ON THE INCLINATION OF THE LUNAR AXIS

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Since the days of Cassini (1693), study of the position of the lunar axis relative to an inertial system of reference has been based on the assumption that "the inclination of the Moon's equator to the plane of the ecliptic is constant". And, although the actual value of that inclination has been subject to continuous changes and modifications, reduced from the originally suggested $4\frac{1}{2}°$ to $1°32'4''$ (Koziel, 1967), no sufficient attention has been paid to the fact that, as the lunar globe moves within the field of varying external forces, the inclination of its axis cannot remain constant.

Moreover, certain confusion seems to have been involved in works dealing with the subject, and several 'axes' of the Moon appear to be interchanged with each other and used quite inconsistently. It must be understood that the shortest inertial axis of the lunar dynamical configuration, the instantaneous axis of rotation of the Moon, and the rotation axis which the Moon would possess if it could obey precisely Cassini's laws of motion, do not coincide with each other (Habibullin, 1968). Therefore, whenever measurements of reference points of the lunar surface are reduced for the determination of the position of the lunar 'axis', either the selenographic coordinates of those points or the inclination of the axis they are referred to, should be expressed as time-dependent functions. Which one would be considered as constant depends on the definition; but they cannot both possess constant values.

The Eulerian equations of motion provide the required relations between the system of principal axes of the Moon, $Oxyz$, and a system fixed in space. If we adopt as our fixed system the ecliptic system of coordinates and we represent the longitude of the descending node of the Moon's equator, the angular distance of the direction Ox from the descending node of the Moon's equator, and the inclination of the lunar equator to the ecliptic, with ψ, φ and θ, respectively, the kinematic equations take the form:

$$\omega_x = -\frac{d\psi}{dt}\sin\theta\sin\varphi - \frac{d\theta}{dt}\cos\varphi \tag{1}$$

$$\omega_y = -\frac{d\psi}{dt}\sin\theta\cos\varphi + \frac{d\theta}{dt}\sin\varphi \tag{2}$$

$$\omega_z = \frac{d\psi}{dt}\cos\theta + \frac{d\varphi}{dt}. \tag{3}$$

where ω_x, ω_y and ω_z stand for the components of the angular velocity along the x, y, z axes.

Urey and Runcorn (eds.), The Moon, 13–21.

The system of Equations (1)–(3) solved for $\mathrm{d}\psi/\mathrm{d}t$, $\mathrm{d}\varphi/\mathrm{d}t$ and $\mathrm{d}\theta/\mathrm{d}t$ gives:

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -\frac{\omega_x \sin\varphi + \omega_y \cos\varphi}{\sin\theta} \tag{4}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{(\omega_x \sin\varphi + \omega_y \cos\varphi)\cos\theta}{\sin\theta} + \omega_z \tag{5}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_y \sin\varphi - \omega_x \cos\varphi\,. \tag{6}$$

We can introduce into the right-hand side of the Equations (4)–(6) the components σ, τ and ϱ of the physical libration, which, as is known, represent deviations in the position of the $Oxyz$ system from that prescribed by Cassini's laws, and therefore are connected to the angles ψ, φ and θ by means of the relations:

$$\psi = \Omega + \sigma \tag{7}$$

$$\varphi = 180° + l - \psi + \tau \tag{8}$$

$$\theta = I + \varrho\,. \tag{9}$$

where Ω is the longitude of the ascending node of the lunar orbit, l the mean longitude of the Moon and I the mean value of the inclination of the lunar equator to the ecliptic.

Retaining the first-order terms for the small quantities σ, τ and ϱ, and taking into account the relation:

$$l = g + \omega + \Omega \tag{10}$$

which expresses the mean longitude of the Moon in terms of the mean anomaly of the Moon, g, the angular distance of the Moon's perigee from the ascending node of the orbit, ω, and the longitude of the ascending node of the lunar orbit, Ω, we find the system of equations:

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \frac{1}{\sin I + \varrho\cos I} \times \\ \times \{[\omega_x \cos(g+\omega) - \omega_y \sin(g+\omega)](\tau - \sigma) + \omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)\} \tag{11}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\frac{\cos I}{\sin I + \varrho\cos I} \times \\ \times \{[\omega_x \cos(g+\omega) - \omega_y \sin(g+\omega)](\tau - \sigma) + \omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)\} \\ + \frac{\sin I}{\sin I + \varrho\cos I}[\omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)]\,\varrho + \omega_z \tag{12}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = [\omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)](\sigma - \tau) + \omega_x \cos(g+\omega) - \omega_y \sin(g+\omega) \tag{13}$$

or the equivalent to that system:

$$\frac{d\sigma}{dt} = \frac{1}{\sin I + \varrho \cos I} \times$$
$$\times \{[\omega_x \cos(g+\omega) - \omega_y \sin(g+\omega)] \times$$
$$\times (\tau - \sigma) + \omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)\} - \frac{d\Omega}{dt} \tag{14}$$

$$\frac{d\tau}{dt} = \frac{1-\cos I}{\sin I + \varrho \cos I} \{[\omega_x \cos(g+\omega) - \omega_y \sin(g+\omega)] \times$$
$$\times (\tau - \sigma) + \omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)\} + \frac{\sin I}{\sin I + \varrho \cos I} \times$$
$$\times [\omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)]\, \varrho + \omega_z - \frac{dg}{dt} - \frac{d\omega}{dt} - \frac{d\Omega}{dt} \tag{15}$$

$$\frac{d\varrho}{dt} = [\omega_x \sin(g+\omega) + \omega_y \cos(g+\omega)]\,(\sigma - \tau) + \omega_x \cos(g+\omega) - \omega_y \sin(g+\omega). \tag{16}$$

Solution of the system of Equations (11)–(13), or its equivalent (14)–(16), will give the value of the true inclination, θ, of the Moon's axis to that of the ecliptic, as a function of time.

The velocity components ω_x, ω_y and $\tilde{\omega}_z$ are derived from Euler's dynamical equations:

$$A\frac{d\omega_x}{dt} = (B - C)\left(\omega_y\omega_z - \frac{3Gm_\oplus}{R^5}\, y_E z_E\right) \tag{17}$$

$$B\frac{d\omega_y}{dt} = (C - A)\left(\omega_z\omega_x - \frac{3Gm_\oplus}{R^5}\, z_E x_E\right) \tag{18}$$

$$C\frac{d\omega_z}{dt} = (A - B)\left(\omega_x\omega_y - \frac{3Gm_\oplus}{R^5}\, x_E y_E\right) \tag{19}$$

where A, B, C are the principal moments of inertia of the Moon, G is the gravitational constant, $m_\oplus$ is the mass of the Earth, R the distance between the centre of mass of the Moon and that of the Earth, and x_E, y_E, z_E the rectangular coordinates of the centre of the Earth in the selenocentric system $Oxyz$, which are related to the true selenocentric longitude of the Earth, υ, the true geocentric latitude of the Moon, $B_☾$, and the Eulerian angles, by means of the expressions:

$$x_E = R\cos B_☾ [\cos(\upsilon - \varphi) - \sin\upsilon \sin\varphi\,(1 - \cos\theta) + \tan B_☾ \sin\varphi \sin\theta], \tag{20}$$

$$y_E = R\cos B_☾ [\sin(\upsilon - \varphi) - \sin\upsilon \cos\varphi\,(1 - \cos\theta) + \tan B_☾ \cos\varphi \sin\theta], \tag{21}$$

$$z_E = R\cos B_☾ [\sin\upsilon \sin\theta - \tan B_☾ \cos\theta]. \tag{22}$$

As is known, the true geocentric latitude of the Moon, $B_☾$, can be expressed in terms of the inclination of the lunar orbit to the ecliptic, i, and the true selenocentric longitude of the Earth, υ, by means of the formula:

$$\tan B_{☾} = -\tan i \sin \upsilon . \tag{23}$$

We can, moreover, express the coordinates x_E, y_E, z_E in terms of the true geocentric longitude and latitude of the Moon, L and $B_{☾}$, its mean anomaly, g, the angular distance of the lunar perigee from the ascending node of the orbit, ω, and the physical libration components σ, τ, ϱ, taking into account the relations (7)–(9) and the fact that

$$\upsilon = 180° + L + \psi . \tag{24}$$

Then the system of Equations (17)–(19) takes the form:

$$\begin{aligned}\frac{d\omega_x}{dt} + \alpha\omega_y\omega_z = \frac{3Gm_{\oplus}\alpha \cos^2 B_{☾}}{R^3} & \{\sigma [\sin(L-l)\cdot\cos(L-l+g+\omega) - \\ & - (1-\cos I + \tan i \sin I)\{\tfrac{1}{2}\sin 2(L-l+g+\omega)\cos(g+\omega) + \\ & + \sin(L-l+g+\omega)\cos(L-l+2g+2\omega)\}]\sin(I+i) + \\ & + \tau[\cos(L-l)\sin(L-l+g+\omega) - (1-\cos I + \tan i \sin I) \times \\ & \times \sin^2(L-l+g+\omega)\sin(g+\omega)]\sin(I+i) + \varrho\{[(1-\cos I + \\ & + \tan i \sin I)\sin^2(L-l+g+\omega)\cos(g+\omega) - \sin(L-l) \times \\ & \times \sin(L-l+g+\omega)]\cos(I+i) + (\sin I + \tan i \cos I)\sin(I+i) \times \\ & \times \sin^2(L-l+g+\omega)\cos(g+\omega)\} + [(1-\cos I + \tan i \sin I) \times \\ & \times \sin^2(L-l+g+\omega)\cos(g+\omega) - \sin(L-l) \times \\ & \times \sin(L-l+g+\omega)]\sin(I+i)\}\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{d\omega_y}{dt} - \beta\omega_z\omega_x = -\frac{3Gm_{\oplus}\beta \cos^2 B_{☾}}{R^3} & \{\sigma [\cos(L-l)\cos(L-l+g+\omega) - \\ & - (1-\cos I + \tan i \sin I)\{\tfrac{1}{2}\sin^2(L-l+g+\omega)\sin(g+\omega) + \\ & + \sin(L-l+2g+2\omega)\sin(L-l+g+\omega)\}]\sin(I+i) - \\ & - \tau[\sin(L-l)\sin(L-l+g+\omega) - (1-\cos I + \tan i \sin I) \times \\ & \times \sin^2(L-l+g+\omega)\cos(g+\omega)]\sin(I+i) + \\ & + \varrho[\{(1-\cos I + \tan i \sin I)\sin^2(L-l+g+\omega)\sin(g+\omega) - \\ & - \cos(L-l)\sin(L-l+g+\omega)\}\cos(I+i) + (\sin I + \tan i \cos I) \times \\ & \times \sin(I+i)\sin^2(L-l+g+\omega)\sin(g+\omega)] + \\ & + [(1-\cos I + \tan i \sin I)\sin^2(L-l+g+\omega)\sin(g+\omega) - \\ & - \cos(L-l)\sin(L-l+g+\omega)]\sin(I+i)\}\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{d\omega_z}{dt} + \gamma\omega_x\omega_y = \frac{3Gm_{\oplus}\gamma \cos^2 B_{☾}}{2R^3} & \{2\sigma(1-\cos I + \tan i \sin I)\cos 2(g+\omega) - \\ & - 2\tau[(\cos I - \tan i \sin I)\cos 2(L-l) - (1-\cos I + \tan i \sin I) \times \\ & \times \cos 2(g+\omega)] - \varrho(\sin I + \tan i \cos I)[\sin 2(L-l) + \sin 2(g+\omega)] + \\ & + (\cos I - \tan i \sin I)\sin 2(L-l) - \\ & - (1-\cos I + \tan i \sin I)\sin 2(g+\omega)\}\end{aligned} \tag{27}$$

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where α, β and γ represent the ratios of mechanical ellipticities of the Moon:

$$\alpha = (C - B)/A, \quad \beta = (C - A)/B, \quad \gamma = (B - A)/C. \tag{28}$$

We are therefore led to the simultaneous solution of the Equations (14), (15), (16), (25), (26) and (27), which will produce the values of the corrections ϱ for the inclination of the Moon's shortest axis of inertia to the ecliptic, as well as the libration components σ and τ.

The variation in the inclination of the lunar axis during the year 1971, obtained by numerical integration of this system of equations, is presented in Table I. It has been assumed that at the beginning of the year the librations were zero and the inclination possessed the mean value of 1° 32′ 4″. In order to establish an ephemeris for the inclination, it now only remains to obtain by observational detection a set of measurements which will provide the correct initial conditions for the solution, while the periodic nature and other general characteristics of the nutational motion of the lunar axis are apparent in the values presented here.

TABLE I

Variation of the inclination of the lunar axis to the ecliptic over the period of a year

Day	Inclination	Day	Inclination
1	1° 32′ 4″	28	1° 32′ 5″
2	1° 32′ 1″	29	1° 31′ 59″
3	1° 31′ 57″	30	1° 31′ 55″
4	1° 31′ 51″		
5	1° 31′ 42″	31	1° 31′ 46″
6	1° 31′ 31″	32	1° 31′ 38″
7	1° 31′ 18″	33	1° 31′ 25″
8	1° 31′ 2″	34	1° 31′ 13″
9	1° 30′ 43″	35	1° 30′ 55″
10	1° 30′ 31″	36	1° 30′ 41″
		37	1° 30′ 23″
11	1° 30′ 18″	38	1° 30′ 12″
12	1° 30′ 10″	39	1° 30′ 4″
13	1° 30′ 6″	40	1° 30′ 2″
14	1° 30′ 7″		
15	1° 30′ 10″	41	1° 30′ 5″
16	1° 30′ 21″	42	1° 30′ 14″
17	1° 30′ 34″	43	1° 30′ 26″
18	1° 30′ 52″	44	1° 30′ 42″
19	1° 31′ 10″	45	1° 31′ 0″
20	1° 31′ 30″	46	1° 31′ 19″
		47	1° 31′ 38″
21	1° 31′ 44″	48	1° 31′ 55″
22	1° 31′ 58″	49	1° 32′ 7″
23	1° 32′ 4″	50	1° 32′ 16″
24	1° 32′ 11″		
25	1° 32′ 11″	51	1° 32′ 20″
26	1° 32′ 12″	52	1° 32′ 20″
27	1° 32′ 8″	53	1° 32′ 17″

Table I (continued)

Day	Inclination	Day	Inclination
54	1° 32′ 13″	102	1° 32′ 12″
55	1° 32′ 6″	103	1° 32′ 25″
56	1° 32′ 0″	104	1° 32′ 39″
57	1° 31′ 52″	105	1° 32′ 41″
58	1° 31′ 43″	106	1° 32′ 44″
59	1° 31′ 33″	107	1° 32′ 35″
60	1° 31′ 21″	108	1° 32′ 31″
		109	1° 32′ 16″
61	1° 31′ 7″	110	1° 32′ 8″
62	1° 30′ 53″		
63	1° 30′ 38″	111	1° 31′ 52″
64	1° 30′ 22″	112	1° 31′ 43″
65	1° 30′ 9″	113	1° 31′ 26″
66	1° 30′ 0″	114	1° 31′ 15″
67	1° 29′ 57″	115	1° 30′ 56″
68	1° 30′ 2″	116	1° 30′ 43″
69	1° 30′ 12″	117	1° 30′ 25″
70	1° 30′ 28″	118	1° 30′ 14″
		119	1° 30′ 1″
71	1° 30′ 46″	120	1° 29′ 57″
72	1° 31′ 8″		
73	1° 31′ 27″	121	1° 29′ 53″
74	1° 31′ 49″	122	1° 29′ 57″
75	1° 32′ 5″	123	1° 30′ 4″
76	1° 32′ 21″	124	1° 30′ 20″
77	1° 32′ 29″	125	1° 30′ 38″
78	1° 32′ 33″	126	1° 31′ 3″
79	1° 32′ 31″	127	1° 31′ 27″
80	1° 32′ 28″	128	1° 31′ 52″
		129	1° 32′ 11″
81	1° 32′ 20″	130	1° 32′ 28″
82	1° 32′ 12″		
83	1° 32′ 2″	131	1° 32′ 38″
84	1° 31′ 53″	132	1° 32′ 47″
85	1° 31′ 41″	133	1° 32′ 46″
86	1° 31′ 30″	134	1° 32′ 44″
87	1° 31′ 17″	135	1° 32′ 34″
88	1° 31′ 4″	136	1° 32′ 24″
89	1° 30′ 48″	137	1° 32′ 9″
90	1° 30′ 34″	138	1° 31′ 57″
		139	1° 31′ 41″
91	1° 30′ 20″	140	1° 31′ 28″
92	1° 30′ 10″		
93	1° 30′ 0″	141	1° 31′ 10″
94	1° 29′ 58″	142	1° 30′ 55″
95	1° 29′ 58″	143	1° 30′ 36″
96	1° 30′ 8″	144	1° 30′ 20″
97	1° 30′ 21″	145	1° 30′ 2″
98	1° 30′ 45″	146	1° 29′ 53″
99	1° 31′ 5″	147	1° 29′ 44″
100	1° 31′ 31″	148	1° 29′ 45″
		149	1° 29′ 48″
101	1° 31′ 50″	150	1° 30′ 1″

Table I (continued)

Day	Inclination	Day	Inclination
151	1° 30′ 12″	200	1° 29′ 50″
152	1° 30′ 36″		
153	1° 30′ 56″	201	1° 29′ 38″
154	1° 31′ 26″	202	1° 29′ 33″
155	1° 31′ 47″	203	1° 29′ 33″
156	1° 32′ 13″	204	1° 29′ 43″
157	1° 32′ 27″	205	1° 30′ 2″
158	1° 32′ 43″	206	1° 30′ 25″
159	1° 32′ 45″	207	1° 30′ 53″
160	1° 32′ 50″	208	1° 31′ 21″
		209	1° 31′ 49″
161	1° 32′ 44″	210	1° 32′ 15″
162	1° 32′ 40″		
163	1° 32′ 26″	211	1° 32′ 37″
164	1° 32′ 16″	212	1° 32′ 54″
165	1° 31′ 58″	213	1° 33′ 5″
166	1° 31′ 46″	214	1° 33′ 6″
167	1° 31′ 27″	215	1° 33′ 3″
168	1° 31′ 14″	216	1° 32′ 52″
169	1° 30′ 53″	217	1° 32′ 40″
170	1° 30′ 38″	218	1° 32′ 23″
		219	1° 32′ 8″
171	1° 30′ 17″	220	1° 31′ 49″
172	1° 30′ 3″		
173	1° 29′ 45″	221	1° 31′ 32″
174	1° 29′ 38″	222	1° 31′ 11″
175	1° 29′ 35″	223	1° 30′ 53″
176	1° 29′ 42″	224	1° 30′ 33″
177	1° 29′ 52″	225	1° 30′ 17″
178	1° 30′ 11″	226	1° 30′ 0″
179	1° 30′ 32″	227	1° 29′ 48″
180	1° 30′ 57″	228	1° 29′ 38″
		229	1° 29′ 32″
181	1° 31′ 23″	230	1° 29′ 33″
182	1° 31′ 50″		
183	1° 32′ 14″	231	1° 29′ 41″
184	1° 32′ 35″	232	1° 29′ 56″
185	1° 32′ 47″	233	1° 30′ 18″
186	1° 32′ 56″	234	1° 30′ 48″
187	1° 32′ 56″	235	1° 31′ 17″
188	1° 32′ 52″	236	1° 31′ 48″
189	1° 32′ 43″	237	1° 32′ 13″
190	1° 32′ 33″	238	1° 32′ 38″
		239	1° 32′ 56″
191	1° 32′ 18″	240	1° 33′ 9″
192	1° 32′ 4″		
193	1° 31′ 46″	241	1° 33′ 14″
194	1° 31′ 30″	242	1° 33′ 13″
195	1° 31′ 12″	243	1° 33′ 2″
196	1° 30′ 56″	244	1° 32′ 49″
197	1° 30′ 37″	245	1° 32′ 30″
198	1° 30′ 21″	246	1° 32′ 13″
199	1° 30′ 4″	247	1° 31′ 51″

Table I (continued)

Day	Inclination	Day	Inclination
248	1° 31′ 33″	296	1° 33′ 19″
249	1° 31′ 10″	297	1° 33′ 12″
250	1° 30′ 51″	298	1° 33′ 3″
251	1° 30′ 28″	299	1° 32′ 45″
252	1° 30′ 9″	300	1° 32′ 27″
253	1° 29′ 51″	301	1° 32′ 3″
254	1° 29′ 38″	302	1° 31′ 42″
255	1° 29′ 28″	303	1° 31′ 17″
256	1° 29′ 26″	304	1° 30′ 56″
257	1° 29′ 29″	305	1° 30′ 32″
258	1° 29′ 36″	306	1° 30′ 11″
259	1° 29′ 52″	307	1° 29′ 46″
260	1° 30′ 13″	308	1° 29′ 29″
261	1° 30′ 41″	309	1° 29′ 10″
262	1° 31′ 11″	310	1° 29′ 4″
263	1° 31′ 44″	311	1° 29′ 4″
264	1° 32′ 11″	312	1° 29′ 16″
265	1° 32′ 37″	313	1° 29′ 32″
266	1° 32′ 54″	314	1° 29′ 59″
267	1° 33′ 8″	315	1° 30′ 27″
268	1° 33′ 15″	316	1° 31′ 0″
269	1° 33′ 16″	317	1° 31′ 34″
270	1° 33′ 9″	318	1° 32′ 7″
271	1° 32′ 57″	319	1° 32′ 38″
272	1° 32′ 40″	320	1° 33′ 2″
273	1° 32′ 19″	321	1° 33′ 18″
274	1° 31′ 58″	322	1° 33′ 27″
275	1° 31′ 36″	323	1° 33′ 28″
276	1° 31′ 14″	324	1° 33′ 22″
277	1° 30′ 51″	325	1° 33′ 9″
278	1° 30′ 29″	326	1° 32′ 56″
279	1° 30′ 6″	327	1° 32′ 35″
280	1° 29′ 45″	328	1° 32′ 14″
281	1° 29′ 27″	329	1° 31′ 49″
282	1° 29′ 19″	330	1° 31′ 26″
283	1° 29′ 13″	331	1° 31′ 0″
284	1° 29′ 18″	332	1° 30′ 38″
285	1° 29′ 28″	333	1° 30′ 15″
286	1° 29′ 48″	334	1° 29′ 55″
287	1° 30′ 6″	335	1° 29′ 35″
288	1° 30′ 37″	336	1° 29′ 19″
289	1° 31′ 5″	337	1° 29′ 5″
290	1° 31′ 41″	338	1° 29′ 1″
291	1° 32′ 9″	339	1° 29′ 3″
292	1° 32′ 39″	340	1° 29′ 17″
293	1° 32′ 57″	341	1° 29′ 43″
294	1° 33′ 13″	342	1° 30′ 12″
295	1° 33′ 17″	343	1° 30′ 48″

Table I (continued)

Day	Inclination	Day	Inclination
344	1° 31′ 22″	355	1° 32′ 25″
345	1° 31′ 58″	356	1° 31′ 57″
346	1° 32′ 30″	357	1° 31′ 35″
347	1° 32′ 59″	358	1° 31′ 6″
348	1° 33′ 20″	359	1° 30′ 43″
349	1° 33′ 35″	360	1° 30′ 17″
350	1° 33′ 35″		
		361	1° 29′ 57″
351	1° 33′ 33″	362	1° 29′ 35″
352	1° 33′ 20″	363	1° 29′ 22″
353	1° 33′ 6″	364	1° 29′ 10″
354	1° 32′ 44″	365	1° 29′ 3″

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