10th to the 90th percentile is to decrease the predicted number of patents three years ahead by 1.160 (1.298) and the predicted number of citations received by the patents filed three years ahead by 1.102 (1.666). These marginal effects are statistically significant at the 1% level, and they are also economically important, given that the median (mean) firm in the *Patent Sample* files 3 (32.233) patent applications during a sample year and receives 2.388 (31.750) citations for these patents, as reported in Panel C of Table IA.II in Section C of the Supplementary Material.

Similar to the R&D regressions, in the patent regressions, we also control for ANALYST_DISPERSION (which is insignificant in Table 4), and for institutional

TABLE 4

Determinants of Patenting Activity

Table 4 presents the coefficient estimates of count models for patenting activity. The dependent variable is PATENTS in Panel A, and CITATIONS in Panel B. The sample period is 1993–2016. In Panel A (B), the sample consists of all firms with at least four quarters of data for all variables of interest, four years in the pre-sample period, and *at least one patent application* (*at least one cited patent*) filed during the sample period (i.e., the *Patent Sample* (*Citation Sample*)), excluding penny stocks, very small firms and very young firms. Marginal effects are calculated as differences in predicted counts at high (the 90th percentile of the estimation sample) and low (the 10th percentile of the estimation sample) AMBIGUITY$_t$ and RISK$_t$, while keeping all other variables at their sample means. All regressions include the following control variables: INSTOWN_DED$_t$, INSTOWN_TRA$_t$, INSTOWN_QIX$_t$, ln(SALES)$_t$, Q$_t$, ln(K_L)$_t$, CASH_FLOW$_t$, LEVERAGE$_t$, ln(AGE)$_{t+1}$, ln(RD_CAPITAL)$_t$, NASDAQ$_t$, as well as three-digit SIC code fixed effects, Blundell et al. (1999) pre-sample firm fixed effects and quarter–year fixed effects. Sample construction is detailed in Section III.A. Variable definitions are in Table IA.I in Section B of the Supplementary Material. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	Poisson				Negative Binomial			
	1 Quarter	Year 1	Year 2	Year 3	1 Quarter	Year 1	Year 2	Year 3
	$t+1$	$t+1:t+4$	$t+5:t+8$	$t+9:t+12$	$t+1$	$t+1:t+4$	$t+5:t+8$	$t+9:t+12$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Patents								
Coefficients								
AMBIGUITY$_t$	−1.868*	−2.192*	−2.880**	−3.983***	−0.066	0.599	−0.764	−1.494
	(1.120)	(1.156)	(1.233)	(1.379)	(1.177)	(1.150)	(1.154)	(1.240)
RISK$_t$	−34.969***	−40.358***	−52.206***	−54.260***	−16.003**	−13.617**	−21.350***	−15.332**
	(13.015)	(13.221)	(14.274)	(14.885)	(6.613)	(5.712)	(5.999)	(7.746)
ANALYST_DISPERSION$_t$	3.614	3.852	4.784	2.518	0.492	0.220	−1.567	−3.733
	(4.174)	(4.253)	(4.196)	(3.858)	(2.424)	(2.389)	(2.544)	(2.660)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	55,538	52,534	47,892	43,572	55,538	52,534	47,892	43,572
No. of firms	1,815	1,815	1,755	1,670	1,815	1,815	1,755	1,670
Pseudo R^2					0.190	0.157	0.155	0.151
Marginal Effects								
(1) Low ambiguity	1.574	6.669	6.864	7.012	1.418	5.958	6.244	6.290
(2) High ambiguity	1.437	6.001	5.995	5.852	1.413	6.132	6.024	5.877
Marginal effect (2)–(1)	−0.136*	−0.668*	−0.869**	−1.160***	−0.005	0.174	−0.220	−0.413
	(0.081)	(0.349)	(0.368)	(0.398)	(0.081)	(0.335)	(0.332)	(0.342)
(3) Low risk	1.597	6.786	7.036	7.091	1.449	6.149	6.357	6.268
(4) High risk	1.416	5.888	5.820	5.793	1.372	5.861	5.882	5.920
Marginal effect (4)–(3)	−0.180***	−0.898***	−1.216***	−1.298***	−0.077**	−0.288**	−0.475***	−0.348**
	(0.069)	(0.301)	(0.339)	(0.362)	(0.032)	(0.121)	(0.135)	(0.176)
Panel B. Citations								
Coefficients								
AMBIGUITY$_t$	−1.650	−1.790	−1.932	−3.265***	0.795	1.148	0.026	−1.612
	(1.131)	(1.126)	(1.215)	(1.253)	(1.382)	(1.289)	(1.361)	(1.356)
RISK$_t$	−1.540	−21.994	−57.552***	−60.951***	−7.492	−8.654	−17.841**	−13.537*
	(31.088)	(22.994)	(13.740)	(14.064)	(8.076)	(6.932)	(7.234)	(8.027)
ANALYST_DISPERSION$_t$	1.025	1.870	1.941	−0.648	1.771	0.700	−1.103	−4.917
	(3.535)	(3.808)	(3.801)	(3.876)	(3.040)	(2.910)	(2.966)	(3.023)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	54,459	51,544	47,037	42,846	54,459	51,544	47,037	42,846
No. of firms	1,762	1,762	1,704	1,624	1,762	1,762	1,704	1,624
Pseudo R^2					0.141	0.126	0.125	0.122
Marginal Effects								
(1) Low ambiguity	1.893	7.830	7.791	7.963	1.559	6.412	6.630	6.764
(2) High ambiguity	1.747	7.181	7.111	6.861	1.620	6.779	6.638	6.285
Marginal effect (2)–(1)	−0.146	−0.649	−0.680	−1.102***	0.062	0.366	0.008	−0.479
	(0.099)	(0.403)	(0.424)	(0.420)	(0.108)	(0.414)	(0.426)	(0.401)
(3) Low risk	1.838	7.815	8.193	8.239	1.600	6.639	6.813	6.706
(4) High risk	1.828	7.235	6.651	6.572	1.560	6.441	6.387	6.378
Marginal effect (4)–(3)	−0.010	−0.579	−1.541***	−1.666***	−0.040	−0.198	−0.426**	−0.328*
	(0.195)	(0.601)	(0.379)	(0.394)	(0.044)	(0.159)	(0.174)	(0.196)

ownership and other determinants of patenting activity (unreported in the tables).[58] In the models reported in Table 4, we find a significant negative effect of transient institutional ownership, and a significant positive effect of quasi-indexing institutional ownership on both patents and citations, which might be due to longer-term investors favoring investments in longer-term projects, as reflected in patents. However, dedicated institutional ownership is insignificant in our sample. These findings are different from those in Aghion et al. (2013), who find a significant positive effect of both dedicated and transient institutional ownership on citations.[59]

C. Further Robustness Tests

Next, we re-estimate the patent and citation models for subsamples of high-tech and non-high-tech firms. Tables IA.IX and IA.X in Section C of the Supplementary Material show stronger statistical significance, as well as higher marginal effects, for both AMBIGUITY and RISK in the subsample of high-tech firms compared with non-high-tech firms. ANALYST_DISPERSION, while insignificant in the full sample (Table 4), becomes positive and significant for high-tech firms (Table IA.IX in Section C of the Supplementary Material) and negative and significant, reflecting a higher value of the option-to-wait, for non-high-tech firms (Table IA.X in Section C of the Supplementary Material). Some studies suggest that the dispersion of analysts' earnings forecasts is a proxy for ambiguity. We find that, when ambiguity is defined precisely and measured through market expectations, it is a very different concept from the variance of analysts' forecasts, which is based on different motivations and behavioral precepts. The latter is closely related to outcomes rather than probabilities, as captured by our ambiguity measure. The coefficient estimates on RISK and AMBIGUITY do not materially change when we include ANALYST_DISPERSION (untabulated results), indicating that, indeed, all three measures capture different dimensions of uncertainty.

Furthermore, it is important to note that the set of seven three-digit SIC codes used to identify high-tech firms (Brown et al. (2009)) is geared toward the most R&D-intensive industries. However, a significant number of patenting firms do not report positive R&D expenditures (Koh and Reeb (2015)), and thus could be classified as low-tech. To address this potential misclassification, in Table IA.XI in Section C of the Supplementary Material, we analyze the subsample of *patent-intensive firms*, which are defined as the firms ranking in the top tercile according to the average number of patents (Panel A) or citations (Panel B) filed during the

[58]Specifically, in addition to AMBIGUITY, RISK, and ANALYST_DISPERSION, the patent regressions also include the following unreported variables: INSTOWN_DED$_t$, INSTOWN_TRA$_t$, INSTOWN_QIX$_t$, $\ln(\text{SALES})_t$, Q$_t$, $\ln(\text{K\_L})_t$, CASH_FLOW$_t$, LEVERAGE$_t$, $\ln(\text{AGE})_{t+1}$, $\ln(\text{RD\_CAPITAL})_t$, and NASDAQ$_t$.

[59]Table IA.VI in Section C of the Supplementary Material shows that the R&D findings are robust to controlling for institutional ownership, but the institutional ownership variables themselves are insignificant. In the patent regressions, we do find a significant effect of institutional ownership, so we include these variables in the main tables. The main results for AMBIGUITY in the patent regressions are not affected by the inclusion of the institutional ownership variables. Furthermore, our patent findings are also robust to controlling for total institutional ownership instead of dedicated, transient, and quasi-indexing institutional ownership separately in the regressions. Total institutional ownership is itself insignificant.

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sample period. Table IA.XI in Section C of the Supplementary Material shows that, for *patent-intensive firms*, the effect of AMBIGUITY on patents is negative and significant in both the Poisson and the Negative Binomial regressions.[60] These findings suggest that ambiguity matters more for patent-intensive firms, regardless of the industries they operate in. We note that our hypotheses should apply to any firm that creates real options. These are often high-tech firms, but other firms that research and develop products may also behave similarly.[61]

We obtain our strongest results when we further restrict the sample to *patent-intensive high-tech firms* (Table 5). In this subsample, AMBIGUITY is negative and highly significant at most horizons, for both patents and citations, in both Poisson and Negative Binomial models.

One concern with the findings in Table 5 is that, since we define *patent-intensive firms* based on the average number of patents filed, the sample would tend to include larger firms that potentially file more patents. To address this concern, Table IA.XII in Section C of the Supplementary Material considers high-tech firms in the top tercile according to the *average size* (quarterly sales) during the sample period (instead of the average number of patents). In Table IA.XII in Section C of the Supplementary Material, the effect of AMBIGUITY is insignificant in most specifications, in stark contrast with the negative and significant effect of ambiguity on patents and citations in Table 5 (where we focus on patent-intensive high-tech firms). These results prove that the finding documented in Table 5 is not a size effect, and that there is an inherent significant relation between AMBIGUITY and patenting activity.[62]

In summary, Tables 4 and 5 show that both ambiguity and risk have a negative and significant effect on patents and citations up to three years into the future, especially for patent-intensive high-tech firms, supporting Hypotheses 1 and 3a (for ambiguity) and Hypothesis 4a (for risk).

Interestingly, while ambiguity has an overall negative effect on both R&D and patents, the effect of ambiguity on R&D is stronger for low- and medium-knowledge-capital high-tech firms (Table 2), whereas the effect of ambiguity on patents and citations is stronger for patent-intensive high-tech firms (Table 5).

[60]Note that the sample size in Table IA.XI in Section C of the Supplementary Material (22,004 firm-quarters for one-quarter-ahead patents) is comparable to that in Table IA.IX in Section C of the Supplementary Material (20,059 firm-quarters), so the increased significance of the coefficient estimate of AMBIGUITY in Table IA.XI in Section C of the Supplementary Material is not due to the sample size.

[61]In untabulated analysis, we find that patent-intensive firms have higher ambiguity than patenting firms in high-tech industries. The mean (median) ambiguity is 0.028 (0.022) in the subsample of patent-intensive firms used in Table IA.XI in Section C of the Supplementary Material, and only 0.021 (0.016) in the subsample of patenting firms in high-tech industries used in Table IA.IX in Section C of the Supplementary Material. The standard deviation of ambiguity is also higher for patent-intensive firms (0.022) than for patenting firms in high-tech industries (0.019). The higher variation in ambiguity could provide a partial explanation for why the coefficient estimate on ambiguity is, in general, more significant in the subsample of patent-intensive firms (Table IA.XI in Section C of the Supplementary Material) than in the subsample of patenting firms in high-tech industries (Table IA.IX in Section C of the Supplementary Material).

[62]The within-firm correlation between AMBIGUITY and RISK is -0.285 in the subsample of patent-intensive firms (Table IA.XII in Section C of the Supplementary Material) and -0.347 in the subsample of patent-intensive high-tech firms (Table 5), compared with -0.158 in the sample used in Table 4. For robustness, we estimate the regressions in Tables IA.XII and IA.V in Section C of the Supplementary Material including only AMBIGUITY, without RISK. The findings for AMBIGUITY are similar to those reported in the tables, indicating that the correlation between AMBIGUITY and RISK does not drive the results.

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TABLE 5

Determinants of Patenting Activity in Patent-Intensive High-Tech Firms

Table 5 presents the coefficient estimates of count models for patenting activity. The dependent variable is PATENTS in Panel A, and CITATIONS in Panel B. The sample period is 1993–2016. The sample is the same as in Table 4, restricted to firms with three-digit SIC codes 283, 357, 366, 367, 382, 384, or 737 (high-tech firms), and further restricted to either firms in the top tercile according to the average number of patents per quarter filed during the sample period – patent-intensive firms (Panel A); or to firms in the top tercile according to the average number of citations-weighted patents filed during the sample period – citation-intensive firms (Panel B). Marginal effects are calculated as differences in predicted counts at high (the 90th percentile of the estimation sample) and low (the 10th percentile of the estimation sample) AMBIGUITY$_t$ and RISK$_t$, while keeping all other variables at their sample means. All regressions include the following control variables: INSTOWN_DED$_t$, INSTOWN_TRA$_t$, INSTOWN_QIX$_t$, ln(SALES)$_t$, Q$_t$, ln(K_L)$_t$, CASH_FLOW$_t$, LEVERAGE$_t$, ln(AGE)$_{t+1}$, ln(RD_CAPITAL)$_t$, NASDAQ$_t$, as well as three-digit SIC code fixed effects, Blundell et al. (1999) pre-sample firm fixed effects and quarter–year fixed effects. Sample construction is detailed in Section III.A. Variable definitions are in Table IA.I in Section B of the Supplementary Material. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	Poisson				Negative Binomial			
	1 Quarter	Year 1	Year 2	Year 3	1 Quarter	Year 1	Year 2	Year 3
	$t+1$	$t+1:t+4$	$t+5:t+8$	$t+9:t+12$	$t+1$	$t+1:t+4$	$t+5:t+8$	$t+9:t+12$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Patents								
Coefficients								
AMBIGUITY$_t$	−3.598**	−3.710**	−4.265**	−4.175**	−5.408***	−5.283***	−6.348***	−6.273***
	(1.455)	(1.558)	(1.728)	(2.041)	(1.690)	(1.698)	(1.762)	(1.971)
RISK$_t$	−24.802	−31.416	−40.951	−37.497	−51.596***	−54.142***	−53.956***	−38.601**
	(23.944)	(24.901)	(27.838)	(28.599)	(15.437)	(14.983)	(15.469)	(18.328)
ANALYST_ DISPERSION$_t$	9.517***	9.380***	8.158***	6.169*	10.990**	10.434**	8.159*	7.726
	(2.969)	(2.828)	(2.920)	(3.287)	(4.475)	(4.520)	(4.905)	(5.115)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	8,964	8,449	7,686	6,994	8,964	8,449	7,686	6,994
No. of firms	263	263	252	241	263	263	252	241
Pseudo R^2					0.115	0.098	0.095	0.091
Marginal Effects								
(1) Low ambiguity	13.974	57.808	61.000	63.311	15.505	64.507	69.552	72.579
(2) High ambiguity	11.670	48.150	49.882	52.474	11.828	49.726	51.549	54.740
Marginal effect (2)–(1)	−2.303**	−9.658**	−11.118**	−10.837**	−3.677***	−14.781***	−18.003***	−17.840***
	(0.920)	(3.995)	(4.440)	(5.207)	(1.189)	(4.900)	(5.192)	(5.760)
(3) Low risk	13.265	55.150	58.266	60.643	14.505	60.792	64.582	66.996
(4) High risk	12.641	51.819	53.641	56.118	13.120	54.602	57.914	61.855
Marginal effect (4)–(3)	−0.624	−3.332	−4.625	−4.525	−1.384***	−6.190***	−6.668***	−5.140**
	(0.608)	(2.672)	(3.185)	(3.495)	(0.421)	(1.743)	(1.963)	(2.462)
Panel B. Citations								
Coefficients								
AMBIGUITY$_t$	−3.088*	−3.582**	−2.756	−3.478	−3.992**	−4.000**	−4.743**	−4.829**
	(1.685)	(1.803)	(2.287)	(2.342)	(1.929)	(1.890)	(1.923)	(2.170)
RISK$_t$	−43.838	−50.925*	−56.507**	−58.355**	−28.276	−26.659	−29.786	−27.554
	(26.859)	(28.214)	(27.977)	(27.630)	(18.133)	(16.514)	(18.115)	(16.969)
ANALYST_ DISPERSION$_t$	7.189**	8.374**	7.488**	5.147	11.330**	11.935**	10.339**	9.133*
	(3.595)	(4.075)	(3.617)	(4.106)	(4.694)	(4.815)	(4.471)	(4.715)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. of obs.	8,468	7,979	7,236	6,576	8,468	7,979	7,236	6,576
No. of firms	257	257	243	232	257	257	243	232
Pseudo R^2					0.067	0.067	0.064	0.060
Marginal Effects								
(1) Low ambiguity	16.598	69.243	71.420	75.579	17.611	73.273	78.427	82.292
(2) High ambiguity	14.297	58.432	63.115	65.072	14.521	60.618	63.398	66.850
Marginal effect (2)–(1)	−2.301*	−10.812**	−8.305	−10.507	−3.090**	−12.655**	−15.029**	−15.443**
	(1.265)	(5.462)	(6.904)	(7.082)	(1.500)	(5.986)	(6.077)	(6.909)
(3) Low risk	16.256	67.752	71.603	75.193	16.715	69.581	73.977	77.634
(4) High risk	14.849	60.783	63.317	66.056	15.767	65.737	69.333	73.027
Marginal effect (4)–(3)	−1.406	−6.969*	−8.287**	−9.137**	−0.948	−3.844	−4.644	−4.607
	(0.872)	(3.918)	(4.186)	(4.411)	(0.610)	(2.389)	(2.842)	(2.864)

These findings imply that, for firms that do not yet have a solid track record of successful innovations, ambiguity affects mainly R&D. For these firms, R&D investment is perhaps the dimension of innovation activities over which the managers exercise more control relative to patenting decisions, because the number of inventions that these firms can patent is constrained by their limited knowledge capital. In contrast, in patent-intensive, high-knowledge-capital firms, managers have more discretion over which innovations to patent and over the timing of patent filings, perhaps delaying patenting for strategic reasons or until uncertainty, as reflected in ambiguity and risk, decreases; for these firms, R&D investments are likely to be incremental and less affected by ambiguity.

VII. Conclusion

A number of recent studies document the impact of various factors on innovation. One of the most important questions in innovation research is the effect of uncertainty on investment in R&D and in patenting, which by definition are both paths into the unknown. We analyze two different types of uncertainty (ambiguity and risk) which ex ante may lead to very different firm decisions. We focus on the distinction between ambiguity and risk as drivers of innovation.

We view innovation as a real option and argue that, since the value of a real option increases with risk and decreases with ambiguity, firms should *increase* investment in innovative projects as risk (the uncertainty of outcomes) increases, but *decrease* investment as ambiguity (the uncertainty of probabilities) increases. Allowing for flexibility regarding the *timing* of innovation investments, uncertainty may delay innovation investments if the benefit of delaying outweighs its cost. Empirically, we find broad support for the hypothesis that firms facing high ambiguity decrease and delay both R&D and patents. This is particularly true for high-tech, high-growth firms and firms that file many patents, which are the types of firms expected to be particularly concerned about ambiguity in addition to risk. However, we find that riskier firms indeed invest more in R&D, but they also file fewer patents and receive fewer citations, which is consistent with the idea that in uncertain times delaying investment and the option-to-wait are more valuable. The different effects of risk on R&D and patent investments may be attributed to the different nature of these two types of innovation activities and to measurement issues.

Our findings may be generalized to other types of investment as well. Ignoring the effects of ambiguity in addition to risk may lead to erroneous characterizations of under-investment or over-investment.

Supplementary Material

To view supplementary material for this article, please visit http://doi.org/10.1017/S002210902300128X.

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