Correspondence

"Faster! Faster!"

DEAR EDITOR,

Before Note 63.34 could be published, it was already seriously out of date.

At the time of writing, the latest news is that between October 1978 and April 1979 three further Mersenne primes were discovered, namely M_{21701} (reported in my Note), M_{23209} (with 6987 digits) and M_{44497} (13 395 digits). This brings the total number of known Mersenne primes M_p to 27 (all values of p up to 50 000 having now been tested).

Of course, by the time this letter appears in print it too may have been overtaken by events!

Yours sincerely,

A. R. G. MACDIVITT

Paisley College of Technology, High Street, Paisley, Renfrewshire PA1 2BE

An alternative route to the exponential series

DEAR EDITOR,

In a recent Note (*Mathl Gaz.* **63**, 123–4 (No. 424, June 1979)) D. H. Armitage presents an "elementary" proof of the series expansion for e^x which avoids use of Taylor's theorem.

A very similar series to that used by Armitage, which will give the series expansion for e^x and the irrationality of e by using methods identical to those of Armitage, can be obtained by applying repeated integration by parts as follows.

$$\frac{1}{n!}\int_0^x (x-t)^n e^t dt$$

$$= \left[\frac{1}{n!}(x-t)^n e^t\right]_0^x + \frac{1}{(n-1)!}\int_0^x (x-t)^{n-1} e^t dt$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$= -\frac{x^n}{n!} - \ldots - \frac{x}{1!} + \int_0^x e^t dt.$$

Thus we obtain

$$e^x = 1 + \frac{x}{1!} + \ldots + \frac{x^n}{n!} + \frac{1}{n!}\int_0^x (x-t)^n e^t dt.$$

Furthermore, if we now replace e^t in the first integral by $f^{(n+1)}(t)$, the above calculation will carry through to give Taylor's theorem in the "integral error" form.

Yours faithfully,

G. N. THWAITES

Rugby School, Rugby, Warwickshire