Structuring Spectra in Electroacoustic Music

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This article describes the ways in which the spectra of electroacoustic music compositions can be structured coherently. It begins by describing the straightjacket that composers and listeners are constrained by when using the concept of 'source bonding' and how this needs to be discarded for effective listening. It then describes the concept of spectral merging and how ideas of musical timbre are formed, and finally discusses the many ways that spectra can be structured with both harmonic and inharmonic components. Examples are given from the author's own music and other well-known works in the literature of electroacoustic music.

1. THE TYRANNY OF SOURCE BONDING

Until the development of recording, the only way that people had to hear music was through the physical actions of a performer. All sounds that they heard were produced by musical instruments or the human voice, and it made perfect sense to associate specific sounds with the objects and actions that produced them. Recorded music changed these perspectives in significant ways. Now people had to imagine what was producing the sounds, but of course these were the same traditional instruments that they were hearing before, except that there may have been some ambiguities or confusion about some of those objects and actions. Also, the introduction of percussion music as a thing in itself, which did not happen until the twentieth century, brought to mind new sounds and images.

Electroacoustic music changed these perspectives much further. Since the earliest music was made by altering recorded sounds so that they did not have the character that they started with, confusion was more prevalent. Nevertheless, people still underwent the same process of grappling with this as they had with other recorded music, by assuming that the sounds arose as a result of specific actions or instruments, even as the sounds themselves began to include a significant amount of materials that had not been associated with music in the past, such as noises and natural sounds.

This issue was recognised by early composers of this genre, and one of the first to write about it was Pierre Schaeffer ([1966] 1977) in his book *Traité des objets musicaux*. He felt that identifying the source was not relevant, and he recognised that new types of listening were called for. He described several 'listening modes' that we could use, which included detecting the sound (listening), deciphering its character (perceiving), then describing it (hearing), and finally, comparing it to other sounds, abstracting ideas from it, and working out its meaning (comprehending).

Denis Smalley (1986) expanded on these ideas considerably in his article 'Spectro-Morphology and Structuring Processes'. Even though it does not arise from actions of a performer, Smalley nevertheless feels that we can assign the types of gestural movements and sound shapes to the music that we hear in much the same way, even though these may have to be created in our imagination. We do this intuitively. He coined the term 'source bonding' to describe our processing of associating sounds with the things that produced them, even though this has to be inferred in electroacoustic music. He went on to describe several different types of musical motions and textures, and applied Schaeffer's notion of 'reduced listening', in which we block out some aspects of a sound in order to concentrate on its spectral components.

In describing spectra, Smalley notes that there is a continuum between harmonic spectra, inharmonic spectra and noise. There are many types of noise. *Granular noise* is associated with many kinds of phenomena such as wind, static and friction. *Saturate noise* can arise from spectral compression, where a frequency area becomes over saturated and we can no longer hear the spectra. Noise can also be narrowly filtered to create the sensation of pitch. Inharmonic spectra can be ambiguous because it cannot be resolved into a note. An inharmonic spectrum can move into a harmonic one or it can move into noise.

Smalley says that we need a vocabulary to describe the *occupancy of spectral space*, and he lists root, centre and canopy as describing the 'frame' of spectral space. These are further qualified by characteristics such as 'emptiness-plenitude, diffuseness-concentration, streams-interstices' and 'overlap-crossover'. *Spectral densities* can therefore by 'filled, packed-compressed, opaque, translucent, transparent,' or 'empty'.

Smalley also made several observations about physical space or sound location, which is another property that has become more prominent because of electroacoustic music.

In all these processes, both Schaeffer and Smalley felt that true understanding of music required developing

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a greater level of abstraction that was removed from the source bonding that gave rise to it. Smalley is undoubtedly right that we ultimately make sense out of the music in similar ways to how we comprehend instrumental and other music, and he is also correct that we need to use many of the same mechanisms that we use to comprehend instrumental music in doing the same with electroacoustic music.

2. SPECTRAL MERGING

It is a fundamental fact of musical perception that the harmonic partials of a note fuse into a single pitch and are perceived as the tone colour of the sound. It is remarkable that we do not have a term to describe this, so I propose that we call this *spectral merging* or *timbral merging*. All timbre identification depends on understanding this quality, and all timbre experimentation depends on exploiting that quality in different ways.

Spectral *merging* is not to be confused with spectral *fusion*, which is the structuring of the components so that they are clearly identifiable as a particular object, such as an instrument or voice. This term is usually employed to describe the 'morphing' of one sound with a distinct timbral quality into another. Examples are in John Chowning's *Stria* (1977) and Michael McNabb's *Dreamsong* (1978).

The harmonic partials of a note fuse into a single pitch and are perceived as the tone colour of the sound. One consequence of this is that *just intonation*, in which the individual notes are tuned by whole number ratios, which amount to harmonic partials, has an inherent conflict with this process. If the frequencies of the notes in a chord are accurately tuned in this manner, then they could fuse into a single tone. This does not happen in live performance because in order for this to happen, the instruments would need to be played with some significant deviations and mistunings, although it could definitely happen at moments.[1]

Analyses of musical instruments have shown that the timbres of almost all the sounds we hear are quite complex. First, they may contain numerous different partials, the amplitudes of which may constantly vary over the course of the duration of a sound. There may be *transient* elements that have a very short duration, and these can include both harmonic and inharmonic components. In sound mixtures, where two or more sounds are blended together, some of the components of one sound may interact with the other, changing its timbre to something different.

It was not until the advent of electroacoustic music, particularly computer music synthesis, where composers had complete control over all aspects of the sound, that the concept of timbre could be fully explored. The necessity of having to specify everything in itself encouraged experimentation with these qualities. Numerous composers have written 'timbre studies' of different kinds.

The development of electronic music synthesisers from the 1960s onward gave rise to instruments that allowed composers to explore timbre and other musical characteristics in great detail. However, synthesiser manufacturers soon became preoccupied with developing keyboard instruments that reproduce the timbres of orchestral and other instruments, and thus derailing any experimentation gained from users programming their own sounds, although there were a few exceptions, such as the Korg Wavestation.

Instead of merely trying to imitate instrumental timbres or even shift between them, some composers have tried to find completely new sounds. In part, that exercise reveals that there may not be so many new timbres as there are old ones in new contexts. That is partly because the composers focus on qualities in the way that traditional instrumentalists produce them, without exceeding those limitations or imagining new ones.[2]

Spectral merging itself is a property that can be explored in detail in musical compositions. I myself have done this in my composition *Emergence* (2012). In this work, the harmonics of individual tones are introduced one at a time so that they do not merge into a single timbre until several of them are sounding, and individual harmonics pervade the surface of the music. Another work which exploits this is my Improvisation No. 2 (1995). This work is based entirely on the overtones of chords, which are all controlled individually. At about 6′30″ into the piece, a group of tones that have been undergoing long glissandos suddenly converge on the harmonics of A_3, resulting in the sudden emergence of that pitch, after which they all continue to different destinations.

It is not a requirement that electroacoustic music imitate the sounds of instruments, voices, or nature, and in fact we can create entirely new categories of sounds. We have to understand that we may need a new vocabulary to describe these, and that they may bear little resemblance to traditional sounds.

While this is true, I do not think it matters much how listeners hear many of these things. Different

[1]Things like this happen occasionally in live performances, where, if you listen carefully to a recording, sometimes a pitch that is not present in the score suddenly appears. It is also the case that single instruments can take on the timbre of a different instrument for single notes in a complex polyphonic passage.

[2]Some very interesting ideas about tone colours are explored in Wayne Slawson's book *Sound Color* (Slawson 1985). These mostly have to do with resonances and perception, but these ideas are definitely worth further exploration.

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images will be conjured up by different people. As long as each listener engages with the music, they can imagine whatever they want in interpreting it.

The problem with source bonding is that is prevents us from hearing sounds in terms of their purely musical characteristics – pitch, amplitude, timbre and, more importantly, *changes* in these qualities.

3. STRUCTURING SPECTRA

In this article, I will describe several ideas that can be employed to structure spectra in electroacoustic music. I have tried many of them myself, but it is clear that there is much more that can be done with them. My focus is on creating a *coherent relationship* between the different frequencies of a complex tone. I will also mention my own compositions that have addressed some of these issues.

In traditional acoustics, harmonic partials are integral multiples of the fundamental frequency, which are produced by instruments that vibrate in complex ways. Overtones are also harmonic partials, albeit the numbering is $n+1$, since the first overtone is the second partial and so on, but all are 'over' the fundamental. I often use the term *fundamental* to describe the lowest tone, or the tone from which the sound originates, which in many contexts is *not* harmonically related to the fundamental. I use the term *harmonic* partials when describing integral multiples of the fundamental, but I use the term *overtone* to describe any sound components that are *above* the fundamental. *Undertones*, similarly, are components which are *below* the fundamental. *Partials* are either harmonic or inharmonic, and sometimes I simply use the term *component* for those tones.

Since all our considerations involve producing frequencies of all kinds, some audible and some not, we also need to have an idea of the *usable range of frequencies* in music. People can perceive tones from about 20 to 25 Hz at the low end; below this limit, the vibrations are heard individually and do not merge into a tone. The upper limit varies with the individual. Young people can hear frequencies up to the range of about 18 KHz, but as people age, they lose their sensitivity to high frequencies, and the upper limit is reduced to about 12 KHz. This is equivalent to somewhat less than G_9,[3] five octaves and a fifth above middle C, but this is a useful upper limit for high partials of tones.

[3]It is customary to notate the octave position of notes in relation to the piano keyboard. The first C is thus indicated as C_1, even though the piano has notes below this, so they are referred to as being in the '0th' octave. The highest note is C_8.

4. OVERTONE MUSIC

Since harmonic partials are individual frequencies above a fundamental, one way of dealing with them is to use them as a source for introducing melodies and harmonies on another level. It goes without saying that such processes may introduce constantly shifting or diffuse timbres that never last long enough to be compared to more traditional instruments of voices.

Figure 1 shows closest pitches to the first 16 partials of C_2.

Any such process involves a consideration of the individual pitches that each of the overtones creates. In order to explain this, I first want to explain the *octave point pitch-class* or '8ve.pc' method of notating pitches,[4] which is used in the Music*n* synthesis programs. The pitch of middle C is near the frequency of 2^8 or 256, and the frequencies of the tempered scale can be generated by raising 2 to the power of the octave plus n/12 where n is the semitone above C that you want. The result is then multiplied by 1.012975, which brings the A above middle C to 440 Hz, which is the international standard. (Some European orchestras use a slightly higher frequency such as 444 Hz.)

In 8ve.pc, the pitch of middle C is 8.00 and each successive semitone is raised by .01. Thus A440 would be 8.09, C above middle C would be 9.00, and the value 8.12 would also work. The lowest note on the piano would be 4.09, and the highest note 12.00.

Fractions of semitones are possible, such that 8.005 would denote a quarter tone between C and C♯. The main difference between 8ve.pc and the usual notation is that the octaves are numbered 4 higher, but this has the advantage that it provides a notation for subsonic and supersonic frequencies. This notation also works for intervals, where 1.00 would represent an octave and 2.07 would mean two octaves and a fifth.

In considering the pitches of overtones above a fundamental, Table 1 shows, in 8ve.pc notation, the interval above the fundamental in equal temperament for the first 32 harmonic partials, spanning five octaves above the fundamental.

As this shows the perfect fifths (3, 6, 12, 24 and so on), major seconds (ninth partial) and minor third (nineteenth partial) are 2%, 4% and 2% apart from equal-tempered pitches. The major thirds (fifth, tenth, 14% flat), minor sevenths (seventh, fourteenth, 31% flat), major seventh (fifteenth partial, 12% flat) and minor sixth (twenty-seventh partial, 6% sharp) are also within one-tenth of a semitone of their tempered counterparts. The one that is most out of tune[5] is the

[4]This notation was invented by J. K. Randall when he began working with Max Mathews's Music4 program, later expanded to Music4B by Godfrey Winham and myself.

[5]In this context, being 'out of tune' means with respect to equal temperament. In other contexts, people often are comparing the intonation with natural harmonics.

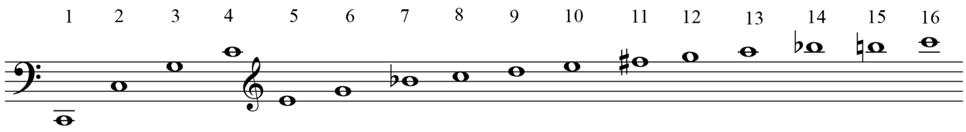

Figure 1. The closest pitches to the first 16 partials of C_2.

Table 1. Interval above the fundamental for the first 32 harmonic partials

Partial	Interval	Partial	Interval
1	.00	17	4.0105
2	1.00	18	4.0204
3	1.0702	19	4.0298
4	2.00	20	4.0386
5	2.0386	21	4.0471
6	2.0702	22	4.0551
7	2.0969	23	4.0628
8	3.00	24	4.0702
9	3.0204	25	4.0773
10	3.0386	26	4.0841
11	3.0551	27	4.0906
12	3.0702	28	4.0969
13	3.0841	29	4.1030
14	3.0969	30	4.1088
15	3.1088	31	4.1145
16	4.00	32	5.00

tritone (six semitones above the note), the eleventh partial being about halfway above the perfect fourth (49% flat). This is one of the reasons why this note was so out of tune with medieval instruments, and the interval was referred to as the 'devil in music'.

I have shown harmonic partials up to the thirty-second. Partials above this certainly exist, but they are so close together that their pitches cannot be perceived independently unless isolated from their immediate neighbours or amplified. While some composers have employed these components, it is my feeling that they are not useful for overtone melodies or other operations like this.

Melodies and harmonies built from harmonic partials have to accept these problems as inherent. But if we accept that, we can see that this technique could work reasonably well. Harmonically, the first six partials state a major triad, and the first eight partials a dominant seventh chord. This fact has been cited in harmony textbooks as a 'justification' of using the major triad as a basis for tonal music, although it has also been pointed out that the overtone series is horrendously dissonant with the minor triad, which is equally important in tonal music. This may also explain how the dominant seventh can be accepted so readily. Melodically, all notes of the chromatic scale are available above the sixteenth partial, and many of these have lower octaves. It is thus easy to see how melodies and harmonies can be created out of overtones.

This is a process that I have used extensively in my compositions such as *Improvisation on the Overtone Series* (1977) and Harmonic Fantasies (2018). The first work is created entirely from overtone melodies. Written in 1976–7 when I had access only to a primitive computer that used punched cards and a digital-to-analog converter that worked at 20 KHz (10 KHz stereo), it uses only the first 16 partials. Timbre Study No. 5 (1991) is similar but uses the first 24 partials. In these works, the fundamentals only appear sporadically and sometimes not at all.

My Harmonic Fantasies 3 and 4 (2018) use a more complicated system of overtone melodies to create patterns that extend the contextual harmonies. All trichords, tetrachords, pentachords and hexachords used in the pieces are stated with overtone sequences of the first 24 partials that begin with the sixteenth partial and then introduce each of the notes in a sequence, followed by the doublings in lower octaves. When possible, a transposition of the harmony is stated next, and then the remaining partials. The harmony 035, for example, with which the piece begins, is stated by the sequence 16, 19, 21, 8, 4, 2, 1 (the basic chord), followed by 17, 20, 22, 10, 55, 5 (T_1) and then 24, 23, 18, 15, 14, 13, 12, 9, 7, 6, 3. The pentachord 02345 employs the sequence 16, 18, 19, 20, 21, 8, 9, 10, 4, 5, 2, 1 (the basic chord), followed by 24, 23, 22, 17, 15, 14, 13, 12, 11, 7, 6 and 3 (the remaining partials). All harmonies in the work use different sequences such as these. In these pieces the fundamental frequencies are always heard, but we also hear the overtones individually, certainly at the onset of the tones, and the timbres are constantly shifting.

Music created by manipulating individual overtones may occasionally veer into sounding like instrumental or vocal sounds, but they usually do not resemble them, and instead create the impression of a continuously shifting palette.

5. FILTER MUSIC

Anyone who has had the opportunity to work with an analog synthesiser soon realises that the filter is one of the most useful devices on the instrument. If you narrow the band, you can move up and down the overtone series and hear each overtone individually.

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Experimentation uncovers a wide range of timbral effects.

This was the strategy I employed for my composition *Mosaic* (2001), and I once had the great pleasure of playing this work on a concert attended by Robert Moog, who designed the synthesiser that I first used for this purpose.

A band pass filter can be given a very narrow bandwidth to define a resonance of a tone, and if the fundamental is low enough, harmonic partials in the frequency area of the filter can be brought out as individual 'tones' that are part of the overall sound. The band can also be expanded to include many partials, and it can be combined with additional filters to add additional formants to the sound. Noise can also be filtered to create the impression of pitched noise.

To be effective, filter bands should mostly be at least four octaves above the fundamental, where overtones close to any note in the scale exist. Thus, there are two levels of pitch organisation in this context: one created by the low fundamentals and one created among the resonances of the upper partials.

I employed this idea in my Timbre Study No. 6 (1997). In this work, all the fundamental frequencies are between $A\sharp_0$ and F_4, and all the filter frequencies are between C_5 and B_7 (the filters are always above the sixteenth partial of the fundamentals). The filters are both fixed and variable, so at times they simply open up creating variable melodic filters among the overtones, and at other times they make a glissando from one frequency to another, similar to vowel diphthongs, although these timbres rarely resemble vowels. There are numerous works in the literature that use filtering in many different ways.

6. INHARMONIC SPECTRA

Inharmonic spectra are unusual in the context of musical instruments, because the main ones that produce them are bells and other metallic object that vibrate in complex ways due to their irregular shape, although inharmonic spectra may exist in some natural sounds. The difficulty in dealing with inharmonic spectra is that we need a way of organising elements that do not have a simple relationship to each other. In fact, there are several processes that can be used for this, and we will consider several of them here.

One of the problems with inharmonic sounds is that we customarily hear them associated mainly with percussion instruments, so that they have an instantaneous attack, die away quickly and do not give us much chance to concentrate on the components. The exceptions are bells, which can have a relatively longer decay times with components decaying at different rates, but these still do not have envelopes that increase any components. One advantage of using these sounds in electroacoustic music is that any kind of envelope can be used, allowing the components to be brought into the foreground in many ways.

6.1. Nonlinear ratios

Nonlinear ratios occur when the relationship between different components is a fraction or irrational number. There are two ways in which these spectra can be generated: using a fractional number as a multiplier for successive partials and frequency modulation (fm) synthesis.

6.1.1. Fractional partials

If we consider the harmonic series as 1, representing the fundamental frequency, multiplied by a series of integers from 1 up to however high we wish to go, that process produces harmonic partials. If we think of replacing the octave by something other than the 2:1 ratio, we get an inharmonic series. If we use a ratio such as 1.001 (which would produce 2.002, 3.003, 4.004, 5.005, 6.006, . . . , 16.016), we would get a series that is technically out of tune, but in practice would simply result in the same fundamental but with beating.

If the numbers are a bit further out of tune, for example, 1.09, we would get 1.09, 2.18, 3.27, 4.36, 5.45, 6.54, 7.63, 8.72, 9.81 and 10.9 for the first 10 partials. This would expand the tenth partial from 10 to 10.9. If we use a ratio lower than 1, we would compress the series. For example, 0.91 would yield 1, 1.82, 2.73, 3.64, 4.55, 5.46, 6.37, 7.28, 8.19 and 9.1. We have to be careful that the fraction is not something simple, like .5, because that would then yield a series 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and so on, which results in the spectra of a tone an octave below the original frequency, missing the fundamental. If that fundamental was in the subaudio range, however, we would not have this problem, but if all the frequencies were very low, the sound would be very muddled.

6.1.2. Frequency modulation synthesis

Frequency modulation (FM) synthesis produces a similar range of frequencies as in our immediately preceding examples. In FM synthesis, a carrier signal c is modulated by a modulator signal m producing sidebands above and below the carrier at intervals of the modulator. While the lower sidebands result in negative frequency values, these amount to positive frequencies 180 degrees out of phase, so they are audible as well, although they sometimes cancel out some of the amplitude of positive frequencies. FM synthesis was developed by John Chowning in the 1970s, and the process was patented by Yamaha and

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implemented in electronic instruments such as the DX7 that they introduced in the 1980s.[6]

When simple integers are used as c:m ratios, harmonic spectra are generated. The modulator determines the partials in the series, and the carrier determines the starting positions of the partials in their orders. A modulator of 1 produces all partials, starting from the carrier. Modulators of 2 and 4 produce spectra of only odd-numbered partials, as in square and triangle waves. Other modulators produce spectra that are clustered around multiples of the modulator without including the modulator. For example, 1:3 produces 2 and 4, 5 and 7, 8 and 10, 11 and 13, and so on, or upper and lower neighbours of multiples of 3; 1:5 produces 4 and 6, 9 and 11, 14 and 16, 19 and 21, 24 and 26, and so on, or upper and lower neighbours of multiples of 5. These spectra are not produced by acoustic resonators such as those in musical instruments.

When spectra contain a number of low harmonic partials that are fairly close together, they produce spectra that have a simple timbre. If the components are widely spaced and do not contain consecutive elements of a harmonic series, they do not merge into a timbre but are perceived as a cluster of separate tones. Such clusters can have a quality of 'timbre' but lack a fundamental frequency. For example, a ratio of 9:11 produces harmonics 2, 9, 13, 20, 31, 24, 35, 42, 46, 53 and 64 in the first five orders; one of 7:11 produces 4, 11, 15, 18, 26, 29, 37, 40, 48 51 and 62. While they are strictly speaking harmonic spectra, they often sound more like inharmonic spectra.

When fractions are used, something resembling real inharmonic spectra are produced. A ratio of 1:1.41 (roughly, the square root of 2), the resulting frequencies are in the ratio of .41, 1, 2.41, 1.82, 3.23, 3.82, 4.64, 5.23, 6.05, 6.64 and 8.05. A ratio of 1:1.05 produces .05, 1, 1.1, 2.05, 2.15, 3.1, 3.2, 5.2 and 6.25. These both strongly resemble inharmonic clusters.

Thus, FM synthesis has a simple process for producing a wide variety of inharmonic spectra, which is why the musical instruments based on frequency modulation can imitate all kinds of percussion instruments as well as many imaginary ones. By developing fairly realistic imitations of acoustic instruments in their presets, Yamaha's have been the most successful of any electronic instruments manufactured.

The real problem with FM synthesis is that the spectra are always generated with lots of partials in one sound, and the only way to change them is to vary the modulation index and the spectral envelope, which changes them in a flexible manner but which rapidly becomes stale, or to construct more elaborate networks of modulation that can change the frequencies in other ways. It is only when we generate the partials separately that we can control their amplitudes in any way possible.

[6]Yamaha has also leased the technology to other companies, so they are not the only manufacturer of such instruments.

6.2. Pitch compression

In pitch compression (or expansion), all the (equal-tempered) pitches of a musical passage are squeezed into a different interval, and then these frequencies are used as overtones. It is important to think of these as *pitches* rather than as *frequencies*, because that is how we intuitively hear sounds, and this gives us a better perception of the distances between them in a musical context. If all the notes in the passage are within a single octave, then they are compressed into twelfths of that interval. If they occupy two or three octaves, then they would be compressed into twenty-fourths or thirty-sixths of that interval. It is best to list these in 8ve.pc form, because that notation is easy to understand. Also, it is easy to compute frequencies, but in order to understand what we have, we need to consider the pitches that they form, always keeping in mind how they relate to the intervals of the overtone series.

For example, let us suppose we want to divide the interval of a perfect fifth or .07 into 12 steps, and the pitch we are using for this is D above middle C, or 8.02. The 12 pitches we obtain would be 8.02, 8.0258, 8.03167, 8.0375, 8.0433, 8.0492, 8.055, 8.0608, 8.0667, 8.0725 and 8.0842; the next value would be 8.09, completing the span. If we divide it into 24 intervals, we would get 8.02, 8.0229, 8.0258, 8.0288, 8.0317, 8.0346 and so on. These may appear to be very close, but we can easily hear the difference between them.

If we want to compress 12 intervals into an octave plus a fifth, or a twelfth, or 1.07 in 8ve.pc form, and the starting note were 9.03, the series would be 9.03, 9.0458, 9.06, 9.0775, 9.0933, 9.1092, 10.005, 10.0208, 10.0367, 10.0525, 10.0683, 10.0842 and 10.10, completing the span. This means that the notes of a single octave would be expanded to an octave and a fifth. If the starting pitch were 8.09, then the series would be 8.09, 8.0979, 8.1058, 8.1138, 9.0017, 9.0096, 9.0175, 9.0254, . . . , 10.0163, 10.0242, 10.0321, 10.04, thus compressing the 24 pitches into an octave and a fifth.

These processes give rise to a logical succession of inharmonic partials that are unlike natural sounds. They make the most sense when used with envelopes that sustain the sounds over a long duration, so that the different components can be faded in and out over the course of the durations.

This technique was used in several of my Inharmonic Fantasies. No. 3 (2014) uses the interval of an octave and a fifth or a twelfth, No. 4 (2014) uses a perfect fifth (the two examples described earlier), and No. 5 (2015) two octaves and a fifth. No. 6 (2017) uses the interval of a minor seventh.

6.3. Frequency shifting

Since pitch compression generally squeezes all the harmonics within a given interval, this process does not work well for low pitches, since our ears have a hard time discriminating sine tones in low octaves. A more promising method for low frequencies is *frequency shifting*. This occurs when a constant value is added to the harmonic frequencies of a tone, which preserves the difference between the frequencies but not their ratio, thus creating inharmonic partials.

Frequency shifting has different consequences if the frequencies are shifted up or down. Shifting frequencies up causes the partials to be shifted into an area where they span a smaller interval than the natural harmonics originating from that note, and shifting them down moves them into a wider range. To give examples just using frequencies to clarify this, imagine moving the first 10 partials or middle C or 261 Hz (leaving out the fractional part for simplicity) up by 100 Hz. This would produce 361, 461, 561, 661, 761, 861, 961, 1061, 1161, 1261. The natural partials beginning on 361 would go from 361 to 3610, which is considerably higher than 1261. Shifting partials down causes the partials originally spanning a wider range to start from a lower level. For example, shifting the first 10 partials of 1000 Hz, which would go to 10 KHz down to 100 Hz would produce 1000, 2100, 3100, 4100, …, 10100, whereas the harmonic partials of 100 Hz would only go as high as 1000 Hz. These considerations have far-reaching consequences on the range of usable frequencies when using this technique.

Turning now to an example where we consider the pitches of the frequencies produced by the partials using 8ve.pc form, let us suppose that we take a ratio of 19/24, which amounts to about a major third, equivalent to the difference between the nineteenth and twenty-fourth partials. Remember that the partials follow the harmonic series, so that the pitches would be an octave, octave plus fifth, two octaves and so on higher. If we start from the pitch 5.00 (C_1), the pitches we obtain would be 5.00, 5.101, 6.0433, 6.0906, 7.0071, 7.0372, 7.0628, 7.0852, 7.1049, 8.0027, 8.0188 and 8.0335 for the first 12 components. This means that the twelfth partial, instead of being about 8.07, has been shifted down about 3.65 semitones.

For another example, let us choose the ratio of 15/24, which is approximately a minor sixth, the difference between the fifteenth and twenty-fourth partials. If we start from the pitch 7.02 (D_3), the pitch series we obtain would be 7.0200, 7.1041, 8.0404, 8.0828, 8.1169, 9.0253, 9.0498, 9.0712, 9.0902, 9.1074, 10.0030, 10.0173, 10.0305, 10.0428, 10.0542, 10.0650, 10.0751, 10.0847, 10.0938, 10.1024, 10.1106, 10.1184, 11.0059, 11.0131, 11.0173. The twenty-fourth partial, which in a harmonic series would be at about 11.09, has been shifted down by a bit more than a perfect fifth.

I have used this technique in several of my Inharmonic Fantasies. No. 8 (2018) uses the ratio of 19/24, described earlier, which is about a major third. Other works use the ratios of 11/24 or about a minor ninth and 15/24, about a minor sixth.

6.4. Undertones

A final way of creating inharmonic spectra is by the *undertone series*. This is analogous to the overtone series, but instead of multiplying each successive partial by an integer from 1 to n we divide it by that interval. The series yields the inversion of the same intervals of the overtone series, so if we want to generate audio and not subaudio tones, we need to start from a high note. The first 16 intervals of the undertone series starting from the pitch 10.00 in 8ve.pc form would be roughly 10.00, 9.00, 8.05, 8.00, 7.081, 7.05, 7.023, 7.00, 6.10, 6.081, 6.065, and 6.05, 6.04, 6.02, 6.01 and 6.00.

Figure 2 shows these in musical notation. Note that these are *not* equivalent to natural harmonics or frequencies in just intonation. The lower we go, the closer the elements get until they approach zero at the limit. For this reason, it is not practical to use higher elements of this series. Undertones are not produced by any simple natural process; this is basically an imaginary idea that can mainly be realised by computer synthesis.

In the overtone series, the higher you go, the closer together the partials become, but this is not a problem because the ear resolves them into the perception of the timbre because of spectral merging. With the undertone series, the greater the partials, the lower the frequencies and the closer they become. Frequencies in the low range are very indistinct. For undertones to produce a useful series, the fundamentals have to be very high, and it is practical to think of using fundamentals above the highest note on the piano, in the twelfth and thirteenth octaves in 8ve.pc form. These frequencies are audible, although we do not usually hear them as fundamentals but as extreme upper harmonics. Fundamentals as low as middle C are not practical at all.

If we start with the highest note on the piano, the resulting series in 8ve.pc form would be 12.00, 11.00, 10.05, 10.00, 9.081, 9.05, 9.023, 9.00, 8.10, 8.081, 8.065 and 8.05. These frequencies are all clearly audible, and in fact would be quite distinct.

Undertones can also be compressed to produce a series that does not have such a great span and thus

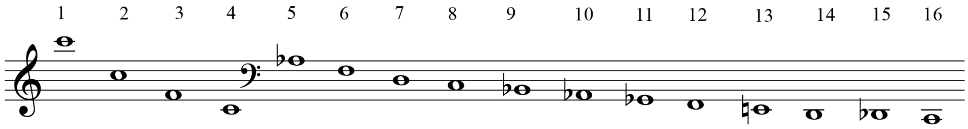

Figure 2. The notes closest to the first 16 undertones of the pitch C_6.

bring the 'upper' partials into a higher range, allowing lower fundamentals to be used. If we compress the undertones by a factor of 11/24 or a little bit less than half, the resulting series for the pitch 10.00 would be 10.00, 9.065, 9.042, 9.030, 9.022, 9.016, 9.012, 9.009, 9.006, 9.004, 9.003, 9.001, 9.00. Notice how they get closer and closer the further they go down, until they would ultimately coalesce at 11/24th of the starting pitch. If we extend the series another octave, the notes would span less than a semitone. However, the differences between all those frequencies are still be clearly audible.

Undertones are perhaps not as useful as the other ideas discussed here, but they remain a wide open area for further exploration. I have used undertones in my composition *Improvisation on the Undertone Series* (2019).

6.5. Irrational numbers

Another way of organising inharmonic spectra are harmonics related by irrational numbers, which are values that cannot be expressed as the ratio of two whole numbers. These include the square roots of prime numbers, the base of natural logarithms, e (approximately 2.718), π (approximately 3.1416), or the *golden mean* (approximately 1.618), or other physical constants.

These have actually been used quite often in electroacoustic music. John Chowning used the golden mean in his composition *Stria*. This work has been analyzed and discussed in the literature. I have used the square roots of both 2 (1.4142) and 3 (1.732) in my composition Fantasy on the Square Roots of Two and Three (2000) I have also used π in my composition *Pi* (2011).

One issue that must be dealt with when using these ideas is that the partial gamut is expanded rather than compressed, so that this imposes limitations on how high the pitches can go (see discussion of the usable range of frequencies in music earlier). For example, in using π this means that the highest usable frequency would be 5.6 KHz. To get 32 partials in this system would mean using no pitch higher than F_3, or 24 partials, $B\flat_4$. Similar computations for the square root of 2 would be middle C for 32 partials and F_5 for 24 partials. Of course, many pieces do not need partials that go as high as these.

6.6. Unrelated frequencies

A final way of organising inharmonic spectra is randomly or with frequencies that are completely unrelated to the kinds of logical sequences discussed in this article. Modeling sounds on the analysis of percussion instruments or other objects might produce something like this. At least, the advantage of the methods described in this article is that there is a coherent relationship between the components.

7. CONCLUSIONS

Structuring spectra in a coherent and intelligible way is clearly a major concern in electroacoustic music. I have thought of everything I could imagine, but I am sure people will find other methods in the future.

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