

who lived (not "flourished", as is stated on p.4) ca. 971 - ca. 1029. This work seems to be the oldest reckonbook surviving in the original Arabic and using Hindu numerals in the algorithms. The editors have published the Arabic text in facsimile, together with an English translation and excerpts from a Hebrew commentary of the 15th century. Kūshyār explained the four basic operations, the extraction of square and cube roots, methods of approximation for these and the use of sexagesimal tables. The operations are carried out both in the decimal and the sexagesimal systems! In an extensive introduction the editors not only give a survey about related Arabic texts but also discuss carefully the various procedures used by Kūshyār. The volume is equipped with a glossary of Arabic mathematical terms and an index.

C. J. Scriba, University, Hamburg

Evolution of Mathematical Thought, by H. Meschkowski. (Translated by Jane H. Gayl), Holden-Day, 1965. Original German edition, 1956. 147 pages.

The thesis of this book is presented in chapter I and again, in more detail, in the concluding chapter. It is that "... mathematical thinking leads to a new kind of philosophy which makes all previous systems seem 'pre-scientific' ... understanding of the philosophical consequences of basic research in mathematics need not always be reserved for a little band of the 'initiated' ... scientific knowledge can definitely be brought closer to interested laymen in a serious manner."

The titles of the remaining chapters are: Foundations of Greek Mathematics, The Road to non-Euclidean Geometry, The Problem of Infinity, Cantor's Foundation of the Theory of Sets, Antinomies and Paradoxes, Intuitionism, Geometry and Experience, Problems of Mathematical Logic, Formalism, Decision Problems, Operative Mathematics, The Philosophical Harvest from Research in the Foundations of Mathematics. There is also an appendix: Education in the Age of Automation.

With reservations noted below, it is a successful book: the mathematical examples are well chosen, and are nicely balanced by readable discussions of just what is supposed to be significant about them.

Non-Euclidean geometry is shown to have led to a detachment from the Platonic view that mathematics is an exploration of the realm of ideas which uncovers universal truths. The irrationality of  $\sqrt{2}$ , here proven both algebraically and geometrically, had already shown the Pythagoreans that insight, gained mathematically, demonstrated that the search for such truths led to some surprising results.

The paradoxes of the infinite are claimed to serve an important educational function in that they put bounds to the applicability of given intuitions and hence clarify the function of intuition generally. The procedure given for constructing a plane set which is congruent by rotation

and translation to each of two subsets into which it is divided, unfortunately (and unnecessarily) uses the transcendentalism of  $e^i$  without making this clear to the average reader.

The Goldbach conjecture is used very convincingly to explain how one can be led to doubt the universal applicability of tertium non datur.

The desirability of "Cantor's Paradise" is used as motivation for developing the doctrines of Formalism, which is introduced in a sound historical fashion by a discussion of Hilbert, geometry and experience.

The chapter on Mathematical Logic is, unhappily, very weak, and must be very confusing indeed to the untutored reader. It is stated that "each proposition A, B, C, ... corresponds to a truth-function  $f(A)$ ". Material implication is poorly handled - "as long as there is no case where A is true and B is false, then the implication holds". Tautologies are "correct for arbitrary substitutions of sentential variables". In predicate calculus "the written form separates the proposition from the object ... the proposition is then written as a mathematical function". There are also numerous confusions of names with what they stand for. It is cautioned that in reference to mathematical logic as the "metaphysics of today" one should not "pour new wine into old bottles".

The concept of a formal proof is clearly explained, and the intuitionists' criticism of the philosophy of formalism is briefly mentioned. Much importance is attached to Skolem's 1934 paper on the non-categoricalness of finite or denumerable axiomatizations of arithmetic, but the result is completely spoiled by a total failure to communicate what Skolem means by an axiomatization. (Fund. Math. XXIII. p.160 clarifies the matter succinctly.)

In connection with the axiomatic approach, Bourbaki is used as an example of modern emphasis on "basic structures". The outline of the proof of Godel's incompleteness theorem leans heavily on a formula which "asserts its own unprovability": more discussion of what this means would have been helpful.

The basic ideas of Lorenzen's Operative Mathematics are described, and an attempt is made to show that for "schematic operations" the principle of induction is demonstrable. It is not clear, however, just what it is demonstrable from. On p.127, lines 9, 16, the subscript  $\nu$  is misplaced.

The final chapter re-affirms the thesis of the book, and the appendix affirms that "the educational possibilities which have been opened up by the transformation of scientific thinking are still not being fully utilized".

R. A. Staal, The University of Waterloo