

THE DISTANCE OF THE VIRGO CLUSTER

David A. Hanes
Anglo-Australian Observatory

Introduction

Until the discovery (Corey & Wilkinson, 1976) of the anisotropy of the cosmic microwave background, the Virgo cluster represented something like a Rosetta Stone for many observational cosmologists: in the absence of a significant peculiar velocity component for the Local Group in the direction of the Virgo cluster, its distance, accurately measured, might reveal the global expansion rate and the Hubble age. Although this simple picture has changed, the distance of the Virgo cluster remains important, partly for a sharper understanding of the properties of rich clusters and the galaxies they contain, but more importantly (for my purposes here) as an interesting distance over which we may test various constructions of the extragalactic distance scale.

Virgo cluster distance moduli in the wide range from $\mu = 30.4$ mag ($d = 12$ Mpc) to $\mu = 31.7$ mag ($d = 22$ Mpc) have been derived, with the best-known advocates of these positions being de Vaucouleurs (1977a) and Sandage & Tammann (1976). The fundamental differences between the approaches and analyses of these scientists have been described by de Vaucouleurs (1980) and Tammann, Sandage & Yahil (1979). In the last five years, various methods with differing degrees of interdependence have led to Virgo cluster distance moduli near 30.8 - 30.9 mag (Hanes, 1979; Mould, Aaronson & Huchra, 1979; Bottinelli et al, 1980). Coupled with reasonable estimates of the local peculiar motion (de Vaucouleurs & Peters 1981), such moduli imply asymptotic values near $H_{\infty} = 100$ km/s/Mpc for the Hubble constant. This value is disquieting to some in that the implied Hubble time may be insufficient to encompass the globular cluster ages, which are independently deduced (Carney 1983). Nevertheless, for reasons which I have described elsewhere (Hanes 1982) it is my belief that this represents the correct state of affairs and that the time-scale arguments will eventually be resolved by other than a 'reflation' of the Virgo cluster distance estimate.

For lack of space, I cannot review here all of the important work which has been done in this area. What I would like to do is first to come to the defence of globular clusters as extragalactic distance indicators (for they have been wrongly criticized and misapplied) and subsequently to remark upon the well-known Sandage & Tammann distance

scale, both in its original and in its present form.

Globular Clusters

In several papers (Hanes 1977a, 1979, 1980a) I have explored and described the use of globular clusters as extragalactic distance indicators. Briefly, as shown in Figure 1(a), one finds the luminosity function (in number of clusters per absolute magnitude interval) in our own Galaxy, perhaps widening the sample to include other Local Group galaxies; this is the fundamental calibrating sample. One then carries out photometry for (say) the populous globular cluster samples in the Virgo cluster elliptical galaxies, deriving functions like that shown in Figure 1(b). Intercomparisons of the functions (either by eye, with the raw empirical data, or analytically, adopting a normal curve as a good representation of the calibrating function) then yield distance estimates: when the functions are superimposed to provide a best fit (with arbitrary

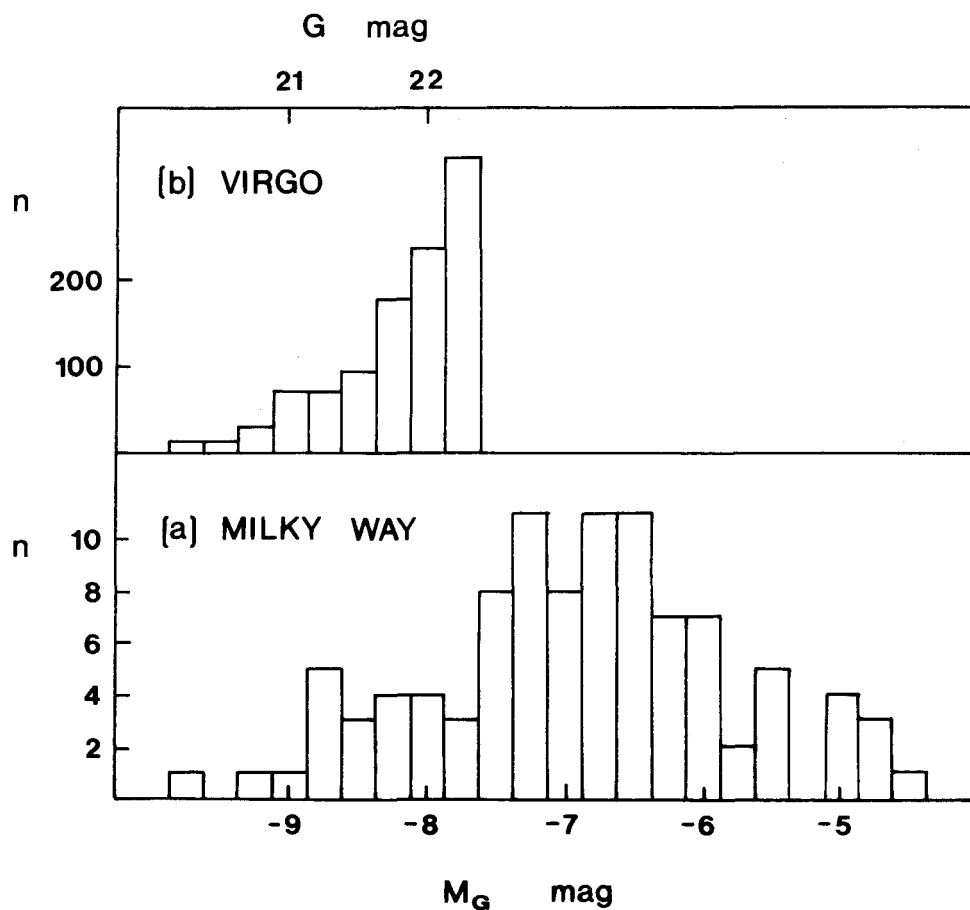


Figure 1: the luminosity function in the G (=103aJ + GG385) system for globular clusters in the Galaxy and in Virgo ellipticals.

population scalings permitted) the difference in abscissae is the apparent distance modulus, just as in main sequence fitting in star clusters. (As it happens, the globular cluster populations inferred are broadly consistent with a direct proportionality to parent galaxy luminosity (Hanes 1977b; Harris & Racine 1979), but the method in no sense depends upon that fact.) Because the method carries us to Virgo in a single stride, and is independent of all of the Population I indicators used by Sandage & Tammann (and others), it may provide an important check on those methods. In fact the Virgo cluster distance moduli derived in this way lie near $\mu = 30.7 \pm 0.3$ mag, very different from the value settled upon by Sandage & Tammann (1976): $\mu = 31.7 \pm 0.08$ mag. However, the conclusions based on globular clusters have been strongly criticized on a number of occasions. Before considering in more detail the power of the approach, let us consider those specific objections.

Sandage & Tammann (1981) have objected to the assumption of a universal globular cluster/luminosity function, stating that "the function is actually fairly well observed only in our own Galaxy, whereas in other galaxies one knows merely its rising branch on the bright side. The basic assumption cannot therefore be tested by observations..." This is not correct. Racine & Shara (1979) have demonstrated that the cluster luminosity function in M31 is photometrically indistinguishable from that in our own Galaxy, from $M_V = -11$ mag right down to $M_V = -4$ mag. Some of the dwarf galaxies in the Local Group (including, importantly, some ellipticals) have been plumbed as deeply, and they too have cluster mean magnitudes and dispersions consistent with the Milky Way sample (but are of course of lower statistical surety because of the numerically smaller samples; see Harris & Racine 1979). I shall return shortly to the question of the form of the luminosity function in the Virgo cluster elliptical galaxies.

Sandage & Tammann (1981) go on to discuss applications of available photometry of the globular clusters associated with Virgo cluster galaxies: they cite three particular uses (Sandage 1968; de Vaucouleurs 1970; Sandage & Tammann 1976), concluding that they lead "...consistently to values of $H_0 \approx 50$." In each case, the derivation relies upon an empirical correlation of the absolute magnitude of a first-ranked globular cluster with the absolute magnitude of its parent galaxy (although in Sandage 1968 it reduces to an assumption that there is no such dependence, that the first-ranked clusters are of the same intrinsic luminosity, at least in M31 and M87).

Unfortunately, each of the cited treatments is erroneous. Sandage (1968) - who concluded incidentally that $H = 75 (+19, -13)$ km/s/Mpc - assumed that the brightest cluster in M87 lay at $B = 21.3$ mag, an estimate based on Racine's (1968) photometry in the field. That photometry, by the way, was quite correct, but the bright tail of the cluster luminosity function was masked by statistical fluctuations: Racine's field star correction depended upon annular subtraction in his small (15 arcmin) M87 field, and by bad fortune the net function mimicked a sharp step at $B \sim 21.3$ mag (see Hanes 1980a). As was subsequently shown (Hanes 1977b) the luminosity function in M87 and other Virgo cluster ellipticals has a bright tail, which in M87 reaches down to a magnitude level near $B \sim 20.5$

mag. Thus Sandage's method, although the best possible at the time and drawing upon the observations then available, gave incorrect results.

De Vaucouleurs' (1970) treatment suffers the same failing - he too made the then quite reasonable assumption that the brightest M87 cluster lay at $B = 21.2$ mag. In retrospect, one sees that this assumption alone invalidates the conclusion, but there are other problems: de Vaucouleurs combined clusters of all types, assigning ~ 3000 to the Galaxy but only ~ 200 to M31, and assumed a universal Gaussian luminosity function with mean magnitude and intrinsic dispersion which are now known to be incorrect. In fact his own recalibration of the method (de Vaucouleurs 1977a), ignored by Sandage & Tammann (1981), took these problems into account: he applied the method to the new photometry of globular clusters in Virgo (Hanes 1977b) to deduce a Virgo cluster distance modulus near $\mu = 30.4$ mag. Elsewhere, Tammann, Sandage & Yahil (1979) have criticized these results, but without foundation, as shown by Hanes (1982).

The third treatment quoted by Sandage & Tammann (1981) is their own (Sandage & Tammann 1976) use of the Hanes photometry: they have applied Hodge's (1974) empirical correlation relating brightest galaxy magnitude to parent galaxy luminosity, using their own estimates of the magnitude of the first-ranked globular clusters in Virgo galaxies, derived in unspecified fashion from the Hanes (1977b) histograms. In this treatment, unlike the other two quoted, the long tail of the luminosity function is explicitly considered and "generous" extrapolations are made, on the basis of which a lower limit of $\mu = 31.5$ mag is found for the distance modulus of the Virgo cluster. Unfortunately the conclusion is incorrect: some numerical error, first pointed out in Hanes (1980b) and perhaps arising because of confusion over the photometric system used in Hanes (1977b), has led Sandage & Tammann (1976) to numerical overestimates of the magnitudes of the first-ranked clusters, by ~ 0.8 mag or more. Thus their apparently stringent lower limit on the Virgo cluster distance modulus must be reduced by at least this amount (see Hanes 1982 for details).

Let me now turn briefly to Tammann & Sandage's (1983, this volume) most recent dismissal of the globular cluster method. Sadly, it too is incorrect. Consider their Figure 5, which shows (i) that Strom et al's (1981) M87 globular cluster luminosity function seems to differ from that of Hanes (1977b), and (ii) that the near-linearity of either representation shows that the function contains no distance information.

Conclusion (i) is surprising. Strom et al (1981) relied upon Hanes' (1971, 1977b) photometry in setting their magnitude scales, and commented upon the gratifying agreement between the two luminosity functions (their own vs. that of Hanes 1977b); see Strom et al's Figure 14. Unfortunately they do not tabulate their luminosity function, which I have therefore read from their figure (and, for completeness, given here in Table 1). Figure 2(a) shows that function, with symbols whose vertical size indicates the Poisson statistics of the counts (but which should of course be somewhat larger because of the field star subtraction). The function has been transformed to the $G (=103aJ + GG385)$ system for comparison with the Hanes function, assuming globular clusters to have an average $B-V$ colour of 0.7 mag (whence $G = B - 0.15$). Figure 2(b)

Table 1 M87 Globular Cluster Luminosity Function
(Strom et al, 1981)

G	n
19.6 mag	21
20.1	29
20.6	58
21.1	107
21.6	194
22.1	275
22.6	358
23.1	414

shows a similar plot for the Hanes (1977b) M87 luminosity function, and the two are compared in Figure 2(c), arbitrarily scaled to cross near $G = 22$ mag. (Strom et al studied an area larger by 60% than that studied by Hanes, including clusters closer to the centre and farther out, and the central concentration of globulars to M87 makes a simple scaling by areas meaningless.) It is clear that the functions agree to within the sampling errors - which are underestimated, as I noted.

Tammann & Sandage's second conclusion - that the near linearity of the function implies that it contains no distance information - would be correct if they were referring to a logarithmic representation. In that case arbitrary population scalings would indeed consist of vertical shifts in the figure; but that is not so in the linear plane; obviously, population scalings change the apparent slopes of the functions in their figure. The intrinsic slope in the calibrating (Local Group) function enforces the population scaling in the sample of unknown distance, and an unambiguous distance estimate emerges. Indeed, Tammann & Sandage's reasonable straight-line fit in the linear plane immediately implies that the corresponding logarithmic representation is non-linear (as my Figure 2 shows), and therefore that the function does in fact have distance information: on this we agree.

My discussion so far has dealt with criticisms of the globular cluster method, but their dismissal is evidence of a negative kind: it reassures us, but provides no proof of the validity of the method. Indeed, there is a fundamental uncertainty: are globulars in elliptical galaxies (the populous Virgo samples) like those in the Local Group spirals (the calibrating sample)? - in particular, are the luminosity functions the same? Until recently, such an assumption simply had to be made perforce, but some recent numerical experiments suggest that the assumption may indeed be justified.

Consider the data shown in Figure 1. The calibrating sample, in panel (a), is well represented by a Gaussian of intrinsic dispersion $\sigma = 1.10 \pm 0.10$ mag (Hanes, 1977a). The question is, does the incomplete function in Figure 1(b) contain sufficient structure to tell us that it too resembles a Gaussian with $\sigma = 1.10$ mag; and (if so) what limits may we put upon the deduced value? Obviously such a question is more easily answered if the population in the sample is large: the sampling errors are reduced, and the unavoidable uncertainties owing to the subtraction

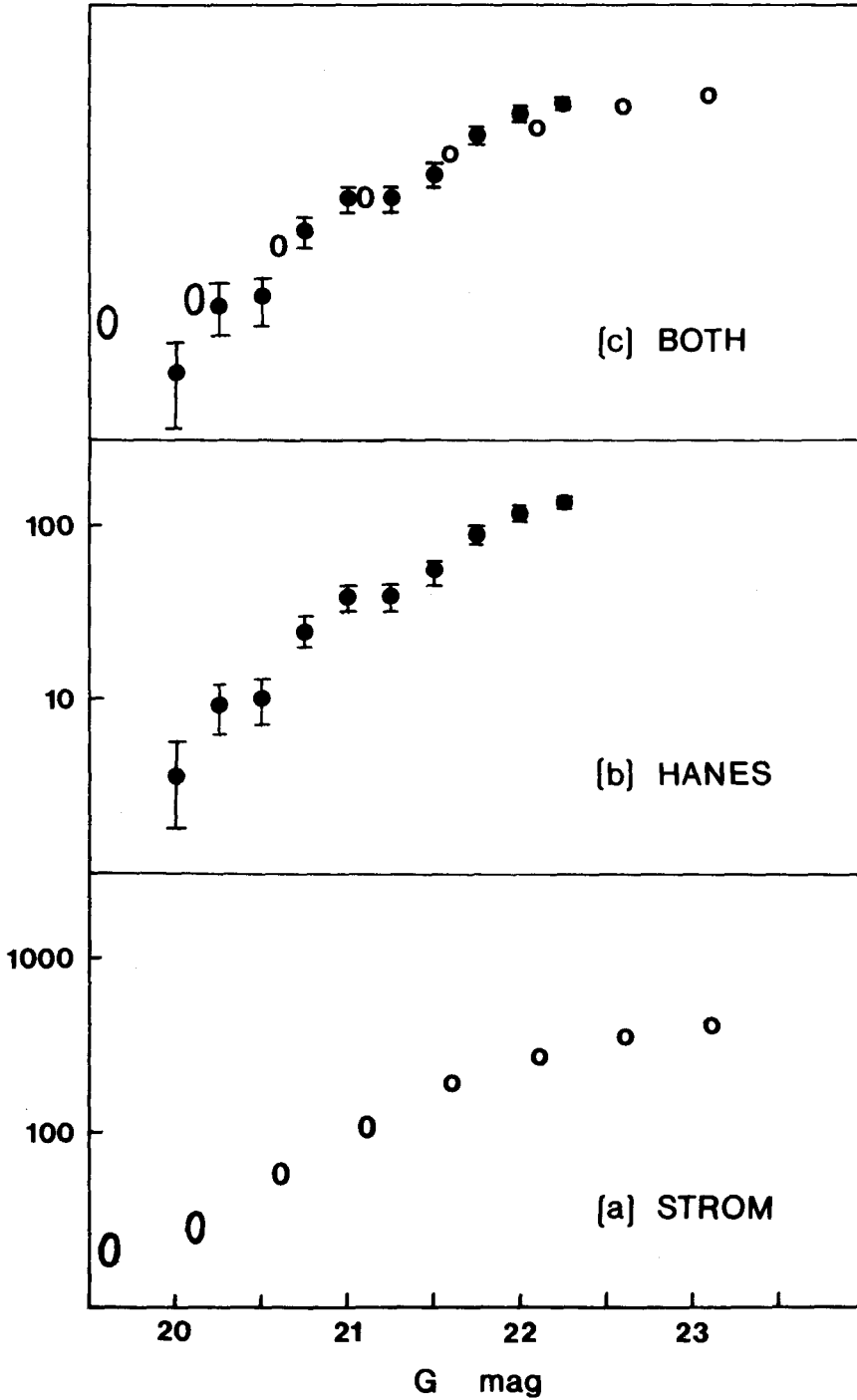


Figure 2: the luminosity function of globular clusters associated with M87 (see text for details)

of field objects are minimized. We can envisage a three-parameter test: adopt an apparent mean magnitude (or, equivalently, a distance modulus) for an assumed Gaussian function and a hypothetical value of σ , then adjust the total population until the scatter about the fitted function is minimized in Figure 1(b). A table of observed scatter as a function of the adopted (mean magnitude, σ) pairs will suggest the allowable intrinsic form of the function. However, it is not enough merely to test the observed Virgo function: some external estimate of the reliability is wanted.

Monte Carlo simulations provide such external estimates. I have computer-generated large numbers of luminosity functions with members drawn from an assumed underlying normal distribution. I have adopted different total populations (from 100 to 30 000 clusters) and assumed different distances (mean apparent magnitudes) and photometric completeness limits, to mimic the observational data. The tests described above were then carried out (with the inclusion of the extra uncertainty of field object subtraction). Of course when the luminosity function is explored well beyond the mean apparent magnitude, the value of the mean and the true intrinsic dispersion are reproduced to good accuracy for even modest samples (hundreds of globulars). Unfortunately we do not reach this level of completeness in the Virgo globular cluster functions, but the intriguing result of the simulations (which will be described in detail elsewhere; Hanes 1983) is that samples greater than a few thousand reproduce the mean magnitude and intrinsic dispersion to typically ± 0.20 and ± 0.10 mag respectively, provided that the function is plumbed to within $\sim 1\sigma$ of the mean. Thus one need not even reach the peak of the function before the statistics are good enough (the tail is well enough delineated) to pin down the underlying distribution precisely.

And in Virgo we have just this situation: the clusters number several thousand and are sampled to adequately deep levels (Hanes 1979). Figure 3 presents the results of the three-parameter test, applied this time to the empirical Virgo cluster data. The absolute minimum residual, shown as a plus sign, occurs at $G = 23.7$ mag and $\sigma = 1.16$ mag: these values provide the absolutely best Gaussian representation of the data. (The implied Virgo cluster distance modulus is 30.6 mag, if the mean absolute magnitude is the same in Virgo as in the Local Group.) The contour lines in the figure represent residuals which are larger by factors of 1.5, 3, and 5. The sense of the 'trough', which also appears in every Monte Carlo experiment, is easily understood: numerically larger mean magnitudes require larger values of σ to match the long tail of the observed distribution. Values of σ outside $\sigma = 1.16 \pm 0.10$ lead to residuals which are larger by more than 50% and by comparison with similar statistics derived from the Monte Carlo simulations I conclude that the value of σ which best characterizes the Virgo globular cluster function lies in this restricted range, which is consistent with that seen in the Local Group galaxies (where $\sigma = 1.10 \pm 0.10$ mag).

What does this mean? It does not and can not convince us of the absolute identity of cluster luminosity functions in spirals (locally) and ellipticals (in Virgo). Even if we could sample the complete Virgo function and (let us suppose) prove that σ is the same there as locally, we would be unable to exclude the possibility that the other crucial

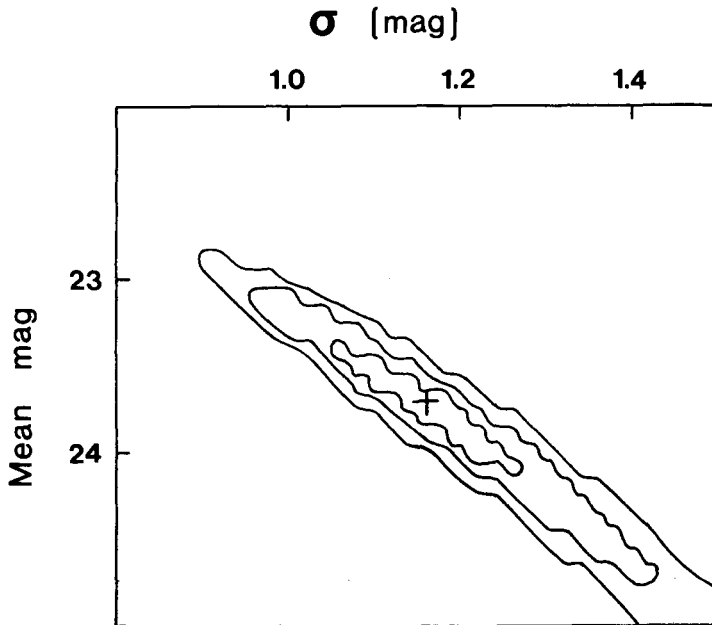


Figure 3: Residuals about fitted Gaussians in the data of Figure 1(b) - see text for a complete description.

parameter, the mean magnitude, differs in the two samples - unless we can derive an independent distance estimate, which begs the question. (Of course we are not alone in this problem.) The experiment described here does show that it is now unnecessary merely to posit the equality of σ ; the Virgo function is well enough sampled to reassure us on that score (unless of course photometric scale errors have conspired to deform an intrinsically quite different function into one which fortuitously reproduces the value of σ seen in the Local Group!).

My optimistic conclusions are (i) that globular cluster luminosity functions are alike, in shape at least (see Harris & Racine 1979); (ii) that the functions have sufficient structure to permit their use as distance indicators at least as far as the Virgo cluster of galaxies (and with real hope for more remote systems: Dawe & Dickens 1976; Smith & Weedman 1976); and (iii) that the Virgo cluster distance modulus is $\mu = 30.7 \pm 0.3$ mag (on a scale where RR Lyrae stars have $M_V = 0.6$ mag).

The Sandage-Tammann Extragalactic Distance Scale

If globular clusters are as useful distance indicators as I have painted them, why do they imply Virgo cluster distance moduli so at variance with the findings of Sandage & Tammann (1976)? Of course, similar questions can be asked with reference to various other analyses which have in recent years also implied distance moduli near $\mu = 30.9$ mag, surprising results in view of the precision claimed in the Sandage & Tammann analysis, where $\mu = 31.70 \pm 0.08$ mag. As is well known, de Vaucouleurs (1980) in particular has been critical of the Sandage-Tammann

construction, citing certain features which he finds unsatisfactory: their model for obscuration internal to our own Galaxy; their calibration and application of the period-luminosity-colour relationship for Cepheids; their measurements of the angular diameters of HII regions in external galaxies; their dependence upon a restricted chain of indicators; and so on. Other criticisms have been voiced by other scientists; see for example Madore (1976), Jaakkola & Le Denmat (1976); Bottinelli & Gouguenheim (1976).

However, taking a slightly different view, I have recently (Hanes 1982) reconsidered the entire Sandage-Tammann chain (1974a,b,c,d; 1976) but have predicated my analysis on their basic assumptions and approach. That is, I assume that their Cepheid distance moduli for Local Group galaxies are correct, that their measurements of HII region diameters and stellar and galaxy magnitudes are of high precision, that their luminosity classifications for galaxies are correct, that their model for absorption in our Galaxy is reasonable, and so on. This is clearly a very restrictive test, but the interesting conclusion I reach is that a careful reworking of their own data leads to a Virgo cluster distance modulus of $\mu = 30.8$ mag, in agreement with the results of many others; see Hanes 1982 for details.

Why is this so? Largely it reflects the one important difference between our analyses: the role played by NGC 2403. Sandage & Tammann adopt a Cepheid-based distance modulus (Tammann & Sandage 1968) for NGC 2403, a galaxy at a distance of a few megaparsecs, and assume that the same distance is valid for five galaxies apparently associated with it. These six galaxies, together with five in the Local Group, then constitute their calibrating sample for a subsequent determination of the dependence upon galaxy luminosity class of brightest star luminosity (1974b) and HII region diameter (1974a), these becoming the distance indicators in the next outward step. The problem is that the distance modulus for NGC 2403 is a matter of dispute: Madore's (1976) analysis suggests that their distance modulus has been overestimated by ~ 0.8 mag. An uncertainty of this size is profoundly important, because an amendment in the NGC 2403 distance modulus applies simultaneously to six galaxies - more than half the calibrating sample - and materially changes the deduced dependences for bright stars and HII region diameters upon galaxy property. This sensitivity to a single distance determination is an unsatisfactory feature of the construction, especially since the determination is a difficult one at best: it relies upon step-scale photographic photometry against often rich stellar backgrounds near the plate limits - indeed some of the Cepheids fade below the limit at minimum light. My own repetition of their construction does not include this assumption, therefore: I take the five Local Group galaxies, within which a variety of distance indicators can be used and for which the distance determinations are independent, to be the fundamental calibrators.

Of course the Sandage-Tammann assumption may be a helpful one, if correct, in that it augments the sample of fundamental calibrating galaxies. A clear test of the validity of the assumption is to search for subsequent inconsistencies in the construction of the distance scale; and such inconsistencies are indeed found (Madore, 1976; Jaakkola & Le Denmat, 1976; Bottinelli & Gouguenheim, 1976; Hanes, 1982). Some

of these were explicitly recognized (but erroneously dismissed; Hanes 1982) by Sandage & Tammann. Space precludes a complete description of these problem areas and of my reworking of their analysis; here I shall merely summarize the important points, as follows:

If the Tammann & Sandage (1968) Cepheid distance modulus for NGC 2403 is correct and the consequent construction is valid, then HII regions are larger by $\sim 50\%$ on average in the NGC 2403 group galaxies than in comparable Local Group galaxies; the galaxies in the NGC 2403 group are themselves more luminous by $\sim 50\%$ than their counterparts in the Local Group; and the Cepheids in NGC 2403 are redder by 0.2 mag on average than those in the Local Group galaxies. Furthermore, M101, the nearest ScI galaxy (and the next calibrator in the outward construction) is atypical of other ScI galaxies in HII region diameter or luminosity; but the remote distance scale depends entirely upon their implicit assumption that all such galaxies are alike in luminosity - that the HII regions in M101 are larger than average. This assumption renders irrelevant their use of HII region diameters in remote galaxies and also has the effect of enforcing an apparently reassuring but possibly spurious agreement between the galaxy magnitude-luminosity class relationship for galaxies within and without rich clusters. Finally, their eventual calibration of the absolute magnitude-luminosity class relationship does not agree with even their own basic calibrators.

The resolution of these inconsistencies requires little more than a recognition of the incorrectness of the Cepheid-based true distance modulus of NGC 2403, as first pointed out by Madore (1976). It seems likely that the Cepheids in NGC 2403 are reddened by 0.2 mag on average by interstellar material within or in the foreground of NGC 2403. (Tammann & Sandage (1968) described explicit tests which led them to dismiss this possibility, but those tests are numerically incorrect as well as powerless to disprove the presence of such obscuration, since they test only for strongly differential effects; see Hanes 1982.) The requisite amendment, a reduction by ~ 0.7 mag of the true distance modulus of NGC 2403, brings into concordance the other indicators - the HII regions and the integrated galaxy luminosities. (It seems certain that a like adjustment for any one of the Local Group galaxies would have little effect in the whole construction; but that is not true for NGC 2403 since it enters with disproportionately high weight through its association with the other galaxies in the group.)

Subsequent steps in the Sandage-Tammann construction lead me to the conclusion that the Virgo cluster distance modulus is ~ 30.8 mag on a scale where the fundamental zero-point is set by their Cepheid period-luminosity-colour relationship. The remote scale remains somewhat uncertain in their treatment because of the residual uncertainty in the atypicality of M101 among ScI galaxies. It may be that M101 possesses atypically large HII regions, as Sandage & Tammann suggest (1974c), but that all ScI galaxies are similar in luminosity; it may be that all ScI galaxies are alike in both luminosity and HII region diameter, but that photo-optical effects (de Vaucouleurs 1977b) led to systematic errors in Sandage & Tammann's (1974b) measurements of HII region diameters in remote ScI galaxies (Kennicutt 1979 presents evidence for this); or it may be that M101 is atypical in both luminosity and HII region diameter,

in which case the construction beyond that point breaks down. None the less, it seems indisputable that the Sandage-Tammann data are consistent with a Virgo cluster distance modulus near $\mu = 30.8$ mag, in good agreement with the findings of others.

However, it must be pointed out that Sandage & Tammann's (1981, 1982; Tammann & Sandage 1983) newly formulated calibration of the extragalactic distance scale implies an asymptotic value of 50 ± 7 km/s/Mpc for the Hubble ratio. This construction bypasses the Virgo cluster (as did the original version, except parenthetically), but clearly must imply a Virgo cluster distance modulus of $\mu \sim 31.5$ mag to be consistent with our present understanding of the local peculiar velocity (de Vaucouleurs & Peters, 1981). Sandage & Tammann were prompted to reestablish their calibration of the distance scale by their disquiet, expressed in Sandage & Tammann (1982), over the imperfect nature of galaxy luminosity classes, which they had earlier treated as essentially scatter-free descriptors of galaxies but which are now known to exhibit large luminosity spreads at constant class. Their new construction (also summarized in Tammann & Sandage, 1982, this volume) follows these steps: distances are derived, via Cepheids, to a number of nearby galaxies within the Local Group and the NGC 2403 group. These distances allow the determination of the absolute magnitude of the brightest red supergiants which, perhaps surprisingly, turn out to be very nearly constant in luminosity in galaxies spanning a vast range of integrated luminosities (and therefore total stellar content) - this is in itself of considerable astrophysical interest because it implies some physical mechanism limiting the luminosity of red supergiants, else larger populations would be expected on simple statistical grounds to contain intrinsically brighter stars. In any event, Sandage & Tammann next carry out photometry of the brightest red supergiants in three galaxies at distance moduli near 28 - 29 mag and establish thereby their distances; these three include NGC 4214 and IC 4182, each of which contained a well-studied Type I supernova (SN1954a and SN1937c, respectively). The mean absolute magnitude at maximum of such supernovae (demonstrated in Sandage & Tammann 1982 to be a good standard candle) follows, and the remote distance scale can be established with reference to remote galaxies which have experienced supernovae, far beyond any anisotropies in the flow.

The method has some very appealing features: few steps are involved; supernovae are visible at great distances, beyond local perturbations; and supernovae of Type I in particular may be little affected by obscuration internal to galaxies. Against that is the philosophical point, often voiced by de Vaucouleurs (1980), that the construction is one-tined: one error or false assumption (such as an erroneous zero-point for the Cepheid period-luminosity-colour relationship) will invalidate the whole construction. Some practical considerations also arise: Sandage & Tammann's magnitudes for red supergiants in the three galaxies in the intermediate step are based upon Argelander step-scale photographic photometry at the $V \sim 21$ mag level, in crowded fields; we may worry about systematic photometric errors at these faint levels, especially as the photometry relies upon extrapolations of bright ($V \leq 17.5$ mag) photoelectric sequences via Racine prism secondaries (as indeed does the photometry of globular clusters in Virgo!); and some assumptions need

strict verification - the reliability of the 1937 and 1954 photometry for the supernovae, Sandage & Tammann's (1982) assumption of the close general similarity of supernova light curves and their particular applicability to SN1937c and SN1954a; and so forth.

These questions notwithstanding, the method seems to provide one more promising weapon in our arsenal and warrants further detailed analysis - as indeed do many methods, both within and beyond the Virgo cluster, in the hope of settling the still incompletely resolved question of the correct extragalactic distance scale.

References

- Bottinelli, L. & Gouguenheim, L. (1976) *Astron. Astrophys.* 51 275
 Bottinelli, L., Gouguenheim, L., Paturel, G. & de Vaucouleurs, G. (1980) *Ap.J. Lett.* 242 L153.
 Carney, B. (1983) This volume: Specialist Session on 'The Distance Scale', p. 255.
 Corey, B.E. & Wilkinson, D.T. (1976) *B.A.A.S.* 8, 351
 Dawe, J.A. & Dickens, R.J. (1976) *Nature* 263 395
 de Vaucouleurs, G. (1970) *Ap.J.* 159 435
 de Vaucouleurs, G. (1977a) *Nature* 266 126
 de Vaucouleurs, G. (1977b) *IAU Colloq. #37 'Descalages vers le Rouge et Expansion de l'Univers'* (CNRS, Paris) p. 70
 de Vaucouleurs, G. (1980) Invited review paper at Tenth Texas Symposium on Relativistic Astrophysics, Baltimore, Maryland
 de Vaucouleurs, G. & Peters, W.L. (1981) *Ap.J.* 248 395
 Hanes, D.A. (1971) M.Sc. thesis, Univ. of Toronto
 Hanes, D.A. (1977a) *MNRAS* 180 309
 Hanes, D.A. (1977b) *Memoirs RAS* 84 45
 Hanes, D.A. (1979) *MNRAS* 188 901
 Hanes, D.A. (1980a) in 'Globular Clusters' ed. D.A. Hanes & B.F. Madore (Cambridge Univ Press), p.213
 Hanes, D.A. (1980b) AAO Preprint #140
 Hanes, D.A. (1982) *MNRAS* 201 145
 Hanes, D.A. (1983) in preparation
 Harris, W.E. & Racine, R. (1979) *Ann.Rev.Astron.Astroph.* 17, 241
 Hodge, P.W. (1974) *PASP* 86 289
 Jaakkola, T. & Le Denmat, G. (1976) *MNRAS* 176 307
 Kennicutt, R. (1979) *Ap.J.* 228 696
 Madore, B.F. (1976) *MNRAS* 177 157
 Mould, J., Aaronson, M. & Huchra, J. (1980) *Ap.J.* 238 458
 Racine, R. (1968) *JRASC* 62 367
 Racine, R. & Shara, M. (1979) *A.J.* 84 1694
 Sandage, A.R. (1968) *Ap.J. Lett.* 152 L149
 Sandage, A.R. & Tammann, G.A. (1974a) *Ap.J.* 190 525
 Sandage, A.R. & Tammann, G.A. (1974b) *Ap.J.* 191 603
 Sandage, A.R. & Tammann, G.A. (1974c) *Ap.J.* 194 223
 Sandage, A.R. & Tammann, G.A. (1974d) *Ap.J.* 194 559
 Sandage, A.R. & Tammann, G.A. (1976) *Ap.J.* 210 7
 Sandage, A.R. & Tammann, G.A. (1981) Paper presented at the Vatican Study Week 1981; Preprint #3 of Astr. Inst. of Univ. of Basel
 Sandage, A.R. & Tammann, G.A. (1982) *Ap.J.* 256 339

- Smith, M.G. & Weedman, D.W. (1976) *Ap.J.* 205 709
- Strom, S.E., Forte, J.C., Harris, W.E., Strom, K.M., Wells, D.C. & Smith, M.G. (1981) *Ap.J.* 245 416
- Tammann, G.A. & Sandage, A.R. (1968) *Ap.J.* 151 825
- Tammann, G.A. & Sandage, A.R. (1983) This volume: Specialist Session on 'The Distance Scale', p. 301.
- Tammann, G.A., Sandage, A.R. & Yahil, A. (1979) Lecture notes for 1979 Les Houches Summer School; Preprint #1 of Astr. Inst. of Univ. of Basel