THE MASSES AND PULSATION MODES OF CLASSICAL CEPHEIDS

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Abstract. Recent general observations pertaining to the masses and pulsation modes of classical Cepheids are reviewed. Certain special ones of these variables that display unusual behavior such as long term amplitude variations, overtone pulsations, and double-mode behavior, including one Cepheid in the first and second overtones as well as those in the fundamental and first overtone, are discussed in some detail. I suggest that the amplitude varying supergiant Cepheid HR 7308, which seems to be pulsating in the second radial overtone, is alternating between states where there is enough helium to barely give kappa and gamma effect driving, and where the helium has settled too deep for this driving. Reestablishment of the helium may be due to rapid levitation of the CNONe elements which cause convection and which then dredge-up the helium again to suppress the convection and also to drive pulsations. The use of Fourier fitting to the light and velocity curves have revealed some features that are interesting and are discussed, such as the pulsation mode discrimination and the derivation of accurate Wesselink radii. The seven methods of determining the masses of these stars are presented with a short critique of each. Evolution and pulsation masses depend on uncertain observed luminosities, and these have been revised recently. Evolution masses also depend on the blue looping yellow giant evolution tracks that may need revision because of convective core overshooting and possible opacity, Z, or Y abundance increases. The most discrepant masses, however, are those that depend on period ratios - the "bump" and "beat" masses. The possibility that the Cepheid internal structure can be modified enough by doubling the material opacity seems unlikely. It is suggested, though, that perhaps the period ratio mass anomaly solution can be found in CNONe element enhancement by radiation absorption levitation that would give higher opacities by abundance effects instead of any revisions in the opacity calculations. Some details for this mass anomaly "solution" are presented.

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INTRODUCTION

A review of the status of research in classical Cepheids is an enormous task. Observational work continues unabated, it seems, and theoretical investigations still center on several unsolved problems. To limit this review properly, I will concentrate only on the aspects of Cepheid research that have to do with stellar structure, evolution, and pulsation, leaving the important features of the period-luminosity-color relations and the distance scale to other galaxies to the other reviewers. Unfortunately, I cannot review the results being presented at this conference, and I am sure that some of the things I say will be shown to be incorrect. Further some of my questions may be already answered.

Cox (1980) in his review at Los Alamos, suggested three problems that he considered at the frontier of the theoretical research on Cepheids. These were the full amplitude behavior of Cepheid pulsations as determined from nonlinear calculations, convection, and the masses of the Cepheids. While I will categorize things a bit differently here, the problems that he discussed remain with us today. Mostly, I will review the problem of the masses, attempting to put some of the recent controversies at least into focus.

One can tell that the Cepheid masses are always a topic of interest, because at the last pulsation meeting in Los Alamos, Norman Simon (1987) reviewed the status of these masses. In fact, his entire review centered about the mass anomalies, especially those for the shorter period ones that display period ratios by the light and velocity curve bumps or directly by having two pulsation modes going simultaneously.

Let me close this introduction by noting that Cepheids are everywhere. The recent loss of one of them about 10,000 years ago (plus 170,000 years) by becoming supernova 1987A (Woosley and Phillips, 1988) reminds us of how important they are to studies of stellar evolution.

GENERAL OBSERVATIONAL DATA

The extensive photometric data of Moffett and Barnes (1987 and others) have been a major advance in the knowledge of Cepheid pulsations. The use of these photometric (B-V) and (V-R) data and simultaneous radial velocity measurements published by Barnes, Moffett and Slovak (1987) will hopefully allow them to derive Baade-Wesselink radii for as many as 112 Cepheids. Results to date for 63 Cepheids give radii that are consistent with masses obtained by conventional methods, but accuracies as high as needed to influence research on Cepheid mass discrepancies are not available. To get radii with uncertainties of only a few percent, and masses with uncertainties of less than 10 percent, new data will have to be obtained.

Observations of Cepheid mass loss have been made by Deasy and Butler (1986), McAlary and Welch (1986), and Deasy (1988). They have used IRAS data together with their spectra to show that the mass loss rate rarely exceeds 10^{-8} solar masses per year. With that rate, it is not likely

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that the evolution in blue loops in the Hertzsprung-Russell (H-R) diagram will be affected much, but work by Brunish and Willson (1987) on evolution with even larger pulsation induced mass loss is continuing, as is presented at this conference. Deasy notes that the mass loss rates for non-variable supergiants are of the same order of magnitude as for the Cepheids. We must be careful not to confuse the observations for the low mass and gravity type II Cepheids, which show considerable mass loss, with those for the classical Cepheids that have apparently low mass loss rates. We also need to realize that close binary stars frequently have large mass loss rates, which are induced by the near companion, and they are not typical of a general process.

These mass loss rates are not that much different from those scaled from the observed solar rates by accounting for the larger Cepheid surface area. Mass loss rates of 10^{-10} solar masses per year, if the Cepheid wind is helium deficient as for the sun, are just those proposed by Cox, Michaud, and Hodson (1978) for considerable surface helium enhancement. Winds 100 times stronger, if they do not destroy any hydrogen-helium fractionation, can give all the surface helium enrichment desired for low mass Cepheids without affecting their evolution.

I would like to comment on another general observation, that of the orbit of U Aql (Welch, et al. 1987). This Cepheid, like several others now known from IUE spectra (Böhm-Vitense and Proffitt, 1985), has an early type companion. With reasonable guesses for the companion the mass of the U Aql Cepheid can be estimated to have an upper limit of between 6.4 and 8.8 solar masses. This star and others with approximate masses such as SU Cyg (Evans and Bolton, 1987) and S Mus and V636 Sco (Böhm-Vitense, 1986) as well as the approximate radii determined by many over the years shows that the masses of Cepheids are not as low as the two solar masses that are needed to explain the double-mode Cepheid period ratios.

Special Cepheids: RU Cam, HR 7308 (V473 Lyr), Polaris, η Aql

What has happened to those two intriguing Cepheids RU Cam (Demers and Fernie, 1966) and HR 7308 (Breger, 1981)? These two have displayed an amplitude change over a few years that is not predicted by pulsation theory. Could it be that somehow the helium kappa and gamma pulsation driving decreased by some internal composition change? Could the helium settle out fast enough, as it does for some δ Scuti variables, to make them pulsationally stable? Under such low helium conditions the CNONe elements might be levitated to large local abundances and even produce a convection zone. And then could convective dredgeup mixing reestablish the helium abundance, destroy the convection zone, and produce enough helium driving to once again barely overbalance the ever-present radiative damping?

I note that RU Cam is actually a type II Cepheid with probably a low mass like 0.6 solar mass. Its gravity must be very low. Also the variable HR 7308, F6 I-IIb supergiant, has a very low gravity. Settling of helium out of the pulsation driving region is quite reasonable for these two stars. Our recent calculations and those of Michaud, Vauclair and Vauclair (1983) show that helium depletion is a real possibility. Since HR 7308 is pulsating in the second overtone, which often, and in this case, is the most driven mode, the variable may be starving for helium. This is especially so because the star is actually very near the instability strip red edge at about 6100K (Burki, Mayor, and Benz, 1982) where usually only the fundamental mode is observed.

HR 7308 has an amplitude behavior that has been varying with a period of 1210 days (Breger, 1981) to maybe as long as 1400 days (Burki, et al. 1986). The varying shape of the light curve with amplitude should produce data for detailed theoretical studies of the light curve skewness.

As Simon (1987) has suggested, the Cepheids are indeed a homogeneous class with no large variations of mass or luminosity from star to star, but these two quantities are global, evolutionary ones. But pulsation depends on the surface layer compositions, which could easily vary somewhat from case to case, giving a range of pulsation behavior.

The Cepheid Polaris is interesting. Its period has been increasing for 40 years, and its amplitude is decreasing. I believe that the suggestion by Arellano Ferro (1983) that it is evolving to the red out of the instability strip is correct. However, it could be losing its pulsation region helium so that its driving is decreasing. At the mass of near six solar masses and luminosity of about a thousand solar luminosities, we apparently have a good marker of the pulsational instability strip red edge (maybe 5800K). It would be interesting to find other quite red Cepheids with period increases so that they might mark the instability strip red edge at other luminosities. T Mon with a 27 day period might be a good example at a much higher luminosity and cooler surface temperature.

The case of η Aql is another one with the pulsation period increasing, and details have recently been given by Jacobsen and Wallerstein (1987). A summary of period changes was given by Fernie (1984), showing that there was not any large discrepancy between theory and observation. Fernie has pointed out that fast evolving stars with large period changes would not be easily observed, and slow evolution at blue loop tips could give very small period changes even with standard evolution tracks. He believes that slow period changes do not necessarily indicate mass loss than can trap a Cepheid's blueward evolution in the instability strip.

<u>Pulsation modes: SU Cas, HD 144972, DT Cyg, EU Tau, EW Sct, CO Aur</u>

The case of SU Cas is a favorite one for me because there seemed to be a continual argument about its pulsation mode. In my Cepheid mass review (Cox, 1979), I concluded that it is likely to be in the fundamental mode, and Turner, et al. (1985) as well as Turner and

Evans (1984) have given observational support by estimating its luminosity. However, Gieren (1976) has attracted the attention of most observers by advocating first overtone pulsation, and the low amplitude sinusoidal light and velocity curves (and the ratio of their first two Fourier amplitude coefficients, Simon and Lee, 1981) show that also. This is now confirmed by IUE observations that see the spectrum of both the Cepheid and the blue companion. The luminosity of SU Cas is then 1530 times that for the sun, at least 50 percent larger than the value I used in 1979. Such a bright and big variable then must be in the overtone to match its observed very short period (Simon and Aikawa, 1986).

Except for SU Cas I have thought that all galactic Cepheids were fundamental mode pulsators, but I do have to realize that the recent data for HD 144972 and DT Cyg point to first overtone pulsation (Moffett and Barnes, 1986). At least they have nearly sinusoidal light curves. EU Tau now has joined this list, with a suggestion by Burki (1985) that it is even in the second overtone. Gieren (1985) derives a radius much smaller so that it even could be a fundamental mode pulsator, but Simon and Lee (1981) from Fourier analysis of the light curve suggest that EU Tau is in the first overtone. Presently there is a discussion by Gieren and Matthews (1987) and Fernie (1987b) about whether there is a second very low amplitude higher order mode present.

There is an expectation that if the Cepheid is hot enough, it will pulsate in the first overtone, just as in the case for the RR Lyrae Bailey c-type variables. Since they have small amplitudes usually, their light curves are nearly sinusoidal. These are the s-Cepheids, but just because the light curves have that shape certainly does not insure that the mode is an overtone. Recently Fernie and Chan (1986) have shown that fundamental mode "bump" Cepheids often have almost sinusoidal light curve shapes, and that is also seen in the ratio of the first two Fourier terms for these stars. Near ten days, several fundamental mode Cepheids have more sinusoidal light curves than the overtone pulsators do as displayed by Simon and Moffett (1985). Overtone pulsators are quite rare in our galaxy and apparently common in the lower metallicity Magellanic Clouds. In their place we have the double-mode Cepheids (Stobie 1977).

Before discussing these double-mode Cepheids, I must comment on the suggestion by Böhm-Vitense (1988) that most short period Cepheids are overtone pulsators. The use of Wesselink radii is very unreliable, as Simon (1988, preprint), recently, and Cox (1979), long ago have shown. Gieren (1982 and 1986b) has derived quite small radii that imply these small masses. The even smaller masses, if the overtone mode is assumed, are much less than those that can barely evolve onto the blue loops in the H-R diagram where the Cepheids are observed. Since masses greater than the four solar masses minimum are actually observed for the binary SU Cyg (3.85 day pulsation period) and implied from the more extensive Wesselink radii obtained by Moffett and Barnes (1987), it seems that the Gieren Wesselink radii are not reliable for mass determinations. Burki (1985) has given Wesselink radii that show, if anything, that masses are

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anomalously low for the longest periods, not the shortest ones. Appeal to stellar models for the anomalously low "beat" masses, and to hydrodynamic calculations by Christy (1968) and Stobie (1969) and others for the low "bump" Cepheid masses is not warranted because of the many theoretical uncertainties in the compositions and opacities of Cepheid envelopes.

The list of galactic double-mode classical Cepheids is growing slowly with EW Sct being added to the eleven that I spoke of at Boulder in 1982 (Antonello et al. 1987). More are sure to be discovered. CO Aur seems to be in the first and second overtones (Mantegazza, 1983 and most recently Babel and Burki, 1987). I suppose that theoretically these modes may be acceptable if they are indeed at the very blue edge of the instability strip. From the blue edges given by Cox and Hodson (1978), CO Aur seems to be just at a possible transition line between first and second overtone pulsation.

It may be good news that Andreasen (1987) has started to find double-mode Cepheids in the Large Magellanic Cloud. One can hope that DV 14 and HV 2345 are confirmed. The period ratio near the galactic value of 0.70 will tell us whether the envelope structure of the short period Cepheids depends on the interior metallicity.

It seems that the high metallicity of our galaxy favors double-mode behavior instead of the very common first and maybe even second overtone pure mode pulsation in the Magellanic Clouds. This should be a clue in trying to understand both the anomalous period ratios for the double-mode Cepheids and how the double-mode behavior arises. Unfortunately, the high galactic metallicity possibly could promote both more helium deficient mass loss to give helium enhanced surface layers (Cox, Hodson, and King, 1979) or be more effective in producing a higher opacity in the pulsating regions (Simon, 1982). We need further clues from observations to understand double-mode Cepheids.

The finding of double-mode Cepheids at a period ratio of 0.70 in the low metallicity Clouds may actually be unwelcome, because it has been hoped for a long time that if the period ratio can be explained, the cause of the double-mode behavior (maybe related to the metallicity?) then would follow naturally. Ostlie in a poster paper at this conference alleges that time dependent convection is an essential ingredient in producing double-mode behavior, and how that relates to metallicity is not clear.

Fourier Fitting

The Barnes and Moffett light and velocity curves have been useful for discussion of the general properties of galactic Cepheids. These data have been used with the Fourier fitting methods by Simon and Moffett (1985) to give us an understanding of how the light curves evolve with changing period. These fits have been useful for the determination of Baade-Wesselink radii, as recently discussed by Simon (1988, preprint), as well as for mode identifications and even variable star classification as for XZ Cet (Teays and Simon, 1985).

SEVEN TYPES OF CEPHEID MASSES

The most basic type of mass that any star can exhibit is that determined from the solution of an orbit for a binary or multiple star system. It seems that there is only one case available to us that has a direct bearing on current mass anomalies. That is the case of SU Cyg, that has been investigated relentlessly by Nancy Evans for almost 10 years. With a luminosity of 1530 solar luminosities, evolution tracks (Z=0.02) would predict that the mass for this luminosity would be about 5.5 solar masses. Using evolution tracks that allow for convective core overshooting would reduce this mass to as low as 4.3 solar mass. The orbital solution (Evans and Bolton, 1987) gives 6.3 solar mass with a lower limit of 5.9, verifying, for this single case, that masses of Cepheids with known luminosities can be approximately determined from standard evolutionary tracks in the H-R diagram. At least a very low mass like 2 solar masses that double-mode Cepheids suggest from their period ratios are definitely ruled out. To compensate for the low evolution mass at the observed luminosity, one could use a larger Z approximately equal to 0.03 that would change the 5.5 solar masses to the observed 6.3 solar masses. Such a Z would not be unexpected for the recently born Cepheids.

We should note that the much brighter overshooting evolution tracks give rather low masses, and the degree of overshooting of the convective core may be too large in those evolution tracks that are becoming available from Chiosi. If anything, the SU Cyg case seems to show that the evolution tracks should be fainter, for a given mass, than the standard Becker, Iben, and Tuggle (BIT, 1977) tracks. SU Cyg may rule out the large overshooting that has recently been popular.

Böhm-Vitense (1986) has given masses for two other Cepheids based on orbital information, but these cases depend on the assumed masses of the blue companions. Therefore their accuracy is not very good. Even so, the masses of V636 Sco and S Mus are near 5 solar masses, rather low, but not in bad conflict with evolution masses for them at respectively, 6.80 and 9.66 days period.

Böhm-Vitense (1985) and Evans and Arellano Ferro (1987) have considered the cases for Cepheids with blue companions. The relative luminosity can be determined for these binaries from IUE spectra, and with the assumption of the blue companion luminosity (from its surface effective temperature and an assumed zero age main sequence), the Cepheid luminosity can be determined. Use of stellar evolution tracks then can allow identification of the mass that goes with this Cepheid luminosity. The sometimes quite low Cepheid luminosity implies a low Cepheid mass. However, Feast and Walker (1987) have pointed out that if the blue companion has evolved to a higher luminosity near the main sequence, the resulting Cepheid luminosity and mass may be considerably larger than assumed.

Standard evolutionary tracks have been studied to see if newer ideas about overshooting or increased opacities change their positions in the H-R diagram. Becker and Cox (1982) have showed that an increase of about 0.7 magnitude in luminosity for a fixed mass (9 solar mass) is certainly appropriate to allow for convective core overshooting. This overshooting was also considered by Huang and Weigert (1983) with similar results. Becker (1985) in another investigation demonstrated that a change in opacity in the temperature range between 100,000K and 500,000K did not have any important effect on the tracks. For an known luminosity of a Cepheid from the relative luminosity discussed above or from a known distance, an evolution mass can be determined by finding the mass of the evolution track that has that luminosity.

Schmidt (1984) has studied the Cepheids in eight galactic clusters and has found that their luminosities are perhaps 0.5 magnitude less luminous than believed when I wrote the Cepheid mass reviews in 1979 and 1980 (Cox, 1980). Others like Turner (1986), studying NGC 6087 and Gieren (1986a), using surface brightness and radial velocity measurements for 30 galactic Cepheids, feel that Schmidt is correct in reducing the Cepheid luminosities, but by a slightly smaller amount than 0.5 magnitude. It does seem that these newer luminosities are being accepted, and that causes severe problems for Cepheid masses.

As we reported at Toronto (Kidman and Cox, 1985), we need to be careful about such low luminosities and masses. It takes a mass of 4 or 5 solar masses merely to evolve into the pulsation instability strip, and that result has been verified by several evolution track studies. Using the standard BIT evolution tracks, the low Schmidt luminosities frequently give masses smaller than this lower limit. The way out of this dilemma seems to be to have tracks that are less bright (larger Z) for masses around 5 solar masses. Another intriguing way is to have surfaces enhanced by helium that will give smaller radii for these models and bluer evolution into the instability strip at slightly lower masses. This effect is limited, however, because the blue looping is caused by the structure of the deep composition gradient set up by prior evolution, and not much influenced by composition variations. Cepheids could have somewhat lower masses, and many problems such as the discrepancy between the observed and theoretical ratios of the numbers of long and short period Cepheids discussed by BIT would be alleviated.

Another way of determining the mass of a Cepheid is to use the observed period and an approximate effective temperature. Four equations involving these observed quantities: the definition of the effective temperature, the evolution mass-luminosity relation, the period-mean density relation, and a theoretical fit for the pulsation constant Q, can be used to derive four values: the mass, radius, luminosity, and pulsation constant. These "theoretical masses" (Cox, 1979) differ little from the evolution masses, determined from the observed luminosity only, even though they do not require a known luminosity. They are similar because of the strong influence of the mass-luminosity relation obtained from theoretical evolution tracks.

The classical pulsation mass first discussed by Cogan (1970) is most often computed for variable stars. It needs the luminosity, effective temperature, and the pulsation period with the mode assumed. These

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masses have been moved around as the luminosity (and effective temperature) scale of the Cepheids have changed. With the increase of distances and luminosities by 0.26 magnitude, when the Hyades distance was increased, the masses increased accordingly. Then with the Schmidt observation that the distances of galactic clusters containing Cepheids needed to be reduced from those luminosities by 0.5 magnitude, the masses have decreased by a factor of two. Even though evolution and theoretical masses may have to be reduced by modified overshooting evolution tracks, the lower pulsation masses are now appreciably out of line now with evolution masses.

Observers have long wanted to settle this mass question by measuring the Baade-Wesselink radii that can be inserted into the period-mean density relation with assumed pulsation mode to quite directly give the Cepheid mass. The problem was, and still is, that the radius, which needs to be taken to the third power in the mass determination, is not accurate enough. If Fourier fitting and other special techniques can give radii to the expected accuracy of a few percent, as Simon (1988, preprint) suggests for a Cepheid like U Sgr, then Wesselink masses of great value can be made available.

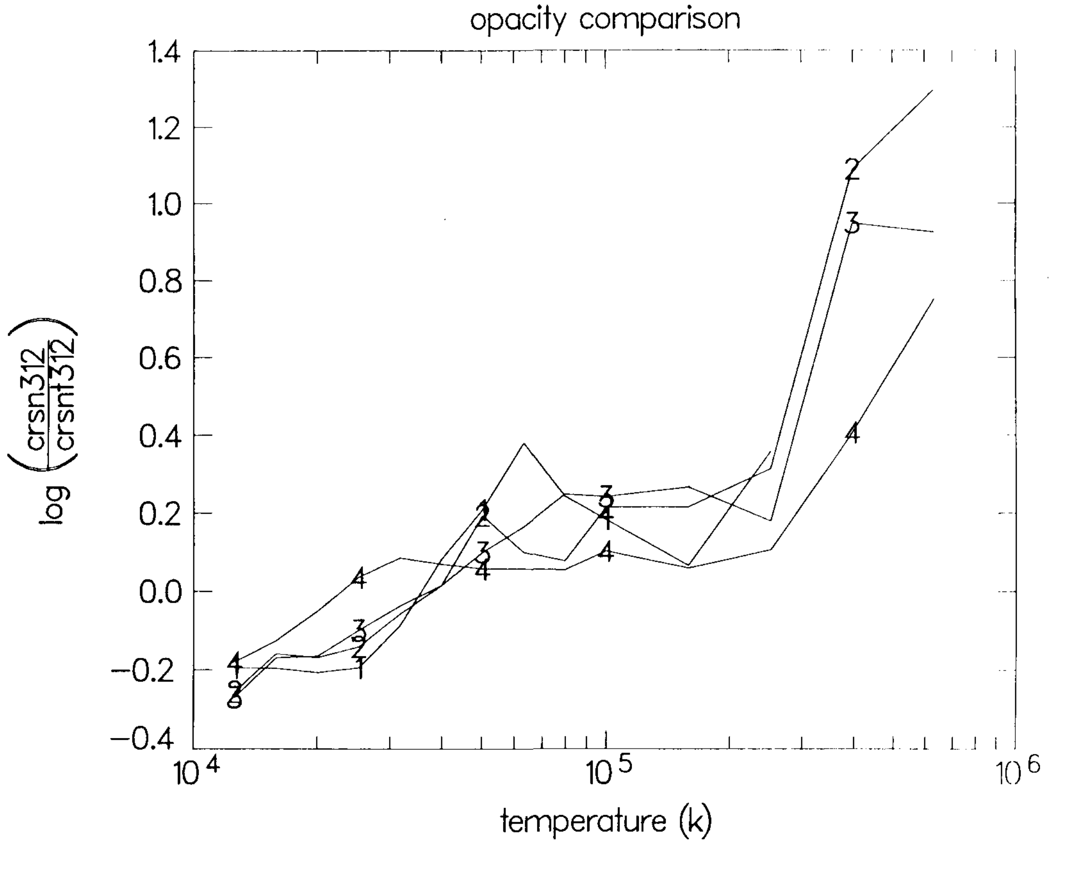
It does seem currently, that these masses are not significantly discrepant with respect to the earlier mentioned dynamical, evolution, and theoretical Cepheid masses, but the accuracy available is often very poor. For example, Gieren (1986b) presents radii and masses for 30 galactic Cepheids using the surface brightness method of Barnes and Evans with simultaneous BVRI photometric and radial velocity measurements. His small radii often give small masses compared to the theoretical masses (Cox, 1979). A possible fix for this problem is to have the theoretical evolution tracks at a higher luminosity for a given mass than those given by BIT, but that is the opposite direction than needed to fix the SU Cyg mass problem. My opinion is that we cannot rely yet on the Wesselink masses for Cepheids.

Carson and Stothers (1988) now claim that masses based on the phase of the bump in the light and velocity curves for Cepheids with periods between about 5 and 21 days are close to all those given by dynamical and evolution methods. Long ago Simon and Schmidt (1976) demonstrated that the bump was a manifestation of the resonance between the fundamental and second overtone modes which appears when the ratio of these periods lies between 0.47 and 0.53. Using the Carson opacities in nonlinear one-dimensional calculations, Carson and Stothers can produce the velocity curve bumps at a mass centered at 6 solar masses. This "bump" mass is only 15 percent smaller than given by Cox in 1979 for evolution, theoretical or pulsation masses for periods near 10 days. This is an unexpected result, because previous authors (Fricke, Stobie, and Strittmatter, 1972, and many others) got "bump" masses near 4 solar masses, just over half the then conventional masses for the 10 day period. The "bump" masses have been anomalously low since Christy (1968) and Stobie (1969) first noticed this effect in the nonlinear calculations for classical Cepheids.

A comparison of the Los Alamos Opacity Library opacities for the Carson 312 mixture (Y=0.25, Z=0.02) shows that the reason Carson and Stothers can get near conventional masses in their nonlinear calculations is that the Carson opacities are just about as much larger than the Los Alamos ones in the 150,000K to 800,000K temperature range as Simon (1982) first suggested to alleviate the mass anomaly. The larger Carson opacities reduce the second overtone to fundamental mode period ratio for six solar mass Cepheids to near resonance.

Figure 1 compares the Carson and Los Alamos opacities, just as previously presented by Cox and Kidman (1984). The line labeled 4 at 10^{-5} $g \cdot cm^{-3}$ indicates that the Carson opacities are approximately twice the Los Alamos opacities in the interesting temperature and density region. Since the Carson opacities in this temperature region have been shown to be incorrect (Carson et al. 1984), and opacities in this region

Figure 1. The logarithm of the ratio of the Carson 312 opacities to the Los Alamos Opacity Library opacities are plotted versus temperature for log density $(g \cdot cm^{-3})$= -8, -7, -6, and -5. The CNO bump is seen at the highest temperatures.



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are important to the "bump" Cepheid structure, we cannot accept this "reconciliation" between theory and observations that they propose.

I cannot support the Carson and Stothers statement that use of the Carson opacities makes no difference; I find that it indeed does. In linear theory studies, 10 day, 7 solar mass models (5420K, 1.536×10^{37} $erg \cdot sec^{-1}$) have a second overtone to fundamental mode period ratio of 0.52 using the Carson opacities and 0.55 with the King4a mixture of Cox and Tabor (1976). This reduction of the period ratio is just about what one would expect for the larger Carson opacities at the sensitive temperatures.

Another problem that Carson and Stothers have is that the amplitudes of the velocity curves, just as for earlier work for the RR Lyrae variables, are too large, probably because of their different helium opacities and their larger derivative with respect to temperature. This may explain why theoretical bumps are seen between 5 and 21 days whereas the observed light and velocity curves have these bumps only between 7 and 12 days. I suggest that the large helium opacities drive these nonlinear models to such a high amplitude that the bumps occur way outside the observed resonance region.

I also want to note that Fernie and Chan (1986) have discovered that in this second overtone to fundamental mode resonance region, all Cepheids seem to pulsate with about the same moderate amplitude, midway between that for many of both shorter and longer period. Apparently the resonance locks-in a ten percent radius amplitude, almost sinusoidal shape behavior that is not displayed for Cepheids at other periods where a wide range of amplitudes is observed.

I must, however, point out that Cogan, Cox and King (1980) were not able to make sense of Cepheid amplitudes. My best opinion is that the Cepheids have a small range of helium composition in the driving region that produces a range of amplitudes for most classical Cepheids. However, when there is the near two to one resonance, the driving becomes limited regardless of the helium abundance.

I very much like the recent paper by Andreasen and Petersen (1988) and another by Andreasen (preprint) who attempt to solve the "bump" and "beat" Cepheid low mass anomaly by increasing the opacity of the stellar material. This is the idea of Simon (1982), and it has had some notoriety because earlier ideas about this "bump" mass and the double-mode Cepheid mass anomalies possibly are not so desirable. The goal is to get the two classes of period ratio Cepheids ("bump" and "beat") to have an envelope structure such that the period ratios decrease for a given mass from those using conventional models.

Currently the popular idea is that the opacity in the temperature range of 150,000K to 800,000K needs to be increased by approximately a factor of 2.5 to change the surface layer structure of Cepheids so that the observed period ratios of the dozen double-mode classical Cepheids can be matched. This idea has been taken up by Andreasen and extended to

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other yellow giant classes such as the "bump" Cepheids, the RR Lyrae and δ Scuti variables. This adjustment has been successful in changing the second overtone to fundamental mode period ratio resonance to near 10 days as observed for the central period for the "bump" Cepheids. It also has increased the mass of the RR Lyrae variables to as much as 0.75 solar mass so that, with the asymptotic giant branch evolution, some additional mass loss can occur to produce the common form of white dwarf at 0.6 solar mass. Finally, the δ Scuti variable period ratios, for those dozen or so that show two radial periods, are predicted closer to those observed.

At Los Alamos, we have had great difficulty in accepting that the opacities in the required temperature range are wrong by a factor of more than two. Magee, Merts, and Huebner (1984) have investigated this problem in detail and find no opacity problem. I must note that the recent Iglesias, Rogers, and Wilson (1987) result that for iron, the same-shell (n=3 to n=3) transitions are larger than expected, giving a big opacity increase for iron at 20ev (232,000K) is not likely relevant to the stellar composition of Cepheids. The number fraction of iron in this composition is 3.7×10^{-5}, and investigations by myself have shown that iron only rarely has any influence on mixture opacities.

But even if the iron proves to be an important component of the opacity at the temperature of kT=20 ev, the Livermore opacity work has shown that at 60 ev, the Los Alamos values are unchanged. Thus the desire to have opacities more than doubled over the entire temperature range of 150,000K to 800,000K does not yet seem in hand. For classical Cepheids, the opacities do not have be increased at these highest temperatures, however. It is only the δ Scuti variable period ratios that dictate opacity increases at 800,000K as Andreasen at this conference demonstrates.

We now propose, however, that instead of an increase in the opacity, the mass anomalies can be solved by an increase in the heavy element abundances in a thin layer hidden below the surface. This then increases the opacities as desired, but without any large error in the atomic physics.

In a poster paper Siobahn Morgan, working with me this summer at Los Alamos, shows that with an effective Z of 0.30, period ratios of double-mode Cepheids can be reduced to the observed level. This indeed is an increase in the opacity by the amount suggested by Simon and by Andreasen. We are now running the Iben evolution program with element diffusion and element levitation to see if such abundance increases actually can be predicted.

We need to be careful about a great increase in the heavy elements because an inverted mu gradient will be Rayleigh-Taylor unstable to downward mixing. Figure 2 shows the Z variation throughout the envelope. This structure is formally stable against downward mixing because the radiative gradient there is so subadiabatic. Any mixing, which will actually be rather rapid, will have to be countered by an

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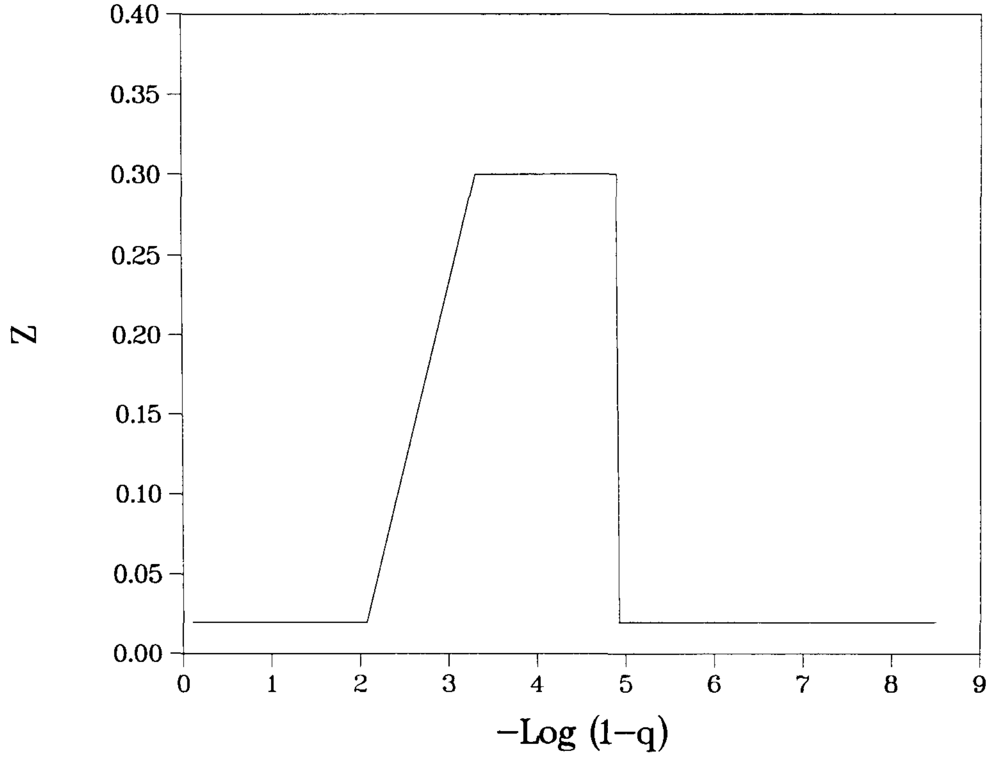
effective and rapid element levitation. This large Z means that actually only 4 percent in the particles are CNONe elements. This last number is important, because Michaud privately to me has noted that it is difficult to get number fraction enhancements of more than a few percent when all the photons are being absorbed by the levitating elements.

This opacity increase causes a thin convection zone to appear in just the region where the increased opacity is desired. While unrelated to this problem, we have suggested that the high luminosity (high photon flux) B stars may also have a thin layer of enhanced Z elements that

Figure 2. The Z mass fraction abundance is plotted versus the mass into a 5 solar mass model for a double-mode Cepheid. Going inward from the surface, the Z abundance starts up at a mass fraction of -log(1-q) of 4.92 at a temperature of 61,000K. The plateau is reached at 4.89 and 63,000K. The inverted mu gradient begins at -log(1-q) of 3.30 and ends at 2.07 with temperatures, respectively, of 200,000K and 420,000K. The hydrogen convection zone lies between -log(1-q) of 6.68 and 6.14 with temperatures of 6000K and 25,800K, and the helium convection zone is between 5.77 and 5.29 with temperatures of 34,000K and 49,000K. A thin convection zone also exists on the plateau between 4.88 and 4.70 with temperatures of 64,800K and 75,000K.



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produces a convection zone. For the B stars, it just may be that this slowly reacting convection is the cause of the β Cephei variable star pulsations that for at least a quarter of a century has remained unexplained.

Cox, Deupree, King and Hodson (1977) suggested long ago an enhanced helium layer at the surface down to a temperature of 250,000K that would make the model appear to the fundamental and first overtone modes to be less concentrated. Thus these modes would have their periods increased by over 5 percent with the fundamental feeling the decreased concentration more than the first overtone. Then the period ratio could be reduced to the observed values with conventional masses. Because of the Rayleigh-Taylor instability of such a composition inversion, this idea has never been generally accepted. Now, however, we are again investigating whether it may be possible that the gradients are shallow enough to be stable in the presence of the very subadiabatic radiative gradient.

Stothers (1979) suggested strong magnetic fields that could deconcentrate the models by having this additional pressure contribution in these same surface layers. Again the idea has not obtained acceptance, because such strong fields are not thought to be possible at a mass fraction depth of 0.001 into the star.

FUTURE TASKS

For the future, it seems to me that there are several observational and theoretical tasks that are appropriate for understanding the pulsations of Cepheids and the structure of the instability strip in the H-R diagram. We are interested in the masses, radii, luminosities, compositions, and pulsation periods and modes.

Observers are indeed busy trying to identify cases where binary stars can measure stellar masses. Is it possible that the eclipsing binary BM Cas (Fernie, 1987a) will prove to be the Rosetta Stone after all?

Baade-Wesselink radii should be available with only a few percent uncertainty, but this status has not yet been achieved for any Cepheid. Investigations now being pursued may make this accuracy a reality with very accurate photometry in many colors including the infrared. Hopefully then the radius and mass problems will be alleviated at least for a few special Cepheids. It would seem appropriate that SU Cyg with its known dynamical mass be included in these Baade-Wesselink radius studies.

The more traditional ways of getting Cepheid masses using observed luminosities needs to be refined. This means that there is a need for an accurate luminosity scale; current uncertainties are greater than 0.1 magnitude. This implies an error of at least 15 percent in the mass, and that is just marginally acceptable for understanding Cepheids.

Double-mode Cepheids are useful for understanding the structure of the instability strip. The placement of the double-mode Cepheids at the transition line between the fundamental and overtone modes (Barrell, 1981) is valuable for guiding theoretical analyses, and with CO Aur at 6700K, it seems that this Cepheid is at the transition line between the first and second overtones. Is it certain that the overtone pulsators, SU Cas, HD144972, DT Cyg, and EU Tau are pure overtone pulsators? If so, as we now believe, the first overtone region must have a significant temperature width in the H-R diagram.

Searches for Magellanic Cloud double-mode Cepheids, and determination of their period ratios would give indications of how the period ratio depends on metallicity if there is any dependence at all. Observational information on what causes the low period ratio is badly needed.

A final need from observers is a clue to the pulsation amplitude changes for HR 7308. My proposed disappearance and reappearance of helium in the 40,000K temperature levels in the star might result in small surface composition changes in levitated elements like CNONe.

An important theoretical task is the calculation of evolution tracks for cases with convective core overshooting and for cases of possible CNONe element levitation with helium being dragged up also. The large SU Cyg mass seems to indicate that we do not need core overshooting, but a decrease in the track luminosities for a given mass such as for a larger Z.

For understanding the double-mode problem, more linear theory models are needed to obtain periods in unconventional composition cases. Clearly the opacity uncertainty needs to be settled.

Full amplitude nonlinear calculations are needed for a variety of cases with varying compositions (probably only helium), so that the large range of amplitudes observed can be understood. And these especially need to be done near the second overtone to fundamental mode resonance to explain the uniformity of light and velocity curves there.

Simple applications of time-dependent convection need to be made. The question is whether this feature is an essential one for obtaining double-mode behavior.

As a final important task for the theoreticians, a demonstration with nonlinear methods, of the double-mode behavior for classical Cepheids is most important. Hopefully, one of the period ratio anomaly solutions, such as surface enhanced helium, increased envelope opacities, or augmented Z in the deeper pulsating layers, will also allow simultaneously a demonstration of double-mode Cepheids.

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