'Further work and the epilogue', in which the achievements of PMGY, Zhang, Maynard, Tao and the Polymath8 teams are summarised. The chapters also include various statements on topics such as the Siegel-Walfisz theorem, Vaughan's identity, speculations on the generalisation of the Elliott-Halberstam conjecture, gaps between almost primes, and consecutive primes in arithmetic progressions with a fixed common difference. There are also brief mentions of topics not directly related to prime gaps: the large sieve, Kloostermann sums, extension of Artin's primitive root conjecture, modular forms and elliptic curves.

There are nine appendices which include the following topics: Bessel functions of the first kind, the Brun-Titchmarsh inequality for multiplicative functions, exponential sums, and the dispersion method of Linnik. There is also a PGpack mini-manual for a set of functions written to assist the reader to reproduce, and possibly extend, the calculations mentioned in the book. There are 215 items of references. Although there are only a few misprints, I read with alarm that my friend Roger Baker is referred to as the "late Roger Baker". I can reassure readers that Roger is still very much alive and enjoying rude health.

Mathematical research, be it the creation of a theory or the solution of a difficult problem, is a human endeavour. It is mainly the activity of sole individuals, with perhaps one or two collaborators, but there is now a new phenomenon: the massive collaborations over the internet on specific projects with the aim of finding solutions to various famous problems. There is a section in the book explaining what a Polymath project is, and also specific information on the contributors to Polymath8a/b, but it is not a book just telling us who did what and when in the pursuit of the twin-primes conjecture. The reader who really wants to know how Zhang's spectacular theorem was arrived at will need to absorb a large amount of technical detail, and such an individual may feel that there is a lack of overall coherence in the presentation. The book is thus more useful for graduate students in number theory who are already familiar with much of the material, whereas other readers should perhaps first read the more easily digestible book [1] by Vicky Neale. Nevertheless, perhaps even a casual reader may find it an interesting read, and will appreciate that mathematical research is a human activity well worthwhile pursuing.

Reference

1. Vicky Neale, *Closing the gap*, Oxford University Press (2017), reviewed in *Math. Gaz.* 102 (November 2018) p. 561.

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PETER SHIU
353 Fulwood Road,
Sheffield S10 3BQ
e-mail: *p.shiu@yahoo.co.uk*

When least is best: how mathematicians discovered many clever ways to make things as small (or as large) as possible by Paul J. Nahin, pp. 392, £20 (paper), 978-0-69121-876-2, Princeton University Press (2021).

I found this an enjoyable and engaging read. It brings to life the maths of optimisation by portraying its creators and the problems they were trying to solve. Woven into the mix are problems for the reader, with solutions, computer explorations, cultural links to films and books where the ideas are used, and references for further reading. I enjoyed Judith Grabiner's quote that "The derivative was first *used*; it was then *discovered*; it was then *explored and developed*; and it was finally *defined*" and the details of Fermat and Descartes' strained relationship. I was previously unaware of how far Fermat had advanced his ideas, and I have sympathy for Laplace and Lagrange's view that he was the true inventor of calculus—although I realise that he just happened, like them, also to have been French.

"This is a history of mathematics book …" declares the preface, but I think it is more a popular book on mathematics, based around ideas whose origins are placed in historical context. A look at the contents makes this clear: here is a summary of the main content of the seven chapters.

1. Minimization, when derivatives won't work, the AM/GM inequality, and using the computer.
2. Queen Dido's problem, Steiner's solution and problems with it, minimum spanning problems in computational geometry.
3. The Regiomontanus problem (more familiar to some in the context of the optimal viewing distance for a painting, or the optimal distance back from the try line to attempt a rugby conversion), an envelope folding problem, the problem of moving furniture round a corner in a corridor, and examining the maximum height attained by mud thrown off a wheel.
4. The argument between Descartes and Fermat; Snell's Law and the rainbow.
5. A miscellany of problems including Kepler's wine barrel problem, the sizes of mailable packages, ideal basketball shots, Halley's gunnery problem and de l'Hôpital's pulley problem.
6. The calculus of variations, the brachistochrone, the Euler-Lagrange equations and the shape of hanging chains.
7. The Fermat-Steiner problem, least cost paths on graphs, the travelling salesman problem, linear programming, and dynamic programming.

The book is an eclectic mix, containing much more besides. The chapters serve as receptacles for collecting together interesting items, which are not always clearly connected with the chapter headings.

What are the weaknesses, and what things might annoy some readers? I thought there were some gaps: in the preface, the author takes the opportunity to include the problem of getting water from a river whilst minimising the distance, but omits to mention the reflection principle which would greatly simplify things—although he does use it towards the end of the book, on page 289, on a different problem; similarly on page 10, a problem of positioning a bridge is easily solved by removing the bridge and river, solving the problem and then putting them back again; the analysis on page 28 of when to cut a corner by swimming leads after four pages to the conclusion we should cut the corner if our walking speed is less than $\sqrt{2}$ times our swimming speed, which I felt was an easy early observation; a similar problem of walking and swimming on page 135 misses the fact that the distance walked is always non-negative, so the failure of the derivative to help us locate the true solution is due to us pretending we could gain time by walking a negative distance. Nonetheless, the conclusion that we cannot always find the minimum by setting the derivative to zero is sound. The author chooses not to show the geometrical solution to the Regiomontanus problem, but provides a reference. But when he then extends the problem to one of viewing Saturn's rings from the planet's surface, the claim on page 79 that "this formulation of the problem literally demands computer analysis" should have a footnote saying "unless you've already solved it exactly using a geometrical approach"! However, all this quibbling shows that the author has clearly succeeded—he has engaged my interest and got me arguing about and trying to solve these problems.

Who is this book for? The use of integration and partial derivatives in places means that in the UK you would need to be an enthusiastic sixth form maths student or a science or maths undergraduate to enjoy it all. There are things such as differentiation from first principles that UK students will have met earlier, but they may enjoy rediscovering the magic now that they have a deeper knowledge of calculus. And students in other education systems will have different states of knowledge. The author himself locates the background needed as "what a science or engineering major learns in the first year of undergraduate calculus and physics".

So in summary, I would recommend this book for a school library or as a gift to a student, or as background reading for teachers and lecturers. It will doubtless greatly enrich their knowledge and appreciation of maths and its creators.

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FRANCIS HUNT
6 Aneurin Bevan Drive,
Church Village CF38 1GD
e-mail: *Francis.H.Hunt@gmail.com*

Trigonometric delights by Eli Maor, pp. 236, £17.95 (paper), ISBN 978-0-691-20219-8, Princeton University Press (2020).

Like algebra and number theory, trigonometry is a central pillar of mathematics that can be traced all the way back to the ancient Babylonians. Any keen mathematician would struggle to get far without a firm understanding of trigonometry, and while it is a key component of school curricula across the world it has gained a reputation for being a dry and cumbersome subject, bogged down by fiddly calculations and repetitive questions. Eli Maor's book aims to address this issue, cutting through the needless formalism and monotonous exercises to try to endear trigonometry to the school and undergraduate students that are his intended audience. A number of topics have been selected for aesthetic reasons or for their applications to other sciences, and they are all examined through a historical lens in an attempt to make the journey more engaging for readers and thus to allow them to share in the author's "love affair" with the subject.

The first nine chapters are intended to require nothing more than basic algebra and trigonometry, while the final six rely on some familiarity with calculus. Alongside the historical sidebars, most of which give brief biographies of influential figures such as François Viète or Abraham De Moivre, there should be plenty of material to interest readers of all levels. Indeed, there are numerous fascinating historical and mathematical gems, such as a demonstration of the ingenious method used by the Egyptians to calculate difficult multiplications in the Rhind Papyrus, or the story of the perilous 18th century expeditions to triangulate Lapland and Peru in order to determine the shape of the earth. A particular highlight for me comes from the sidebar on Johann Müller, alias Regiomontanus. A problem he posed in 1471 concerns finding the maximum apparent size (or "visual angle") of a perpendicularly suspended rod, and can be reinterpreted as finding the ideal distance to stand from a building in order to see into the first floor window. Anyone with some experience of calculus might be tempted to dive straight into a standard optimisation method, but the algebraic and geometric methods that would have been his only recourse at the time are remarkably elegant and significantly more rewarding. Discovering and solving this problem, as well as subsequently presenting it to pupils of my own, was precisely the sort of delightful experience I anticipated on starting this book, and there were many more scattered throughout its fifteen chapters.

Unfortunately, not every section enthralled me as much as those just mentioned, and I imagine that most readers would experience similar peaks and troughs in their interest. For instance, someone hoping to find some engaging applications of trigonometry might