effect on the motion of an artificial satellite. Brouwer (6) discussed analytically the resonance caused by radiation pressure on the motion.

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Critical inclination

Brouwer's theory (1) on the motion of an artificial Earth satellite is based on von Zeipel's method of eliminating all short-period terms by a canonical transformation. Brouwer's elements are transformed by Cain (2) to the mean elements in the ordinary sense. Lyddane and Cohn (3) computed numerically the motion of an artificial Earth satellite by Cowell's integration method for verifying Brouwer's theory. The first-order terms in the semi-major axis are not sufficient. By taking the second-order terms in the semi-major axis they could verify Brouwer's theory satisfactorily. Lyddane (4) referred to Poincaré's canonical variables instead of Delaunay's in order to remedy Brouwer's theory on von Zeipel's method from the difficulty for $e = 0$, $I = 0$. The same difficulty has been dealt with by Smith (5) by ordinary co-ordinate transformation.

The expressions in Brouwer's theory show that the method fails when $1 - 5\cos^2 I = 0$. It corresponds to the inclination $I = 63°26'$, which is called the critical inclination. There have been several hot discussions as regards to the critical inclination whether it is a real existence or it is just an illusion caused by the wrong treatment of the problem. As far as the present method of perturbation theory is concerned, that is, in separating the perturbation into the short-period, the long-period and the secular, the appearance of the critical inclination is essential, although it is tacitly assumed that the perturbation is small enough to be divisible into its parts, then integrated separately and finally summed over the separately integrated results, irrespective of the convergence of the solution.

Hagihara (6) referred to his general theory (7) of libration based on Poincaré-Andoyer's theory on the motion of the Hecuba group asteroids. After carrying out von Zeipel's transformation an integral $F =$ constant is obtained, where F is the new Hamiltonian. A pair of double points of the curve representing this integral corresponds to the critical inclination. It is shown that one of the double points corresponding to the critical inclination is a centre in Poincaré's terminology and is stable, while the other is a saddle point and is unstable. The orbits near the centre are libratory and those near the saddle point are of revolution. The width of the libration in inclination is very narrow. The solution is obtained in elliptic functions in both cases. It is noticed that there appears no critical inclination in Vinti's treatment with the assumption $J_2^2 + J_4 = 0$. Garfinkel (8) obtained similar results by a different method based on von Zeipel's transformation by making Vinti's parameter $J_2^2 + J_4$ explicitly.

Hagihara considered only the terms J_2 and J_4. Kozai (9) extended the discussion to include J_3 and J_5. Aoki (10) included all these terms from the start. By referring to the elliptic functions of Weierstrass he solved the problem completely in each of the different cases arising from the values of the constants in the problem.

Hori (11) expanded the solution in powers of the square root of J_2 in order to solve the problem near the critical point. He obtained the solution in the form of the elliptic integrals of the first and the second kinds.

Izsak (12) noticed that the expansion in powers of $\sqrt{J_2}$ fails when we include higher order

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terms. He answers the question of how far we have to carry out the approximation so that higher order terms shall no longer change the qualitative feature of the motion. Izsak derived the perturbation equations for the modified action-angle variables of Vinti's dynamical problem and computed the secular and the long-period parts of the Hamiltonian, in order to obtain equations simpler than the usual.

Petty and Breakwell (13), and Struble (14) also discussed the problem of the critical inclination.

Message, Hori and Garfinkel (15) have shown that the solution for the critical case agrees with that for the non-critical case as far as the dominant terms in the immediate vicinity of the critical point. This should be interpreted as the fitting of the two asymptotic solutions.

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24-hour Earth satellite

The asymmetric character of the Earth's equator discovered by Kozai, Izsak, Kaula and others leads to observable effects on the orbits of close Earth satellites. This effect is important for stationary communication satellites, as well as for space telescopes or any observable platform designed to stay fixed at a given geographic longitude on the equator. The motion can be discussed in the same way as for a satellite with critical inclination. The influence of the principal longitude-dependent term of the Earth's potential on the orbit of a 24-hour satellite has been studied by Sehnal (1). He noticed the influence of tesseral harmonics on the long-period perturbation and obtained the periodic shift from the stationary point by considering the action of the Moon, then he discussed the long-period terms produced by the Earth's equatorial ellipticity and the secular terms of inclination due to the action of the Moon by the method of the variation of constants.

Blitzer, Boughton, Kang and Page (2) and Blitzer, Kang and McGuire (3) studied the influence of the principal longitude-dependent term by assuming the orbit to be nearly circular. There is a stable equilibrium point on the minor axis of the equatorial ellipse and an unstable equilibrium point on the major axis, the motion in latitude being simple periodic. Libration occurs of the order 850 days around the stable point. The nearby motion to the unstable point is a revolution.

Musen and Bailie (4, 5) studied the condition of stability even for an orbit with high inclination. They referred to von Zeipel's method after Brouwer, and computed the period and the amplitude of the libration and the mean motion of the revolution.