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Letter to the Editor

Ion solitary waves in a dense quantum plasma

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Abstract. The existence of localized ion waves in a dense quantum plasma is established. Specifically, ion solitary waves are stationary solutions of the equations composed of the nonlinear ion continuity and ion momentum equations, together with the Poisson equation and the inertialess electron momentum equation in which the electric force is balanced by the quantum force associated with the Bohm potential that causes electron tunneling at nanoscales. The solitary ion waves are characterized by a large-amplitude electrostatic potential and ion density maxima and smaller amplitude minima on the flanks of the solitary waves. We identify the speed interval for the existence of the ion solitary waves around a quantum Mach number that is of the order of unity.

Dense quantum plasmas are ubiquitous in microelectronics and nanotechnologies (e.g. semiconductors [1] and micromechanical systems [2], quantum dots and nanowires [3], resonant tunneling [4], quantum diodes [5] and nanoelectron tubes/nanotriode [6]), in intense laser–solid density plasma experiments [7], as well as in compact astrophysical objects [8–10] (e.g. giant planetary interiors, neutron stars/magnetars, massive white dwarfs, supernovae). In quantum plasmas, the electrons and anti-electrons are degenerate since the interparticle distance is comparable to the de Broglie wavelength. Here quantum mechanical effects (e.g. electron and positron tunneling through the Bohm potential barrier [11]) play an important role. The effect of quantum force associated with the Bohm potential has been incorporated into studies of linear and nonlinear electrostatic waves [12–15] in quantum plasmas.

In this letter, we present a theory for arbitrary large-amplitude nonlinear ion waves in an unmagnetized quantum plasma. For our purposes, we shall use the nonlinear ion continuity and momentum equations, together with the Poisson equation and the equation of motion for the inertialess electrons. We assume that the quantum force acting on electrons dominates over the quantum statistical pressure, which amounts to assuming that $k_B T_{Fe} n_e \ll (\hbar^2/4m_e)\nabla^2 n_e$, where k_B is the Boltzmann constant, T_{Fe} is the Fermi electron temperature, $\hbar$ is the Planck constant divided by 2π, m_e is the electron mass, and n_e is the electron number density. The latter is then obtained from the inertialess electron momentum equation

$$e\nabla\phi + \frac{\hbar^2}{2m_e}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) = 0, \tag{1}$$

which dictates that the electrostatic force is balanced by the quantum force (minus the gradient of the Bohm potential). Here e is the magnitude of the electron charge and ϕ is the electrostatic potential. The electrons are coupled with ions through the space charge electric field $(-\nabla\phi)$.

The ion dynamics is governed by the ion continuity and ion momentum equations

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \nabla\cdot(n_{\mathrm{i}}\mathbf{v}_{\mathrm{i}}) = 0, \tag{2}$$

and

$$m_{\mathrm{i}}\left[\frac{\partial \mathbf{v}_{\mathrm{i}}}{\partial t} + (\mathbf{v}_{\mathrm{i}}\cdot\nabla)\mathbf{v}_{\mathrm{i}}\right] = -e\nabla\phi, \tag{3}$$

where n_{i} is the ion number density, $\mathbf{v}_{\mathrm{i}}$ is the ion fluid velocity perturbation, and m_{i} is the ion mass. The system of equations (1)–(3) is closed by the Poisson equation

$$\nabla^2\phi = 4\pi e(n_{\mathrm{e}} - n_{\mathrm{i}}). \tag{4}$$

Let us now assume a slab geometry with spatial variations only along the x-axis, so that $\nabla = \widehat{\mathbf{x}}\partial/\partial x$ and $\mathbf{v}_{\mathrm{i}} = \widehat{\mathbf{x}}u_{\mathrm{i}}$, where $\widehat{\mathbf{x}}$ is the unit vector along the x-axis in a Cartesian coordinate system. Furthermore, we look for stationary nonlinear ion wave structures moving with a constant speed u_0. Hence, all unknowns depend only on the variable $\xi = x - u_0 t$ [16, 17]. Defining $\sqrt{n_{\mathrm{e}}} \equiv \psi$, we note that (1) can be integrated once to obtain

$$\frac{\hbar^2}{2m_{\mathrm{e}}}\frac{\partial^2\psi}{\partial\xi^2} + e\phi\psi = 0, \tag{5}$$

where for localized disturbances we have used the boundary conditions [16, 17] $\phi = 0$ and $\partial/\partial\xi = 0$ at $|\xi| = \infty$.

Equations (2) and (3) can be integrated once with the boundary conditions $n_{\mathrm{i}} = n_0$ and $u_{\mathrm{i}} = 0$ at $\xi = |\infty|$, and the results can be combined to give

$$n_{\mathrm{i}} = \frac{n_0 u_0}{\sqrt{u_0^2 - 2e\phi/m_{\mathrm{i}}}}. \tag{6}$$

Inserting (6) into (4) we obtain

$$\frac{\partial^2\phi}{\partial\xi^2} = 4\pi e\left(\psi^2 - \frac{n_0 u_0}{\sqrt{u_0^2 - 2e\phi/m_{\mathrm{i}}}}\right). \tag{7}$$

Equations (5) and (7) are the desired equations for the study of nonlinear ion waves in dense quantum plasmas.

It is convenient to introduce dimensionless quantities (see below (9)) into (5) and (7), and rewrite them as

$$\frac{\partial^2\Psi}{\partial X^2} + \frac{\Phi\Psi}{2} = 0, \tag{8}$$

and

$$\frac{\partial^2\Phi}{\partial X^2} - \Psi^2 + \frac{M}{\sqrt{M - 2\Phi}} = 0, \tag{9}$$

where we have normalized the space variable as $X = k_q\xi$, the electron wave function as $\Psi = \sqrt{n_0}\psi$, and the potential as $\Phi = e\phi/m_{\mathrm{i}}c_q^2$. Here $c_q = \omega_{\mathrm{pi}}/k_q$ is the quantum ion wave speed and $k_q = (2m_{\mathrm{e}}\omega_{\mathrm{pe}}/\hbar)^{1/2}$ is the quantum wavenumber. The quantum 'Mach number' is defined as $M = u_0/c_q$. Furthermore, $\omega_{\mathrm{pe}} = (4\pi n_0 e^2/m_{\mathrm{e}})^{1/2}$ and

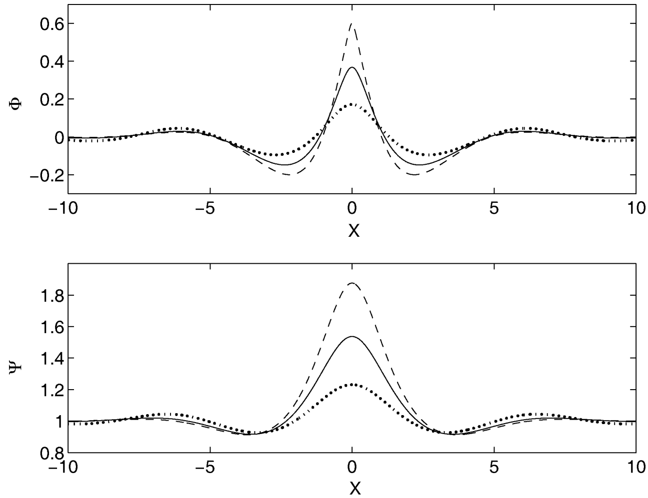

Figure 1. The profiles of the potential Φ (top panel) and the electron density Ψ^2 (bottom panel) as a function of X, for different values of the Mach number: $M = 1.1$ (dashed curves), $M = 0.9$ (solid curves), and $M = 0.75$ (dotted curves).

$\omega_{\rm pi} = (4\pi n_0 e^2/m_{\rm i})$ are the electron and ion plasma frequencies, respectively, where n_0 is the equilibrium electron number density.

We note that the coupled Equations (8) and (9) admit a conserved quantity

$$H = -2\left(\frac{\partial \Psi}{\partial X}\right)^2 + \frac{1}{2}\left(\frac{\partial \Phi}{\partial X}\right)^2 - \Phi\Psi^2 - M(\sqrt{M^2 - 2\Phi} - M) = 0, \tag{10}$$

where we have used the boundary conditions $\Phi = \partial\Phi/\partial X = \partial\Psi/\partial X = 0$, and $\Psi = 1$ at $|X| = \infty$. For a symmetric solitary ion wave structure, we can assume that $\Phi = \Phi_{\rm max}$ and $\Psi = \Psi_{\rm max}$, as well as $\partial\Psi/\partial X = \partial\Phi/\partial X = 0$ at $X = 0$. Hence, at $X = 0$, (10) yields

$$\Phi_{\rm max}\Psi_{\rm max}^2 + M(\sqrt{M^2 - 2\Phi_{\rm max}} - M) = 0. \tag{11}$$

In the wave-breaking limit, where $M = (2\Phi_{\rm max})^{1/2}$, we find that $-\Phi_{\rm max}\Psi_{\rm max}^2 + 2\Phi_{\rm max} = 0$, or $\Psi_{\rm max} = 2$. Accordingly, the electron density will locally rise to twice the background density at wave breaking.

We have numerically solved (8) and (9), and the resulting profiles of the electrostatic potential and electron number densities for different values of M are displayed in Fig. 1. We see that both the electrostatic potential and the electron density have localized and strongly peaked maxima and an oscillatory tail. The latter is in sharp contrast to the classical (non-quantum) case, where the ion acoustic solitary waves have a monotonic profile, which in the small-amplitude limit, where the system is governed by the Korteweg–de Vries equation [18], assumes a secant hyperbolicus shape [16]. We observe from Fig. 1 that $M = 1.1$ is close to the wave-breaking limit above which there do not exist solitary wave solutions. Our numerical investigation also suggests that there is a lower limit of M (slightly lower than 0.75), below which the solitary wave solution vanishes. In the numerical solution of (8) and (9), we used a centered second-order difference scheme to approximate the second derivatives. At the boundaries at $X = \pm 20$ we used $\Phi = 0$ and $\Psi = 1$. The resulting nonlinear system of equations was solved with Newton's method.

To summarize, we have studied fully nonlinear ion waves in an unmagnetized quantum plasma, in which the electrostatic and quantum forces acting on the electrons are in balance. The solitary ion waves arise due to a balance between nonlinearities (coming from the divergence of the ion flux and ion advection) and the dispersion originating from the quantum electron tunneling effect. The associated electrostatic potential and density profiles have localized large-amplitude maxima, and smaller amplitude minima on each side of the maximum. The solitary waves exist in a limited velocity interval around the quantum Mach number $M = 1$. It is expected that the ion solitary wave, as found here, may be observed at nanoscales in dense plasmas.

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References

[1] Markowich, P. A., Ringhofer, C. A. and Schmeister, C. 1990 *Semiconductor Equations*. Berlin: Springer.

[2] Berggren, K. F. and Ji, Z. L. 1996 *Chaos* **6**, 543.

[3] Shpatakovskaya, G. V. 2006 *JETP* **102**, 466.

[4] Kluksdahl, N. C., Kriman, A. M., Ferry, D. K. and Ringhofer, C. 1989 *Phys. Rev.* B **39**, 7720.

[5] Lau, Y. Y., Chernin, D., Colombant, D. G. and Ho, P.-T. 1991 *Phys. Rev. Lett.* **66**, 1446.
Ang, L. K., Kwan, J. T. and Lau, Y. Y. 2003 *Phys. Rev. Lett.* **91**, 208303.
Ang, L. K. and Zhang, P. 2007 *Phys. Rev. Lett.* **98**, 164802.

[6] Driskill-Smith, A. A. G., Hasko, D. G. and Ahmed, H. 1999 *Appl. Phys. Lett.* **75**, 2845.

[7] Marklund, M. and Shukla, P. K. 2006 *Rev. Mod. Phys.* **78**, 591.
Glenzer, S. H. et al. 2007 *Phys. Rev. Lett.* **98**, 065002.
Malkin, V. M., Fisch, N. J. and Wurtle, S. J. 2007 *Phys. Rev.* E **75**, 026404.

[8] Chabrier, G., Douchin, F. and Potekhin, Y. 2002 *Condes. Matter* **14**, 9133.
Chabrier, G., Saumon, D. and Potekhin, A. Y. 2006 *J. Phys. A: Math. Gen.* **39**, 4411.

[9] Harding, A. K. and Lai, D. 2006 *Rep. Prog. Phys.* **69**, 2631.

[10] van Horn, H. M. 1991 *Science* **252**, 384.

[11] Gardner, C. L. and Ringhofer, C. 1996 *Phys. Rev.* E **53**, 157.
Ancona, M. G. and Iafrate, G. 1989 *J. Phys. Rev.* B **39**, 9536.
Manfredi, G. and Haas, F. 2001 *Phys. Rev.* B **64**, 075316.

[12] Pines, D. J. 1961 *Nucl. Energy C: Plasma Phys.* **2**, 5.

[13] Haas, F., Garcia, L. G., Goedert, J. and Manfredi, G. 2003 *Phys. Plasmas* **10**, 3858.

[14] Manfredi, G. 2005 *Fields Inst. Comm.* **46**, 263.

[15] Shukla, P. K. and Eliasson, B. 2006 *Phys. Rev. Lett.* **96**, 245001.
Shukla, P. K. and Eliasson, B. 2007 *Phys. Rev. Lett.* **99**, 096401.
Shaikh, D. and Shukla, P. K. 2007 *Phys. Rev. Lett.* **99**, 125002.

[16] Sagdeev, R. Z. 1966 *Reviews of Plasma Physics* **4** (ed. M. A. Leontovich). New York: Consultants Bureau, p. 23.
Sagdeev, R. Z. 1979 *Rev. Mod. Phys.* **51**, 11.

[17] Schamel, H., Yu, M. Y. and Shukla, P. K. 1977 *Phys. Fluids* **19**, 1286.
Yu, M. Y. and Shukla, P. K. 1977 *Plasma Phys.* **19**, 889.
Shukla, P. K. and Stenflo, L. 1984 *Phys. Rev.* A **30**, 2110.
Shukla, P. K. and Yu, M. Y. 1078 *J. Math. Phys.* **19**, 2506.

[18] Washimi, H. and Taniuti, T. 1966 *Phys. Rev. Lett.* **17**, 996.