

THE  
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SYMBOLIC LOGIC

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ALONZO CHURCH      S. C. KLEENE      ALICE A. LAZEROWITZ

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## TABLE OF CONTENTS

A note on direct products. By L. NOVAK GÁL . . . . .	1
Open sentences and the induction axiom. By J. R. SHOENFIELD . . . . .	7
An extensional variety of extended basic logic. By FREDERIC B. FITCH . . . . .	13
Reviews . . . . .	22
Twenty-second annual meeting of the Association for Symbolic Logic. By BURTON S. DREBEN . . . . .	103
Abstracts of papers . . . . .	103
Elections . . . . .	111
Notice of a meeting of the Association for Symbolic Logic . . . . .	111
United States National Committee of the International Union of the History and Philosophy of Science . . . . .	111
Foundational aspects of theories of measurement. By DANA SCOTT and PATRICK SUPPES . . . . .	113
Überprüfung einer frühen Anwendung der Kombinatorik in der Logik. By KURT-R. BIERMANN and JÜRGEN MAU . . . . .	129
A 12th century paradox of the infinite. By IVO THOMAS . . . . .	133
Peirce's axioms for propositional calculus. By A. N. PRIOR . . . . .	135
Generalization of a lemma of G. F. ROSE. By I. L. GÁL, J. B. ROSSER and D. SCOTT . . . . .	137
Simplified axioms for many-valued quantification theory. By ATWELL R. TURQUETTE . . . . .	139
On closure under direct product. By C. C. CHANG and ANNE C. MOREL . . . . .	149
Mathematical significance of consistency proofs. By G. KREISEL . . . . .	155
Reductions of Hilbert's tenth problem. By MARTIN DAVIS and HILARY PUTNAM . . . . .	183
The classical and the $\omega$ -complete arithmetic. By A. GRZEGORCZYK, A. MOS- TOWSKI and C. RYLL-NARDZEWSKI . . . . .	188
Reviews . . . . .	207

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A note on direct products. By L. NOVAK GÁL . . . . .	1
Open sentences and the induction axiom. By J. R. SHOENFIELD . . . . .	7
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A 12th century paradox of the infinite. By IVO THOMAS. . . . .	133
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Reviews . . . . .	207
Logical reflection and formalism. By P. LORENZEN. . . . .	241
Alternative proof of a theorem of Kleene. By HAO WANG . . . . .	250
A formalization of inductive logic. By R. M. MARTIN . . . . .	251



TABLE OF CONTENTS

Note on Carnap's relational asymptotic relative frequencies. By FRANK HARARY . . . . .	257
Epimenides the Cretan. By A. N. PRIOR . . . . .	261
On Shupecki T-functions. By TREVOR EVANS and P. B. SCHWARTZ . . . . .	267
A problem of Rosser and Turquette. By ANGELO MARGARIS . . . . .	271
Measure-theoretic construction of incomparable hyperdegrees. By CLIFFORD SPECTOR. . . . .	280
Finite axiomatizability using additional predicates. By W. CRAIG and R. L. VAUGHT. . . . .	289
Three theorems on recursive enumeration. I. Decomposition. II. Maximal set. III. Enumeration without duplication. By RICHARD M. FRIEDBERG. . . . .	309
Elementary completeness properties of intuitionistic logic with a note on negations of prenex formulae. By G. KREISEL. . . . .	317
Gödel numberings of partial recursive functions. By HARTLEY ROGERS, Jr. . . . .	331
Reviews . . . . .	342
Notices of two meetings of the Association for Symbolic Logic . . . . .	368
A remark on free choice sequences and the topological completeness proofs. By G. KREISEL. . . . .	369
Degrees of formal systems. By J. R. SHOENFIELD. . . . .	389
An investigation on the logical structure of mathematics (V). Contradictions of Russell's type. By SIGEKATU KURODA. . . . .	393
On recursively enumerable and arithmetic models of set theory. By MICHAEL O. RABIN . . . . .	408
On the formalisation of indirect discourse. By R. L. GOODSTEIN. . . . .	417
Professor Goodstein's formalisation of the policeman. By L. JONATHAN COHEN. . . . .	420
Schémas pour le calcul des propositions fondé sur la conjonction et la négation. By JEAN PORTE. . . . .	421
Reviews . . . . .	432
Twenty-third annual meeting of the Association for Symbolic Logic. By R. M. MARTIN . . . . .	456
Abstracts of papers . . . . .	456
Elections. . . . .	461
Notice of a meeting of the Association for Symbolic Logic . . . . .	461
List of officers and members of the Association for Symbolic Logic. . . . .	462

## ERRATA

### VOLUME 13

Page 112, line 22 from the bottom. The subscript after  $P$  should be  $n + 1$  instead of  $n$ .

### VOLUME 16

Page 286, line 15. For «xxi», read «xii».

### VOLUME 21

Page 89, line 10. Insert right parenthesis before first square bracket.

Page 123, line 11. For the second occurrence of  $\mathfrak{A} \rightarrow \mathfrak{B}$ , read  $\mathfrak{B} \rightarrow \mathfrak{A}$ .

### VOLUME 22

Page iv, line 12. For «FRIEDMANN», read «FRIEDMAN».

Page 293, line 15. For «1953», read «1955».

Page 304, line 13 from the bottom. For «quantifier free», read «quantifier-free».

Page 351, line 12 from the bottom. The first word in the line should be «von».

Page 351, line 8 from the bottom. For «Axione», read «Axiome».

Page 351, line 7 from the bottom. For «widerspruchsfrie», read «widerspruchsfrei».

Page 351, line 2 from the bottom. Supply a period at the end of the line.

Page 351, last line. Add the first  $s$  of «Widerspruchsfreiheitsbeweis».

Page 361, line 14. For the first «to» read «the».

Page 371, lines 2, 3, 6. The inverted circumflexes over  $R$  and  $S$  are intended as breves.

Page 371, line 10 from the bottom. For «mosti mportant», read «most important».

Page 380. Throughout line 23 from the bottom, wherever the letter  $s$  has two subscripts, read the second of these subscripts as a subscript to the first subscript.

Page 386, line 8. For «1925j 26» read «1925/26».

Page 387, line 16. For «ari-stotelischen», read «aristotelischen».

Page 432, column 1, line 15. For «Los», read «Łoś».

Page 435, column 2, line 4. For «Ismann», read «Issmann».

Page 439, column 2, line 46. For «caseu», read «cause».

Page 441, column 1, line 38. For «Fefreman», read «Feferman».

### VOLUME 23

Page 27, line 9. For «thaupt», read «thought».

Page 31, line 6 from the top, and line 8 from the bottom. For «petito», read «petitio».

Page 40, line 10. For «notions», read «concept».

Page 59, line 2. For « $F$ », read « $\phi$ », twice.

Page 73, line 7. For «“almost” every», read «“almost every”».

Page 73, line 7. The last word in the line should be «is».

Page 97, line 14. In *Sectiunea*, there should be a comma below the  $t$  as an accent.

## ERRATA

Page 97, line 23. Delete the dash.

Page 98, line 13. Delete the period after «*élémentom*».

Page 100, line 12 from the bottom. After «(XV 210)», insert: «*Philosophical Analysis* (cf. XVI 298–301), and Pap's *Elements of Analytic Philosophy* (XIX 74)».

Page 102, line 3. For XIV, read XVI.

Page 104. In line 5 of the second column at the top of the page, change  $p$  to  $P$ , and in line 7 of the same column, change the last  $p$  to  $q$ .

Page 224, line 12 from the bottom. For «*množestvo*», read «*množéstvo*».

Page 225, line 22. For «*Trétégo*», read «*Trét'égo*».

Page 230, line 13. For «*isčislénie*», read «*isčisléníé*».

Page 234, line 16 from the bottom. For «*Akadem*», read «*Akadémii*».

Page 239, line 1. For «*Stážkin*», read «*Stážkín*».

Page 321, second line of footnote. For  $\neg(Ex)$ , read  $(Ex)\neg$ .

Page 322, Third line of Theorem 4. After «*quantifier*» insert «*,with each prime formula preceded by at least one negation symbol,*».

Page 357, line 6. For «(1956)», read «(1955)».

Page 361, line 5. Insert «no. 1/2» after «vol. 16».

Page 365, line 8. The comma should be after «1956» rather than before it.