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# ON EXCITATION MECHANISMS OF PULSATION IN β CEPHEI STARS

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The present state of our knowledge concerning the excitation mechanisms of β Cephei pulsation is reviewed. First we shall discuss whether or not oscillations of β Cephei stars can be explained by the conventional pulsation theory. After showing that the explanation by the conventional theory might be impossible, we shall discuss other specific excitation mechanisms suggested so far.

The first question is whether β Cephei stars are unstable to radial pulsation. This problem was examined by Davey (1973). He found that all the models considered are quite stable. His results are reasonable because the usual envelope ionization mechanism is ruled out in β Cephei stars since their surface temperature is high. The excitation of radial pulsation by the so-called $\varepsilon$-mechanism is also not expected in β Cephei stars, since their masses are too small for this mechanism to work.

The next question is whether β Cephei stars are unstable to non-radial oscillations. This problem is not so simple, because the eigen-functions of non-radial oscillations are rather different from those of radial pulsation. In non-radial oscillations, new type of restoring force (or buoyancy force) works on fluid motions, since the motions have the vortex components. Corresponding to this, new type of modes, namely the gravity modes (g-modes), appear in addition to the pressure modes (p-modes). It is these gravity



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modes that are interesting from the point of view of pulsational instability, because the spatial behavior of their eigen-functions is quite different from that of the pressure modes. Recently, the pulsational instability of non-radial modes with $\ell = 2$ (of surface harmonics $Y_\ell^m$) has been investigated numerically by Chiosi (1974), by Aizenman, Cox and Lesh (1975), and also by Osaki (1975). Chiosi and Aizenman et.al. claim independently that they found pulsational instability against some non-radial gravity modes, either during the overall contraction stage (Chiosi 1974) or during the initial stage of the shell hydrogen burning (Aizenman et. al. 1975). On the other hand, Osaki (1975a,b) insists that there is no pulsational instability in any mode and in any model for a 10 solar mass star.

Before discussing the possible cause of this discrepancy, it will be helpful to mention briefly characteristics of gravity modes of non-radial oscillations. In the $\mu$-gradient zone which appears outside the receding convective core, the medium is stratified highly stably to convection. Thus the local characteristic frequency of the gravity oscillations, namely, the Brunt-Väisälä frequency, is high in that region compared with in the outside region. This suggests that gravity modes of non-radial oscillations are partially trapped in the $\mu$-gradient zone. Osaki (1975a) clearly showed this trapping nature of the gravity modes. The propagation diagram and the phase diagram introduced by Osaki (1975a) are very helpful to understand not only this trapping nature but also general characteristics of non-radial oscillations. In addition to the fact that in the $\mu$-gradient zone the gravity modes have large amplitude due to trapping, it is important to note here that in the $\mu$-gradient zone the gravity modes oscillate rapidly in space, because the Brunt-Väisälä frequency is high there. This situation can be seen clearly in Figure 1, which has been reproduced from Osaki's article (1975b).

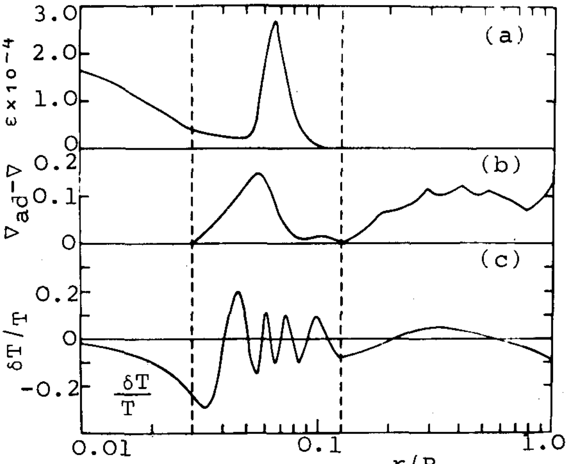


Fig. 1.

Based on the above characteristics of the gravity oscillations, Osaki(1975b) argues that if the μ-gradient zone is treated inaccurately, the radiative damping of oscillations due to rapid spatial variation of temperature in that zone is underestimated. The instability obtained by Chiosi and by Aizenman et.al. might come from this underestimation of the radiative damping. In any way it is difficult to understand why the μ-gradient zone contributes destabilization when the temperature in that zone is stratified sub-adiabatically and there is no shell burning. If we accept Osaki's numerical results and his arguments, we can say that no definite pulsational instability has been found so far within the conventional pulsation mechanism either for radial pulsations or for non-radial oscillations.

Next we shall direct our attention to other specific excitation mechanisms suggested so far. One of them is the "μ-mechanism" proposed by Stothers and Simon (1969). They suppose that the $\beta$ Cephei stars are the mass-accreting components of close binary systems. By accreting helium-rich material on the envelopes, the reversal of the gradient of mean molecular weight might occur in the outer layer of the stars. This reduces the central condensation of mass, and thus the pulsational amplitude of radial oscillations in the central region increases. The relatively large central pulsation amplitudes permit the nuclear-engined pulsational instability. Stothers and Simon showed that a moderate enrichment of the helium content in the outer envelopes is sufficient to produce instability by this mechanism. This mechanism, however, seems to be unlikely both theoretically and observationally. Arguments concerning (mainly against) this mechanism have been summarized recently in a review article by Cox (1974).

The second mechanism we shall discuss here is that proposed by Osaki (1974). The basic point of this mechanism is the resonant interaction between an overstable convection in a rapidly rotating convective core and a mode of non-radial oscillation, in particular a gravity mode. As is well-known, if a rotation is present a large scale convective motion becomes oscillatory by the action of restoring force of vortex lines. Osaki considered, for example, a star which deviates slightly from a uniform rotation with a faster core at the zero age main sequence stage and evolves from the stage conserving angular momentum in each shell. He found that the frequency of the oscillatory convection of $\ell = m = 2$ actually coincides with that of a gravity mode and thus both modes interact resonantly. Furthermore, estimating the coupling of the two modes, Osaki demonstrated that the amplitude of the non-radial mode to be excited is consistent with the observations.

Concerning Osaki s mechanism, there are some points to be discussed here. The first point is whether the large scale convective motion such as $\ell = m = 2$ is overstable. The convective motions are derived by buoyancy force due to a super-adiabatic temperature gradient, while the restoring force resulting from rotation acts against this. So long as the latter effect of rotation is minor, the convection grows oscillatorily (i.e., overstable). However, if the latter effect of rotation surpasses the former, the adiabatic convective modes become purely oscillatory, not overstable. Osaki shows that the critical super-adiabatic temperature gradient necessary for the convective mode of $\ell = m = 2$ to be overstable is $\nabla - \nabla_{ad} \sim 10^{-3}$, which is rather larger than $\nabla - \nabla_{ad} \sim 5.7 \times 10^{-7}$ expected from the mixing length theory. This gives rise to a question that the large scale convective modes might be pure oscillations and could not be excited in the convective core. One possibility to avoid this difficulty is to take account of the effect of dissipative processes. The purely oscillatory convections forced to be so by the effect of rotation can become overstable if thermal conduction is present. In the case of a uniformly rotating homogeneous medium, the condition of this overstability is that the thermometric conductivity $\kappa$ is larger than the kinematic viscosity $\nu$ (e.g., Ledoux 1958). In the convective core $\kappa$ and $\nu$ come mainly from eddy motions and are essentially of the same order of magnitude (e.g., Unno 1970), although $\kappa$ is supplemented by radiative diffusivity. In any case, since the present stage of knowledge of turbulent transport is unsatisfactory, it will be allowed to suppose that large scale oscillatory convections can be excited (overstable) in the convective core by the presence of thermal conductivity.

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The above considerations suggest another possible excitation mechanism of β Cephei pulsations. That is, the observed pulsation is the large scale oscillatory convection itself excited in a rapidly rotating convective core by the presence of thermal (eddy plus radiative) conductivity. The convective motions certainly can not penetrate till the surface of the star in the case of no rotation. In the present case, however, convections are oscillatory (it can be said that convections are inertial waves) by the effect of rotation and the periods of the oscillations are of the order of those of $g_+$-modes in the case of no rotation. Thus the oscillatory convections will rather penetrate to the surface of the star. The problem is whether such oscillations can be excited in the star as a whole. The author think this possibility is worth studying, because the mechanism is simple.

The second point which should be discussed on Osaki's mechanism is that his mathematical formulation concerning the resonant coupling between an oscillatory convection and a non-radial oscillation is inadequate. If there is no resonance in frequencies, both modes behave independently of each other in the frame-work of linear theory even if a rotation is present: there is no coupling. If a resonance occurs, the eigen-functions of both modes are mixed and the convective mode may penetrate to surface. In the frame-work of the linear theory, however, this is still a problem of eigen-value. It will be important to reformulate Osaki's mechanism in this line and to make the condition of excitation of oscillations clear.

The other mechanism which has been suggested so far is a resonant instability of the non-linear coupling between non-radial oscillations and the tidal wave, proposed by Kato (1974). This

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mechanism is based on the assumption that a β Cephei star is one component of a binary system, and the star has two pulsation modes with very close frequencies. More definitely, it is assumed that the beat frequency $f_b$ of the two pulsation modes is very close to the frequency $f_t$ of the tidal wave induced on the star by the revolution of the secondary star. The frequency of the tidal wave is twice that of the revolution frequency of the secondary. The above condition $f_b \sim f_t$ among frequencies is only a necessary condition. For the resonant interaction to be completed, a spatial coupling between oscillations is also required. Since the tidal wave has two waves in the azimuthal direction, i.e., m = 2, a necessary condition for the spatial coupling is that the difference of m's of two pulsation modes is two. For example, one mode is the fundamental radial oscillation, and the other is g modes of $\ell = m = 2$. Frequencies of the above two modes are known to coincide actually at an evolutionary stage of β Cephei type stars (e.g., Osaki 1975a). When the above resonant conditions are satisfied, two pulsation modes and the tidal wave interact resonantly. By this resonant interaction energy exchange between pulsation modes and the tidal wave occurs. The direction of energy flow among the waves depends on the forms of eigen-functions. Kato derived the condition where energy is supplied to the pulsation modes from the tidal wave, and thus the pulsation modes grow spontaneously.

The basic assumption included in this mechanism is that the β Cephei stars are components of binary systems as mentioned before. Even if the mass of the secondary star is rather small, pulsation modes seems to be excited if situations are favorable. Thus it is not inconsistent to the fact that all β Cephei stars are not observed as binaries. A weak point of this mechanism is that there is no definite discussion why this mechanism works preferentially at particular evolutionary stages of the stars.

Next we shall comment on the overstability of non-radial modes in the semi-convection zone. Recently, Shibahashi and Osaki (1975) confirmed that non-radial gravity modes are excited in the semi-convection zone, as suggested by Kato (1966). They show that for a mode to be excited, $\ell$ of the surface harmonics $Y_\ell^m$ must be large: larger than 15 for a 15 solar mass and larger than 8 for a 30 solar mass. This result can not explain directly the β Cephei phenomena, but non-linear coupling of these overstable modes may excite a mode with small $\ell$ such as $\ell = 2$. This possibility might be worth studying if all other possible mechanisms failed in explaining the β Cephei phenomena.

Finally we shall note the effects of thermal imbalance. Kato and Unno (1967) and Aizenman and Cox (1975) suggested that the effects of thermal imbalance on non-radial oscillations might play a role in the instability of β Cephei stars. Recently Aizenman, Cox and Lesh examined this problem and found that these effects are much too small to account for the observed instability[1].

In summary, the instability of β Cephei stars seems to be not explained by the conventional pulsation theory, namely, by the theory concerning non-rotating single stars with normal configurations. Some special excitation mechanisms may be required. Some such mechanisms have been proposed, but they are still unsatisfactory and further examinations or refinements are necessary.

To summarize this article, the author had opportunities to talk with Dr. Y. Osaki. The author heartly thanks him for helpful discussion. The author also thanks Prof. W. Unno and Dr. Y. Osaki for comments on the manuscript.

[1] Footnote of the article by Aizenman and Cox (1975).

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Discussion to the paper of KATO

HALL: Is there a lower mass limit below which you would not expect the Osaki mechanism to be able to operate?

KATO: Yes, at least the presence of a convective core is necessary. Furthermore, the mass of the convective core must be so large that the excitation of oscillations in the core can surpass their damping in the outer zone.

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