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VARIETIES OF NILPOTENT GROUPS OF CLASS FOUR

PATRICK FITZPATRICK

All varieties of nilpotent groups of class at most 3 have been known for many years (Jonsson, Remeslennikov). In a preprint written jointly with L.G. Kovács, we have reduced to two cases the problem of determining all varieties of nilpotent groups of class at most 4 . The first is to deal with all such varieties whose free groups have no elements of order 2 : this is completed in that preprint. Part of the result is that those varieties form a distributive lattice with respect to inclusion order (though some joins in this lattice are different from joins formed in the lattice of all varieties).

The subject of this thesis is the second half of the problem: varieties whose free groups have no nontrivial elements of odd order. These do form a sublattice in the lattice of all varieties, but this sublattice is not distributive and so its description is considerably more complicated than the cases which were handled previously.

The main result assigns to each of our varieties a vector of 16 parameters, each parameter a nonnegative integer or ∞ , subject to simple but numerous conditions. Each parameter vector satisfying these conditions is in fact used (precisely once), and directly yields a (finite) defining set of laws for the variety it labels. One can read off the parameters whether one variety is contained in another.

Indeed, one can calculate the parameters of the join and the meet of two varieties from the parameters of the two components; algorithms for

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these calculations are presented (without proof) in an appendix. Given a variety $\underline{\underline{V}}$ by its parameters, it is easy to write down the parameters of the subvariety generated by the torsionfree groups of $\underline{\underline{V}}$, and to give an upper estimate for the exponents of the torsion subgroups of the free groups of $\underline{\underline{V}}$. (These are necessary for making the reduction described in our preprint fully effective.)

Actually, all this is done in the dual context of fully invariant subgroups of the rank 4 free nilpotent group of class 4. The hardest part of the work is to deal with fully invariant subgroups contained in the last nontrivial term of the lower central series. This part also allows another interpretation, which may be of independent interest. Consider the free Lie algebra L of rank 4 over the ring of rational 2-adic integers, and let W be the homogeneous component of degree 4 in L. Let E be the subalgebra of the endomorphism algebra of W generated by the restrictions of the graded endomorphisms of L. We determine (the Morita-type of) E and the E-submodules of W.

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