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Directed graphs and combinatorial properties of groups and semigroups

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Research on unavoidable regularities in sequences of elements of groups and semigroups is related to Ramsey theory. The present thesis continues work that has been carried out recently by a number of authors, and investigates several new combinatorial properties motivated by a theorem proved by B.H. Neumann [1] as an answer to a question of Paul Erdös.

Cayley graphs are very well known and are the main object of study in the thesis. Several combinatorial properties of sequences defined in terms of Cayley graphs are explored.

The *power graph* of a semigroup S is a directed graph with the set S of vertices, and with all edges (u, v) such that $u \neq v$ and v is the power of u. A combinatorial property of sequences of elements is defined in terms of power graphs, and all groups and monoids satisfying it are characterised. The structure of the power graphs of all finite abelian groups is then described.

The *divisibility graph* of a semigroup S has edges (u, v), where u belongs to the ideal generated by v. For each directed graph D, we describe all commutative and completely 0-simple semigroups with analogous combinatorial property for sequences defined with divisibility subgraphs.

The concept of an annihilator set has been considered in combinatorial semigroup theory, graph theory and the study of formal languages. We consider a combinatorial property concerning sequences of elements and defined using annihilator graphs.

An appendix contains related results on finite state automata defined with labelled graphs.

REFERENCES

[1] B.H. Neumann, 'A problem of Paul Erdös on groups', *J. Austral. Math. Soc.* **21** (1976), 467–472.

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