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On Mexican poverty-trap regimes and struggling to escape them

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Abstract
This paper deals with the phenomenon of poverty-trap regimes in Mexico, that is, self-reinforcing mechanisms in which municipalities which start poor remain poor. We develop a coordination game of poverty traps driven by strategic interactions of economic agents: people choose to complete or not their education levels since it might be excessively costly and unprofitable. A one-shot game is constructed and then converted into a system of differential equations in which strategies that perform relatively better become more abundant in the population. Applying evolutionary games and symbolic-regimes dynamics (nonparametric and nonlinear techniques), we show that Mexican regions are in poverty-trap regimes (stable and dynamically evolving low-level equilibria) characterized by incomplete education and low income since initial conditions (education and income per capita) are such (very precarious) that poverty is the stable steady-state situation. We examine scenarios to show that to overcome the high-poverty regime by the year 2030, it is necessary to reduce incomplete education by 10% in the 5-year periods 2020–2025 and 2025–2030 and increase per-capita income by 10% in both periods.

Keywords: Behavioral macroeconomics; Economic regimes; Poverty traps' stability

JEL classifications: C73; I25; I30; O10

1. Introduction and stylized facts

Poverty traps have been studied extensively from both a theoretical and an empirical point of view [see, e.g., Bowles et al. (2006); Kraay and McKenzie (2014); McKay and Perge (2013); Noritomo and Takahashi (2020); Radosavljevic et al. (2021)]. Theoretically, poverty traps (multiple equilibria) can occur as a result of non-convexities in production technologies in the presence of failures in key markets, such as credit or insurance [e.g., Azariadis (1996); Galor and Zeira (1993)]. Empirically, poverty traps can lead to unintended consequences of anti-poverty programs and adverse effects on productivity.[1]

One of the most prolific lines of research that have emerged in development economics concerns the empirical identification of households facing poverty traps. Many studies have sought to find S-shaped well-being dynamics using both parametric and nonparametric methods [Growiec and Growiec (2014); Barrett and Carter (2013)]. In this research paper, our approach is novel in linking evolutionary game theory, nonparametric dynamics for economic agents such as the municipalities of an economy, and panel-data econometric regressions. We estimate the results from game-theory modeling [Accinelli and Sanchez Carrera (2012); Sanchez Carrera (2019)], combining two nonstandard estimators, namely symbolic dynamics and panel-type regressions [Blundell and Bond (1998); Bollt et al. (2001); Brida et al. (2021); Daw et al. (2003); Risso
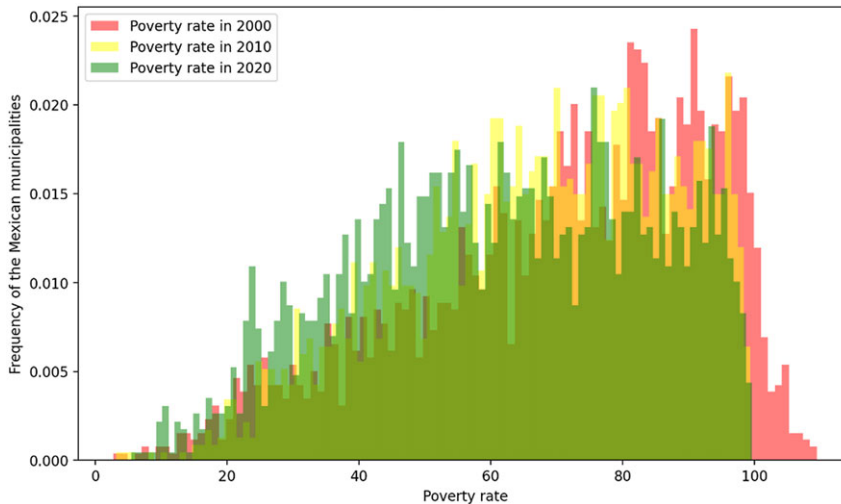
Figure 1. Frequency distribution of poverty in Mexican municipalities for the years 2000, 2010, and 2020. Source: own elaboration.

(2015, 2018); Williams (2004)]. Using these approaches, we find evidence of multiple equilibria or poverty-trap regimes in the case of Mexico. Thus, this paper investigates the existence of poverty-trap regimes in Mexico, that is, self-reinforcing mechanisms through which Mexican municipalities that start poor remain poor. In characterizing the underlying causes of such poverty traps, this paper focuses specifically on the importance of low education levels in the Mexican population. Methodologically, this paper combines panel-data econometric techniques with several nonstandard approaches in economics, such as evolutionary game theory and symbolic dynamics. Applying these methods to data from the National Council for the Evaluation of Social Development Policy (CONEVAL) from 2000 to 2020, this paper argues that there is evidence of poverty traps in Mexican municipalities and that persistently low education levels are a key driver of this phenomenon.

On the topic of poverty and education, Mexico is an interesting case study[2] as poverty, extreme poverty, and economic inequality in Mexico have been the main problems in the last 30 years [Antman and McKenzie (2007); Risso et al. (2013); Brida et al. (2021)]. Antman and McKenzie (2007) find no evidence of poverty traps using a cohort-based approach tracking cohorts in 58 repeated cross-sections. Nevertheless, gender inequalities, ethnic-racial discrimination,[3] corruption, wealth concentration, and, in general, those aspects related to the lack of socioeconomic opportunities constitute structuring axes of being poor in Mexico [Boltvinik and Damián (2020); CEPAL (2020a,b); Iniguez-Montiel and Kurosaki (2018); World Bank Group (2018)]. Taking into account the data available from Mexico's CONEVAL, poverty in Mexico decreased very moderately between 2008 and 2018. The population in conditions of poverty in 2018 had shrunk by 2.5 percentage points compared to 2008. Meanwhile, extreme poverty affected 7% of the population in 2018 and 11% in 2008. In 2018, according to CONEVAL data, 41.9% of the Mexican population lived in poverty (52.4 million people) and 7.4% suffered from extreme poverty (9.3 million people).[4] In 2020, in the context of the global economic crisis resulting from the COVID-19 pandemic and other external economic factors,[5] the Mexican poverty rate rose to 43.9% and the extreme poverty rate to 8.5% [Lustig and Martínez Pabón (2021)]. These facts motivate the present research.

Using CONEVAL data, Figure 1 empirically represents the distribution of poverty at the municipal level, including the characteristics of poverty in the municipalities of Mexico.

Table 1. Descriptive statistics for the Mexican poverty rate, 2000–2020

Statistic	2000	2005	2010	2015	2020
Mean	70.9%	65.6%	67.0%	65.5%	61.9%
Median	74.7%	68.0%	69.1%	67.9%	62.7%
Std. deviation	22.2%	19.7%	20.2%	21.5%	21.9%
30% or below	6%	6%	5%	7%	9%
70% or above	58%	45%	48%	47%	39%

Source: Own elaboration.

Figure 1 shows a shift to the left of the distribution and a decrease in the highest values of poverty. As can be seen in Table 1, 58% of municipalities had poverty rates of 70% or higher in 2000, 48% did in 2010, and 39% in 2020. However, poverty has increased at the bottom (although not with the same impact): the share of municipalities with poverty rates of 30% or lower rose from 6% in 2000 to 9% in 2020. Table 1 presents descriptive statistics such as the mean and median poverty levels in 2000, 2005, 2010, 2015, and 2020. Notably, poverty on the mean has decreased more; however, in the mean (and median), more than 50% of Mexicans are listed as poor. Table 1 also indicates that Mexico's average poverty rate decreased from 70.9% to 61.9% between 2000 and 2020.

CONEVAL (2019) identifies six main social deficiencies feeding back into poverty: (i) the educational gap (incomplete education), (ii) lack of access to health services, (iii) lack of access to social security, (iv) lack of housing quality and space, (v) lack of access to basic services in the home, and (vi) lack of access to food. The number of social deprivations experienced by the population is an indicator of the severity of these deficiencies and vulnerabilities. Not having access to social security alone is not the same as having access to social security and education but another type of deprivation. Between 2008 and 2018, a slight improvement was observed in the social deprivation indicators at the national level and in most states. The average number of deprivations of the population in a situation of poverty went from 2.8 in 2008 to 2.2 in 2018. Similarly, the average number of social deprivations of the population in a situation of extreme poverty decreased from 3.9 to 3.6 in the same period. Specifically, the share of the population with social deprivation in educational matters in Mexico fell from 21.9% to 16.9% between 2008 and 2018. In the same period, lack of access to health services went from 38.4% to 16.2%, lack of access to social security was lowered from 65.0% to 57.3%, lack of housing quality and space shrunk from 17.7% to 11.1%, lack of access to basic services in the home fell from 22.9% to 19.8%, and lack of access to food reduced from 21.7% to 20.4%. In 2020, due to the negative effects of the COVID-19 pandemic,[6] the rates of poverty, extreme poverty, and inequality increased significantly as a result of the sharp increase in unemployment, the contraction of consumption, and the breakdown of commercial chains. Despite compensatory effects on price levels, given the contraction in demand, lower-income families faced an increase in the prices of basic goods. Families whose income depended on informal jobs were the most affected. According to CONEVAL (https://www.coneval.org.mx/Medicion/Paginas/PobrezaInicio.aspx), the number of people living in poverty in Mexico increased by 3.8 million and the poverty rate went from 41.9% in 2018 to 43.9% in 2020. Meanwhile, the number of people in extreme poverty in the country increased by 2.1 million people and the extreme poverty rate rose from 7.0% in 2018 to 8.5% in 2020. In terms of social deprivation, the biggest change between 2018 and 2020 was an increase of 12.0 percentage points in the lack of access to health services, from 16.2% to 28.2%. However, with the arrival of the new progressive leftist government, called AMLO the 4th transformation of Mexico, a decrease of 1.7 percentage points was observed between 2018 and 2020 at the national level in the indicators of lack of housing quality and spaces and lack of access to basic services in housing, as well as a 1.4 percentage-point reduction in lack of access to social security. The percentage of the population living in poverty in rural areas remained at similar levels, changing from

57.7% to 56.8% between 2018 and 2020. In contrast, in urban areas, this percentage increased by 3.2 percentage points in the same period, from 36.8% to 40.1%. According to ECLAC data (which uses a different methodology from CONEVAL), in 2018, the poverty rate was 56.3% in rural areas and 22.3% in urban areas.

In Mexico, educational lag or incomplete education is a problem of multifactorial origin since there are various reasons why people in this situation do not receive literacy or do not complete their basic education. Educational lag is the result of a process in which various events intervene, such as exclusion from the educational system, school achievement below the minimum necessary, overage, and premature abandonment of the studies undertaken, or "school dropout" [INEE (2014); Muñoz Izquierdo and Silva Laya (2012)]. For instance, Tagliati (2022) analyzes the evaluation of the PAL in-kind transfer program in Mexico targeting poor households, and Todd and Wolpin (2006) previously developed a structural household model to evaluate the conditional cash transfer program PROGRESA/Oportunidades to analyze the potentially higher psychic cost of attending a school when a child is behind in school for his age. The existence of such a psychic cost implies that forward-looking parents may forego making their child work when faced with a high child-wage offer that is transitory. Bachhoff et al. (2007) identified particular factors internal to schools that intervene significantly in determining their academic results. They added that the only factors that cannot be solved by the dependencies of the educational sector are those of socioeconomic origin that interfere in the generation and conservation of the school demand. An event that contributes to educational backwardness, where cultural traditions and economic reasons (income poverty) converge, is the fact that these children and young people join the labor market, for economic and/or cultural reasons of their parents. This situation is illegal because, by law, a child under 14 years of age is not allowed to have a job in the labor market. It also violates the rights of the child and is a factor for the child to perform poorly in school, which, in time, leads him to fall behind or, in the worst case, drives him to drop out of school.

Based on all these characteristics, we ask whether the Mexican economy is submerged in an evolutionary poverty trap that may be an institutional, evolutionarily dynamic, and representative in a game where the players/agents have also opted for the strategy of continuing to be submerged in this equilibrium of low levels called the poverty trap. Therefore, does Mexico display poverty-trap features? That is, does a steady-state equilibrium (or equilibria or regimes) of low income, educational lag, and high poverty exist in the country?

Authors such as Bowles (2006)[7] and Polterovich (2008) have developed an institutional view of the notion of poverty traps, studying institutional changes in a dynamic evolutionary framework where inequality and poverty traps emerge along with inefficient endogenous institutional arrangements. Furthermore, neighborhood effects in economic societies arising from social preferences can induce criminal behavior at equilibrium, as shown by Durlauf (1996, 2001). According to this literature, poverty traps are a particular equilibrium in an environment of multiple equilibria and are sustained over time by inefficient or perverse institutions. So evidence on poverty traps has been referred to as "a big question" for development economists [Banerjee (2020)]. An alternative approach that seeks to explain the persistence of poverty traps by combining evolutionary game theory and multiple equilibria, in which agents imitate the behaviors of others, has been developed by Accinelli and Sanchez Carrera (2012) and Sánchez Carrera (2012, 2019).

The contribution of this research paper, however, is broader than a simple game theoretical application as we affirm that it is relevant to both macro- and micro-economic policies that seek to find feasible solutions to poverty traps. Although the game theory model of educational options proposed here can elicit skepticism because it does not consider crucial dimensions that individuals or households take into account when making decisions, such as subsistence consumption levels, the opportunity cost of education, and the existence of liquidity or indebtedness restrictions, the game proposal explains the strategic action of behavioral decision of the

agents/players/citizens about whether to complete education or not. Two important considerations for the development of the game we present here may be pointed out. First, we typically think about education in an inter-generational and intertemporal setting: education is obtained in childhood and, hence, education choices are generally made by parents, whereas returns on education are realized in adulthood (primarily in the form of higher wages). Second, there is an opportunity cost to education, which is particularly important in poor settings: poor households usually face a tradeoff between sending their children to work (without completing education) to increase the household's current income and keep their children in school to maximize their children's welfare and potentially realize higher returns on education in the future. All these ideas have been developed in the literature on child labor, which, in line with this paper's objective, provides theoretical arguments linking poverty with incomplete education. For instance, Basu and Van (1998) develop a model with multiple equilibria: one in which children work (without completing education) because parental earnings are low and the household could not otherwise afford subsistence consumption, and another in which children do not work because parental earnings are sufficient. Although Basu and Van (1998) do not specifically talk about poverty traps in this context, the bad equilibrium is a situation in which households are stuck in a high-poverty and low-education environment. Other authors in this strand of literature have expanded on this idea and specifically modeled inter-generational poverty traps driven by child labor and incomplete education [Emerson and Souza (2003)].

Hence, this paper deals with the phenomenon of persistent poverty (poverty trap) in Mexico, where more than 50% of the population is listed as poor. According to the National Council for the Evaluation of Social Development Policy, poverty is influenced by incomplete education, lack of access to food, health services, and social security, lack of housing quality and space, and lack of access to basic services in the home. In this scenario, we propose and develop a coordination game of poverty traps driven by strategic interactions of economic agents: people choose to complete or not their education because it might be excessively costly and unprofitable. A simple one-shot game is constructed and then converted into a system of differential equations in which strategies that perform relatively better become more abundant in the population. We prove that the system admits four corner equilibria and four interior equilibria, and poverty-trap regimes of different levels might exist. Using a dataset of 2,430 municipalities for 2000, 2005, 2010, 2015, and 2020, we apply symbolic regimes with symbolic time series and econometric panel data to characterize the multiple equilibria of Mexican municipalities in poverty regimes. Then, interesting scenarios are discussed. This paper concludes that incomplete education and low per-capita income are the driving force behind the poverty-trap regimes in Mexico and argues that pushing policies aimed at completing basic education for all Mexicans and increasing their per-capita income would allow them to escape the poverty traps.

Thus, the ultimate goal of this paper is to offer important policy implications in the presence of poverty traps, that is, policies that help people overcome the threshold so that wealth, education, and, thus, economic growth can be part of them. This suggests "big push" policies, analogous to seminal models of industrialization that exhibit multiple equilibria (Hirschman, 1958; Murphy et al., 1989) but focus on the availability of assets (i.e., the completion of basic education and sustained income per capita) to the poor, which can be a powerful means of addressing the problem of poverty in Mexico.

The remainder of this paper is as follows. Section 2 develops the theoretical game of poverty traps driven by strategic interactions of economic agents that make multiple equilibria and evolutionarily stable poverty traps emerge. Section 3 applies the nonparametric technique of symbolic dynamics to characterize the multiple equilibria of Mexican municipalities in poverty regimes. Section 4 complements the above through an econometric exercise that seeks to determine the effects of the elasticities of education and income on poverty reduction. Section 5 concludes the document.

2. The game

In Mexico, the number of children out of school is large enough to feed back into poverty as the trend of potential students out of the classroom is one of sustained increase. This applies to both out-of-school adolescents (the percentage of adolescents of lower secondary school age who are not enrolled in school) and out-of-school children (the percentage of primary school-age children who are not enrolled in primary or secondary school).[8]

The Obligatory Schooling Standard of the Mexican State (NEOEM) establishes that the population with deprivation due to educational backwardness meets any of the following criteria: (i) he or she is between 3 and 15 years old, does not have compulsory basic education, and does not attend a formal education center; (ii) he or she was born before 1982 and does not have the level of compulsory education in force at the time he or should have completed it (complete primary school); and (iii) he or she was born in 1982 and does not have the level of compulsory education (complete secondary school). At the national level, it was observed that between 2018 and 2020, the educational gap increased by 0.3 percentage points (although this change was not statistically significant), from 19.0% to 19.2%.[9] The main problems of the Mexican educational system are coverage, quality, social inequity, and desertion driven by household income [Angulo-Cázares (2018)]. In this socioeconomic environment, Mexicans face the dilemma of completing their studies or not. These are the choices that a Mexican should make.

How the process of preference formation takes place in a society, for example, within the educational system, becomes an important focal point in development economics, stimulating a critical reflection on what preferences are cultivated in it. Generally, preferences can be activated, but they are stable in the short term. However, in the long term, they can change and are specific to socialization (peer effects) and culture and strongly interrelated with the educational levels of a society [Bowles (1998, 2008)].[10] Particularly, in the following one-shot game, the strategies are those in which people or individuals choose to complete or not their levels of education (studies, training, accumulation of human capital). This is strategic simply because this action can be costly when the state of the economy makes studying or going to school unprofitable for individuals [see, e.g., Bloom et al. (2021); Ceroni (2001); Sanchez Carrera (2019)].

In other words, to simplify the game, in what follows, our economy is made up of two population players consisting of individuals, agents, and/or citizens of a representative economy. These agents can be interpreted as potential employees/workers with high or low skill levels depending on their levels of education, for example. That is, the two populations can be interpreted as types of potential employees/workers who have to choose between acquiring or not skills/education levels. The general case must consider n-populations, but these two populations can be generalized. What distinguishes these two populations is simply the asymmetry in neighborhoods, or municipalities, and their compositions. Although the general case is for n-neighborhoods or n-municipalities, it is worth noting the citizens within their populations if they interact with their own kind and then through the dynamic system of the replicator to define the evolution of the entire economy. That is, there are two population agents who face each other (at random), citizen 1 against citizen 2, and have to choose between two strategies: to complete (E) or not complete (N) their educational levels. In other words, the two populations simply represent the interactions between person-populations of a certain type and person-populations of the same or another type. The types are people with complete educational levels and people with incomplete educational levels. In addition, when applying the replicator dynamics, we consider that there are pairing effects in the economy within each population and between the two populations. That is, the more agents of a certain type (i.e., with or without full education) there are within and between the two populations, the more the dynamic evolution of the economy toward one stationary state or another is affected.

We postulate matching effects in the economy: the more agents of a certain type there are, the more the corresponding payoffs to the strategies of the types of agents are impacted. There are

also education costs, which can measure labor-market conditions for skilled/unskilled workers. We assume that the Mexican economy is primarily an unskilled labor market, so educational costs are present only for those agents who opt for the complete education strategy.[11] The normal-form representation of this one-shot game is given by the payoff matrix:

$1\backslash 2$	E	N
E	$\pi_1(E,E); \pi_2(E,E)$	$\pi_1(E,N); \pi_2(E,N)$
N	$\pi_1(N,E); \pi_2(N,E)$	$\pi_1(N,N); \pi_2(N,N)$

That is, this economy is characterized by payoffs defined as follows:

- Player $i = 1, 2$, who opts for the full education strategy E_i, receives when facing a player $j \neq i = 1, 2$, who instead opts for the incomplete-education strategy N_j, player $i = 1, 2$, a payoff given by:

$$\pi_i\left(E_i, N_j\right) = \alpha + bW_i - C_i\left(\cdot\right)$$

where $\alpha \geq 0$ is a constant value of the innate abilities (e.g., minimum or subsistence wages), the earnings (or income) from attaining education are given by $bW_i > 0$, with $b \in [0, 1]$ as the marginal effect that provides the option of a complete education (i.e., a skill premium, human capital wages, etc.).

   The costs of education for player i are given by $C_i(\cdot) = \sigma + \beta N_j$, where N_j is the number of individuals with an incomplete education when player i is different from j and faces them. That is, due to neighborhood effects, education costs $C_i(\cdot)$ increase directly and proportionally when facing players j with incomplete education, $\beta N_j > 0$, since education costs also consider matching effects $\beta \in (0, 1)$ as a weighting element.[12] In other words, these costs increase in such a way by the assumption of either the segregation or segmentation of citizens/players/agents. This is included in the mathematical model through the measures provided by the shares of citizens/agents that are of one profile or another. In addition, by $\sigma > 0$ measuring baseline educational costs driven by, for example, labor-market demand conditions for unskilled labor.

- Player $i = 1, 2$, who opts for the full education strategy E_i, obtains when facing player $j \neq i = 1, 2$, who also opts for strategy E_j, a payoff:

$$\pi_i\left(E_i, E_j\right) = \alpha + bW_i - C_i\left(\cdot\right)$$

In this case, the costs of education, $C_i(\cdot)$, are assumed to take the form:

$$C_i\left(\cdot\right) = \sigma + \frac{\beta}{E_j}$$

where $\sigma > 0$ measures the baseline educational costs. Education costs also consider matching effects, which we define as inversely proportional, $\frac{\beta}{E_j} > 0$, to the number of agents that one faces with complete education levels E_j. This involves neighborhood effects because the greater the number of people one faces with a complete education, the less costly it will be to take the strategy/action of complete education (Barrios-Fernández 2022).[13] Hence, synergies are considered in this case, in which one person with a complete education faces another with a complete education: the positive synergies decrease the costs of education as there are always more people with a complete education.

- Player $i = 1, 2$, who opts for the incomplete-education strategy N_i, receives when facing a player $j \neq i = 1, 2$, who instead opts for the complete-education strategy E_j, a payoff:

$$\pi_i (N_i, E_j) = \alpha + \beta E_j$$

  That is, when player i opts for the incomplete education strategy, she/he will not obtain the income $W_i = 0$ but does not face the costs of education either, $C_i(\cdot) = 0$. The only reward is derived from innate abilities $\alpha \geq 0$ and positive synergies (matching effects) when facing others with a complete education, that is, $\beta E_j > 0$. This case is also related to the neighborhood effects or membership theory: if an uneducated person faces an educated person, then by imitation or with a certain probability, she/he could opt for the complete-education strategy implying an increase in their payoffs. Otherwise facing a person with an incomplete education.

- We assume that when both players $i, j = 1, 2$, opt for the incomplete-education strategy, there are no rewards from education and no positive or negative synergies from playing against each other. Therefore, profits simply come from innate abilities (i.e., subsistence minimum wages):

$$\pi_i (N_i, N_j) = \alpha$$

Given the relationship between the payoffs, $\pi_i(\cdot)$, this game can be categorized as a prisoner's dilemma game or a coordination game (such as the Assurance or Stag Hunt game, the Hawk/Dove or Chicken game, and the Battle of the Sexes game). This game will have a multiplicity of Nash equilibria or some Pareto-dominant equilibrium.

In any case, to simplify and work with the game in an evolutionarily dynamic way, we consider that for both player-populations $i, j = 1, 2$, the number of agents who play the complete education, $E_{i,j}$, or incomplete education, $N_{i,j}$, are defined, respectively, by the fractions of agents in population 1 denoted by x and the fractions of agents in population 2 denoted by y. The population sizes are normalized, which means that according to the given strategy, in population 1, there is a fraction x_E of agents with a complete education, while the rest have incomplete education, $(1 - x_E)$. In population 2, there is a fraction y_E of agents with a complete education, and the rest are incompletely educated, $(1 - y_E)$. With this simplification, we can compute the expected payoffs for agents following one strategy or another.

Hence, the expected payoff of the full education strategy for player-population 1 is given by:

$$\overline{\pi}_1 (E) = y_E [\pi_1 (E, E)] + (1 - y_E) [\pi_1 (E, N)]$$

Or, in full:

$$\overline{\pi}_1 (E) = y_E \left[\alpha + bW_1 - \left(\sigma + \frac{\beta}{y_E} \right) \right] + (1 - y_E) \left[\alpha + bW_1 - (\sigma + \beta (1 - y_E)) \right]$$

The expected payoff of the incomplete-education strategy for player-population 1 is

$$\overline{\pi}_1 (N) = y_E [\pi_1 (N, E)] + (1 - y_E) [\pi_1 (N, N)]$$

Or, in full:

$$\overline{\pi}_1 (N) = y_E \left[\alpha + \beta y_E \right] + (1 - y_E) \alpha$$

Therefore, players from population 1 opt for the full education strategy if their expected payoffs are large enough, $\overline{\pi}_1(E) > \overline{\pi}_1(N)$, which occurs when a certain threshold value is exceeded, namely,

$$y_E^* > \hat{y}_E = \frac{\beta - \sqrt{\beta (2bW_1 - 3\beta - 2\sigma)}}{2\beta}.$$

This threshold value tells us that the higher the level of wages and the marginal effect (the skill premium), bW_1, of a complete education, the easier it will be to exceed the threshold $\hat{y}_E$. Thus, agents will opt for the complete-education strategy. The threshold value is 0 when $bW_1 = 2\beta + \sigma$ because wages cover education costs governed by the characteristics of the unskilled labor market and the negative synergies of the socioeconomic system when there are agents with an incomplete education. Therefore, in this case, the agents of population 1 choose to study.

Analogously, the expected payoff of the full education strategy for player-population 2 is

$$\overline{\pi}_2 (E) = x_E [\pi_2 (E, E)] + (1 - x_E) [\pi_2 (E, N)]$$

Or, in full:

$$\overline{\pi}_2 (E) = x_E \left[\alpha + bW_2 - \left(\sigma + \frac{\beta}{x_E} \right) \right] + (1 - x_E) \left[\alpha + bW_2 - (\sigma + \beta (1 - x_E)) \right]$$

The expected payoffs of the incomplete-education strategy for player-population 2 are

$$\overline{\pi}_2 (N) = x_E [\pi_2 (N, E)] + (1 - x_E) [\pi_2 (N, N)]$$

Or, in full:

$$\overline{\pi}_2 (N) = x_E \left[\alpha + \beta y_E \right] + (1 - x_E) \alpha$$

Thus, players from population 2 opt for the full education strategy if their expected payoffs are large enough, $\overline{\pi}_2(E) > \overline{\pi}_2(N)$, and this holds when a threshold value is overcome, that is:

$$x_E^* > \hat{x}_E = \frac{\beta - \sqrt{\beta (2bW_2 - 3\beta - 2\sigma)}}{2\beta}$$

As in the case of player-population 1, the option to complete education is chosen if the wages or earnings for the complete-education strategy for player-population 2 are large enough to cover the costs of education and the negative synergies $bW_2 = 2\beta + \sigma$.

We can also compute the average expected payoffs as the sum of the weighted expected payoffs for each of these two populations. That is, the average expected payoff for population 1 is given by:

$$\hat{\pi}_1 (E, N) = x_E (\overline{\pi}_1 (E)) + (1 - x_E) (\overline{\pi}_1 (N)) = \alpha + \beta y_E^2 + x_E \left(bW_1 - 2\beta (1 - (1 - y_E) y_E) - \sigma \right).$$

And the average expected payoff for population 2 is

$$\hat{\pi}_2 (E, N) = y_E (\overline{\pi}_2 (E)) + (1 - y_E) (\overline{\pi}_2 (N)) = \alpha + \beta x_E^2 + y_E \left(bW_2 - 2\beta (1 - (1 - x_E) x_E) - \sigma \right).$$

We now consider the *n*-population replicator dynamics [Taylor and Jonker (1978); Taylor (1979); Weibull (1995); Hofbauer and Sigmund (1998)], which is a system of differential equations that reflects the idea that strategies that perform relatively better become more widespread in the population.[14] Hence, considering the higher-than-expected payoffs, we can express the replicator dynamics as the following system:

$$\dot{x}_E = x_E (1 - x_E) \left[bW_1 - 2\beta (1 - (1 - y_E) y_E) - \sigma \right]$$
$$\dot{y}_E = y (1 - y_E) \left[bW_2 - 2\beta (1 - (1 - x_E) x_E) - \sigma \right] \tag{1}$$

This system $(\dot{x}_E, \dot{y}_E)$ is defined by the phase diagram within the unit square $\mathcal{C} = [0, 1] \times [0, 1]$ and admits four corner stationary states or dynamic equilibria, $(0, 0), (0, 1), (1, 0), (1, 1)$ and four interior equilibria $(\hat{x}_E, \hat{y}_E) \in [0, 1]$ given by (see Appendix A):

$$\hat{x}_E^1 = \frac{\beta - \sqrt{\beta (2bW_2 - 3\beta - 2\sigma)}}{2\beta}, \hat{y}_E^1 = \frac{\beta - \sqrt{\beta (2bW_1 - 3\beta - 2\sigma)}}{2\beta}$$

$$\hat{x}_E^2 = \frac{\beta - \sqrt{\beta\left(2bW_2 - 3\beta - 2\sigma\right)}}{2\beta}, \hat{y}_E^2 = \frac{\beta + \sqrt{\beta\left(2bW_1 - 3\beta - 2\sigma\right)}}{2\beta}$$

$$\hat{x}_E^3 = \frac{\beta + \sqrt{\beta\left(2bW_2 - 3\beta - 2\sigma\right)}}{2\beta}, \hat{y}_E^3 = \frac{\beta - \sqrt{\beta\left(2bW_1 - 3\beta - 2\sigma\right)}}{2\beta}$$

$$\hat{x}_E^4 = \frac{\beta + \sqrt{\beta\left(2bW_2 - 3\beta - 2\sigma\right)}}{2\beta}, \hat{y}_E^4 = \frac{\beta + \sqrt{\beta\left(2bW_1 - 3\beta - 2\sigma\right)}}{2\beta}$$

As emphasized by Angeletos et al. (2007), Boehm and Punzo (1994, 2001), and Edmond (2013), an economic regime is a given set of rules and/or institutions that define the game. They are said to govern the economy as a system and its qualitative behavior (i.e., dynamic equilibria), where moving from one equilibrium to another simply constitutes a regime change. The interpretation of these equilibria as regimes is as follows:

1. The regime in which all agents are high profile, that is, the agents' pure strategic profile of complete education is $\{(E_1, E_2)\} = (1, 1)$.

2. The low-level equilibrium is a regime where player-population 1 and player-population 2 are all low-profile economic agents, meaning that their pure strategic profile is characterized by incomplete education $\{(N_1, N_2)\} = (0, 0)$.

3. The interior equilibria $(\hat{x}_E^1, \hat{y}_E^1; \hat{x}_E^2, \hat{y}_E^2; \hat{x}_E^3, \hat{y}_E^3; \hat{x}_E^4, \hat{y}_E^4)$ are regimes composed of marriages among low- or high-profile economic agents.

4. The two remaining corner equilibria $(0, 1)$ and $(1, 0)$ show a mismatch between strategies.

Therefore, if an economic system evolves toward a poverty-trap regime, then a small disturbance in the initial conditions is not enough to take the system out of the basin of attraction of the distribution that defines the poverty trap. This means that the poverty trap is an attractor of trajectories defined by probability distributions that are evolutionarily stable strategic profiles. The following proposition summarizes our main results from this evolutionarily dynamic game (see Appendix A for the general proof).

Proposition 1. *The evolutionary dynamics of the system* $(\dot{x}_E, \dot{y}_E)$*, its orbits in the state space* $\mathsf{C} = [0, 1] \times [0, 1]$*, and the phase portrait composed of these orbits are as follows (see Figure 2):*

  i. *The equilibrium regime* $(0, 0)$ *is an asymptotically stable point and defines the strategic profiles of incomplete education* $\{N, N\}$*. This equilibrium becomes unstable through bifurcations. In contrast, the* $(1, 1)$ *equilibrium regime is unstable but reaches asymptotic stability through the bifurcations, thus defining the steady state of complete education* $\{E, E\}$ *for the economy.*

 ii. *The corner equilibria* $(0, 1)$ *and* $(1, 0)$ *are characterized as nodal sources and unstable fixed points; in particular, these are saddle-point states.*

iii. *The interior equilibria are regimes* $((\hat{x}_E^1, \hat{y}_E^1), (\hat{x}_E^2, \hat{y}_E^2), (\hat{x}_E^3, \hat{y}_E^3), (\hat{x}_E^4, \hat{y}_E^4))$ *that serve as separatrices between the high equilibrium* $(1, 1)$ *and the low equilibrium* $(0, 0)$*, and these equilibria may be center points* $((\hat{x}_E^2, \hat{y}_E^2), (\hat{x}_E^3, \hat{y}_E^3))$ *around which one gravitates, or saddle points* $((\hat{x}_E^1, \hat{y}_E^1), (\hat{x}_E^4, \hat{y}_E^4))$*. However, they never represent a stable situation, but they portray the distributions of both profiles (agents with a complete education and agents with an incomplete education), defining the poverty-trap regimes.*

The appearance of a topologically nonequivalent phase portrait under a variation of parameters is called a bifurcation. A bifurcation is a change in the topological type of the system as its parameters pass through a bifurcation-critical value [Kuznetsov (2004)]. **Fold or saddle-node**
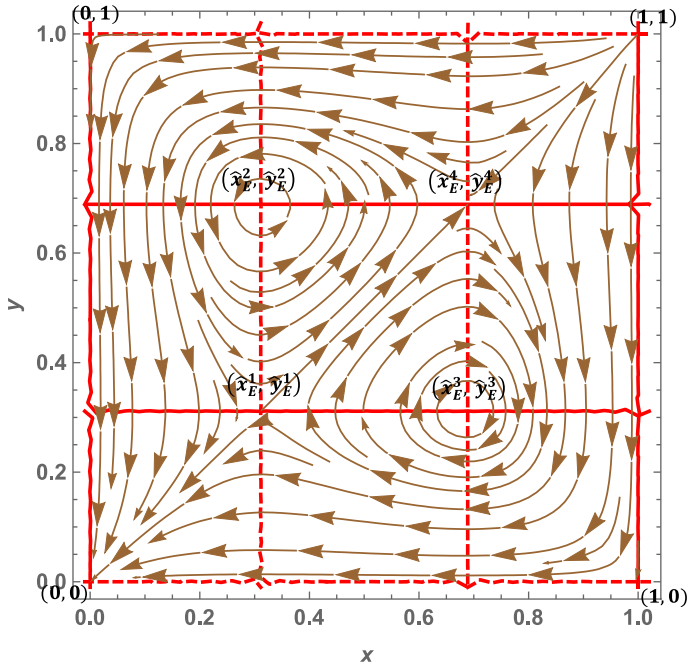
Figure 2. Phase portrait of the replicator dynamic system $(\dot{x}_E, \dot{y}_E)$ for conditions $\frac{3\beta+2\sigma}{2b} < W_1, W_2 < \frac{2\beta+\sigma}{b}$, where the horizontal red lines indicate the nullcline $x_E(1 - x_E)[1.1 - 1.4(1 - (1 - y_E)y_E)] = 0$, and the vertical dashed red lines indicate the nullcline $y_E(1 - y_E)[1.1 - 1.4(1 - (1 - x_E)x_E)] = 0$.
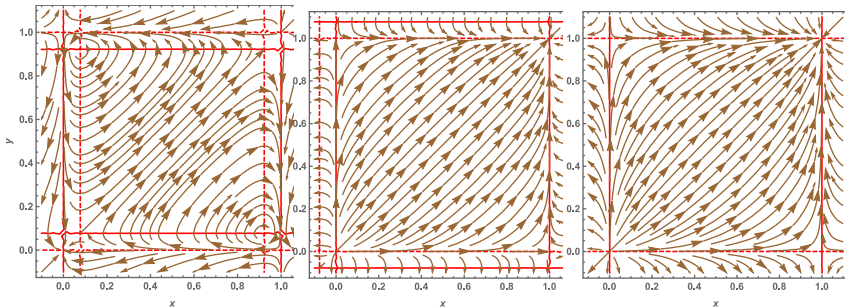

Figure 3. Phase portraits of the replicator dynamic system $(\dot{x}_E, \dot{y}_E)$. From the figure on the left to the one on the right, the bifurcation is shown as the interior equilibrium points merge with the corner equilibrium points (figure on the right). In each figure, the horizontal red lines indicate the nullclines $\dot{x}_E = 0$, and the vertical dashed red lines indicate the nullcline $\dot{y}_E = 0$. The parameters change in such a way that the numerical values of W_1 and W_2 grow more than the ratio $\frac{2\beta+\sigma}{b}$, which implies that the interior equilibria collapse with the corner equilibria, so that the only stable equilibrium is the fixed point $(1, 1)$.

bifurcation occurs when $W_1 = W_2 = \frac{3\beta+2\sigma}{2b}$, in which two equilibria collide and annihilate each other (see Appendix A for the general proof).

Specifically, **transcritical bifurcations** can occur (i.e., one equilibrium overlaps with another, and they exchange their stability), when the interior equilibria collapse with the corner equilibria, that is, when $W_1 = W_2 = \frac{2\beta+\sigma}{b}$, in which the trivial and nontrivial equilibria collide. For example, poverty-trap regimes must disappear when a bifurcation occurs (see Figure 3), which implies that the four interior equilibria collapse with the four nearest corner equilibria. Thus, each of the interior equilibria, $((\hat{x}_E^1, \hat{y}_E^1), (\hat{x}_E^2, \hat{y}_E^2), (\hat{x}_E^3, \hat{y}_E^3),$ and $(\hat{x}_E^4, \hat{y}_E^4))$, collides (shifting and wrapping) at the nearest corner equilibrium $((0, 0), (0, 1), (1, 0), (1, 1))$, namely:

$$(\hat{x}_E^1, \hat{y}_E^1) \rightarrow (0, 0)$$
$$(\hat{x}_E^2, \hat{y}_E^2) \rightarrow (0, 1)$$
$$(\hat{x}_E^3, \hat{y}_E^3) \rightarrow (1, 0)$$
$$(\hat{x}_E^4, \hat{y}_E^4) \rightarrow (1, 1)$$

This happens when $W_1 = W_2 \geq \frac{2\beta + \sigma}{b}$, which means that the per-capita income must be greater than the cost of education determined by the conditions of the labor market and peer effects, and this declines as skill premiums rise. Therefore, to obtain the high-level equilibrium, per-capita income must grow and, thus, incomplete education must decrease.

Considering this evolutionary game, we get the cases of a multiplicity of equilibria or poverty-trap regimes. Poverty-trap regimes exist with different levels of poverty characterized by various levels of complete and incomplete education and per-capita income. We now analyze empirically, with inferential statistics, the poverty of agents in Mexican municipalities as players in the game of poverty in Mexico. We do not estimate the strategies that agents choose but rather the effects of incomplete education and per-capita income. In doing so, we analyze the dynamics and formation of poverty-level regimes in Mexico. To this end, we apply symbolic dynamic techniques that result in equilibria/stable poverty regimes (e.g., moderate poverty or high poverty).

3. Dataset and methodology

The first goal of this section is to describe the dataset that will be used for the statistical inference analysis; the data include the poverty rate, incomplete education, and per-capita income for Mexican municipalities. The second goal is to describe and apply the statistical methodology approach to study the regimes that give the relationship between education and poverty.

The dataset includes three main variables for $N = 2430$ municipalities in 2000, 2005, 2010, 2015, and 2020 ($T = 5$). Thus, we are working with 5-year data and a subset of the 2469 municipalities for which information is available for the entire period. Particularly:

- The poverty rate (***P***) of a municipality[15] is obtained from the multidimensional poverty rate starting in 2010 and is properly combined with a variation of the income poverty (heritage) to obtain ***P*** for 2000 and 2005.
- The percentage of the population aged 15 years or older with basic incomplete education (***IE***) is also obtained from CONEVAL, and the variable is part of the Social Gap Index (IRS).[16]
- The per-capita income (***YL***) is a control variable proxied by the corresponding Mexican state's per-capita GDP at 2013 constant values, which is obtained by dividing the state's GDP by the state's population. It is taken from the National Institute of Geography and Statistics (INEGI).[17]

Then to study the Mexican poverty regime dynamics, we applied two complementary methodologies: symbolic regimes with symbolic time series and econometric panel data.

3.1. Data symbolization and regimes

Municipality poverty in Mexico is studied by identifying different regimes of poverty. To identify these and their dynamics, symbolic analysis is applied. The concept of symbolization has its roots in dynamical systems theory, particularly in the study of nonlinear systems, which can exhibit bifurcation and chaos. Williams (2004) asserts that symbolic dynamics is a method for studying nonlinear discrete-time systems by taking a previously codified trajectory using strings of symbols from a finite set, also called an alphabet. According to Piccardi (2004), symbolic dynamics and symbolic analysis are connected but different concepts. The former is the practice of modeling a dynamical system using a discrete space. However, the latter is an empirical approach to

characterizing highly noisy data by considering a partition, discretizing the data, and obtaining a string representing the very dynamic of the process. For a discussion of symbolization and symbolic time series, see Risso (2018), Piccardi (2004), Hirata et al. (2004), Daw et al. (2003), Kurths et al. (1996), and Bollt et al. (2001).

In practice, symbolization is applied by defining equally probable regimes in time series. With our three variables (poverty rate, incomplete education, and per-capita GDP), the following four regimes are defined:

1. $= \{ x : x \leq \mu_{x25\%} \}$ (low regime)
2. $= \{ x : x \leq \mu_{x50\%}$ and $x > \mu_{x25\%} \}$ (medium regime)
3. $= \{ x : x \leq \mu_{x75\%}$ and $x > \mu_{x50\%} \}$ (high regime)
4. $= \{ x : x > \mu_{x75\%} \}$ (very high regime)

where $\mu_{x25\%}$, $\mu_{x50\%}$, and $\mu_{x75\%}$ are the corresponding percentiles of x at 25%, 50%, and 75%, respectively. Considering the 25th, 50th, and 75th percentiles as limits, four poverty regions are defined in Mexico, given by the following four symbols (1,2,3,4):

1. **Low poverty ($p \leq 52\%$),**
2. **Medium poverty ($52\% < p \leq 69\%$),**
3. **Medium-high poverty ($69\% < p \leq 84\%$),**
4. **High poverty ($p > 84\%$),**

where p is the municipal poverty rate.

Four regimes were chosen for two reasons. First, it is a manageable number of regimes to study the dynamic: two symbols would be a simple binary variable (low, high), and more than four would complicate the sequential analysis (e.g., analyzing patterns of three periods implies eight possible cases, four implies 64 cases, and five implies 125 cases) and reduce robustness in a sample of 2430 municipalities and five different years. Second, there are four equilibria in the theoretical model, and we want to allow the possibility of obtaining the four equilibria in the different regimes.

This symbolization will enable us to study the temporal transition from one regime to another or permanence in one regime, that is, the multiple-equilibria transition. For example, in a given municipality, temporal sequences of three periods of poverty could be (1,1,1), (4,4,4), (1,2,3), and so on. If the dynamics are independent, all the sequences should have the same probability. We test independence using the symbolic independence test proposed by Risso (2014) and Monte Carlo simulations. In addition, we can study a type of symbolic dependence of poverty (p) on per-capita GDP (YL) and incomplete education (IE) by analyzing symbolic sequences such as (IE($t-1$), YL($t-1$), $p(t)$) using the symbolic causality test proposed by Risso (2015), where previous determinant variables (IE and YL) affect the present poverty rate.

3.1.1. Symbolic dynamics for Mexican municipalities
Considering the model described in Section 2 and the symbolic methodology, we empirically analyze the distribution of the players' profiles (i.e., the Mexican municipalities). The map in Figure 4 shows the distribution of poverty regimes in Mexican municipalities in 2020. As can be seen, the low- (or lower-) poverty regimes are concentrated in the north of the country and on the northwest coast, whereas higher-poverty regimes are found in the south.

Figure 5 shows the percentage of municipalities that are in temporal sequences of three 5-year periods between 2000 and 2020. Being 3 symbols the combinations of 3, we find 64 different dynamics of ($p(t-2)$, $p(t-1)$, $p(t)$). One of the first observations is the high persistence of municipalities in regions: 51.1% of the municipalities have remained in the same region for three 5-year periods between 2000 and 2020. More precisely, 16.6% have remained in the low-poverty region
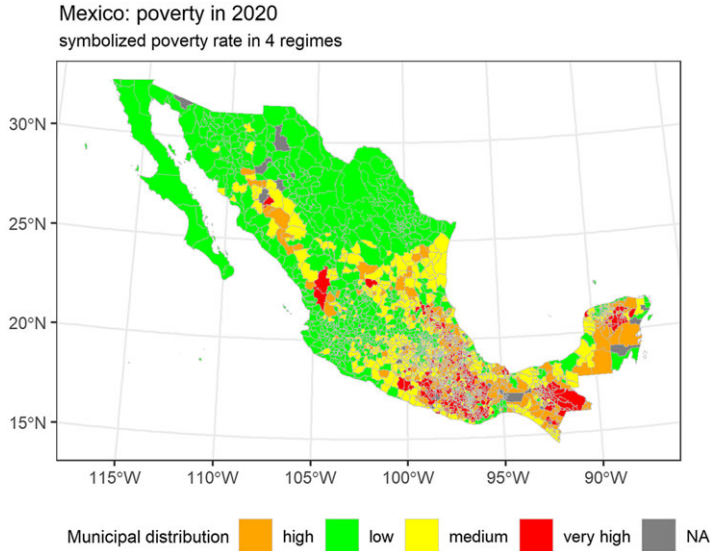
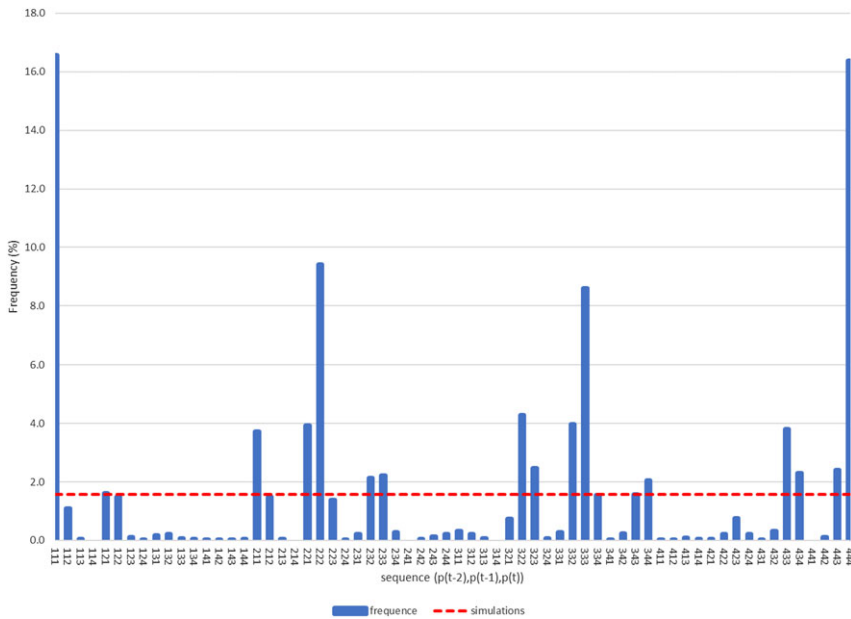**Figure 4.** Municipal poverty in 2020. Source: own elaboration.

Figure 5. Poverty sequence frequencies $\{p(t-2), p(t-1), p(t)\}$. Source: own elaboration.

$(1, 1, 1) = (111)$, 9.5% in the medium-poverty region $(2, 2, 2) = (222)$, 8.6% in the medium-high-poverty region $(3, 3, 3) = (333)$, and 16.4% in the high-poverty region $(4, 4, 4) = (444)$. This empirical evidence indicates that a stable low-level equilibrium may be verified in the Mexican case, that is, a steady state of stable poverty in Mexico.

Conversely, some municipalities have gone from high levels of poverty to lower levels, namely (211), (221), (231), (241), (311), (312), (321), (322), (331), (332), (341), (342), (411), (412), (413), (421), (422), (423), (431), (432), (433), (441), (442), and (443), representing 25.8% together. This

gives rise to multiple steady-state equilibria for the Mexican case. Moreover, emphasizing the multiple equilibria and, in particular, the stability toward the low-level equilibrium (the poverty trap), Figure 5 also shows that 11.1% of the municipalities have moved from low to higher levels of poverty: 112, 113, 114, 122, 123, 124, 132, 133, 134, 142, 143, 144, 213, 214, 223, 224, 233, 234, 243, 244, 314, 324, and 334. The remaining 12.2% represent oscillating sequences since they started and ended in the same region. This multiplicity of equilibria as a distribution of sequences is not random; we run 10,000 Monte Carlo simulations of the same sample size, and the distribution would be like the red dotted line if it were random.

In addition, an independence test for symbolic series is performed as in Risso (2014). The test considers a finite time series generated by an independent or random process sized T. We define a partition in the series in "a" equiprobable regions, obtaining the symbolized time series $\{s_t\}$. Each symbolized variable s_t takes a symbol value from the alphabet $A = \{A_1, A_2, \ldots A_a\}$. We can define a rolling window w over the symbolized time series T, and we obtain $n = a^w$ possible events. S_i being the sum of each possible event for $i = 1, 2, \ldots, n$, since the series is random, each S_i should have the same probability or $S_i/T = 1/n$. For simplicity, we imagine a time series from a dice $A = \{1,2,3,4,5,6\}$ and a rolling window of 3 should define $n = 6^3 = 216$ possible events (where 1,5,3 is a possible event), and, due to randomness, each S_i/T should have a probability of 1/6. Because the time series is finite and sized T, a statistic is needed. Each error $((S_i/T) - (1/n))$ in a finite random process should be near 0 and is multivariate normal distributed for each i. Finally, considering the distribution of quadratic normal variables explained by Mathai and Provost (1992), we can consider the statistic $Tn\{\sum ((S_i/T) - (1/n))^2\}$ asymptotically distributed as a Chi-squared with $n-1$ degree of freedom.

Applying the independence test for symbolic series, we obtain a statistic of 9119.86 which, compared with a Chi-squared of 63 degrees of freedom at 95%, 82.53 rejects the hypothesis that the sequence is random. In addition, the mean of the simulations gives a statistic of 0.01, well below the critical value, indicating a random sequence.

The map in Figure 6 shows that between 2010 and 2020, most municipalities remained in the same poverty regime (gray). However, a large number of municipalities concentrated in the center of the country descended toward lower poverty regimes. Among those that increased, for example, the Felipe Carrillo Puerto municipality in Quintana Roo stands out: despite having reduced the levels of incomplete education (as a proxy for illiteracy) and being characterized by high average income levels, it has not been able to overcome the issue of poverty, which has instead worsened.

With the above, we have shown the multiplicity of equilibria and the diversity of poverty (high, medium, and low) existing in Mexico. We now consider what was shown by the evolutionary game model described above, which revealed the primary role of the cost of education (educational backwardness) and per-capita income (wages, prices, and bonuses). Hence, we analyze the role of incomplete education and income.

3.1.2. The role of incomplete education and income (GDP per capita)

Other deficiencies that increased, albeit to a lesser extent, are the educational gap (0.25 percentage points) and the lack of access to nutritious and quality food (0.31 percentage points). Figure 7 shows the municipal histogram of the percentage of the population aged 15 years or older with incomplete basic education in 2000, 2010, and 2020. The movement to the left from 2000 to 2020 is clearer than in the case of the poverty rate, indicating a general reduction of this indicator. The average rate of incomplete education was 72.3% in 2000 and 45.7% in 2020, and the percentage of municipalities with a rate of incomplete education of 70% or above decreased from 63% in 2000 to 4% in 2020. At the same time, the percentage of municipalities with 30% of incomplete education or less increased from 0% to 15% in the same period. Low levels of incomplete education are concentrated in the north of the country and part of the Yucatan peninsula.

In the following, we characterize once again the multiple equilibria (steady states) for the Mexican municipalities. We consider data for the following variables: (i) incomplete education
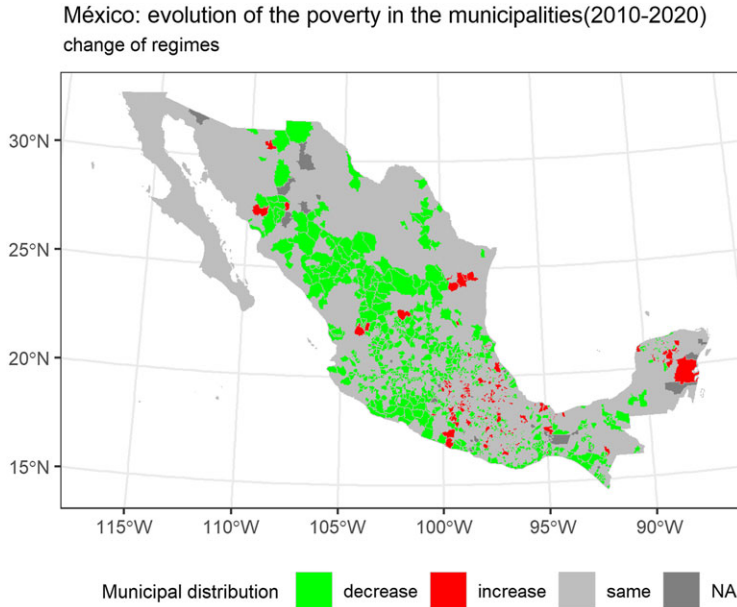
México: evolution of the poverty in the municipalities(2010-2020)
change of regimes

Figure 6. Variation of poverty in municipalities (2010–2020). Source: own elaboration.
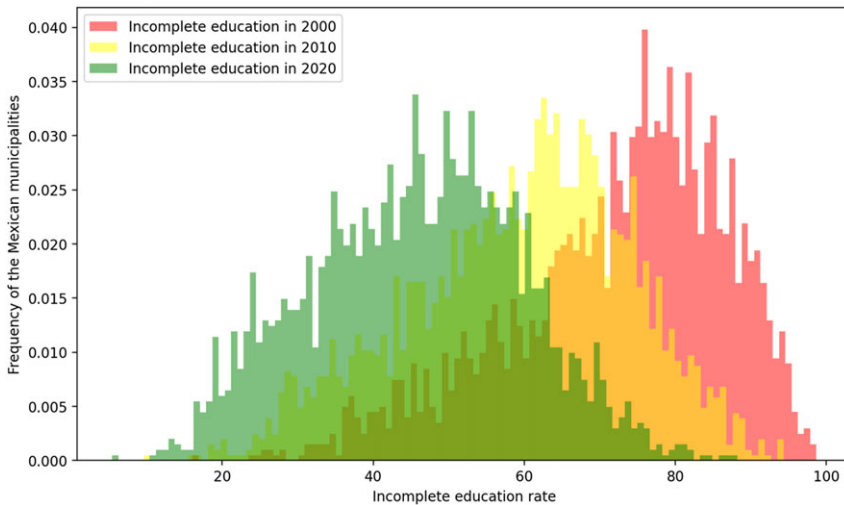

Figure 7. Distribution of incomplete education in the municipalities. Source: own elaboration.

rate (E), (ii) GDP per capita of the federal state in which the municipality is located (Y), and (iii) poverty rate of the municipality (P). Four regions were defined for each variable:

1. Low (below the 25% percentile)
2. Medium (between 25% and 50%)
3. Medium-high (between 50% and 75%)
4. High (above the 75% percentile).

We thus obtain the following characterizations:

Table 2. Sequences of E(*t*−1), Y(*t*−1), P(*t*), and the frequency in municipalities

Sequence	(%)	Sequence	(%)	Sequence	(%)	Sequence	(%)
111	0.7	211	0.3	311	0.1	411	0.1
112	0.3	212	1.2	312	1.0	412	0.8
113	0.1	213	0.8	313	2.2	413	3.2
114	0.0	214	0.2	314	2.4	414	10.4
121	2.8	221	0.7	321	0.3	421	0.2
122	1.8	222	2.6	322	2.3	422	1.4
123	0.3	223	1.7	323	3.9	423	3.2
124	0.0	224	0.5	324	2.1	424	3.0
131	2.7	231	2.0	331	0.8	431	0.2
132	1.5	232	3.2	332	2.9	432	1.2
133	0.4	233	1.7	333	3.3	433	2.2
134	0.0	234	0.3	334	1.1	434	1.9
141	6.9	241	5.0	341	2.5	441	1.0
142	0.7	242	2.1	342	2.2	442	1.1
143	0.1	243	0.6	343	0.7	443	0.5
144	0.0	244	0.2	344	0.2	444	0.2

Source: Own elaboration.

- E1 (low incomplete-education rate), E2 (medium incomplete-education rate), E3 (medium-high incomplete-education rate), E4 (high incomplete-education rate).
- Y1 (low income), Y2 (middle income), Y3 (upper middle income), Y4 (high income).
- P1 (low-poverty rate), P2 (medium-poverty rate), P3 (medium-high poverty rate), P4 (high-poverty rate).

With this symbolization of the variables, the causality sequence of predetermination (*E*(*t*−1), *Y*(*t*−1), *P*(*t*)) is analyzed. It shows how a municipality is affected by being in a region of poverty at time *t* when it was in a region with a given incomplete-education rate and per-capita income at time *t*−1.

Table 2 shows that some frequencies are much higher than the frequency that would be found if the sequences were random (i.e., 1.56%). For example, the sequence (414) has a 10.4% appearance. That is, if one was in a region of very high incomplete education (E4) and low income (Y1) in period *t*−1, one will be at a very high-poverty level (P4). Likewise, the sequence (141) has a probability of 6.9%, which means that if one is in a region with low incomplete education (E1) and a very high-income level (Y4), at time *t*, one is at a low-poverty level (P1). We perform 10,000 Monte Carlo simulations for this process and find that this sequence is far from independent and, therefore, significant. The causality test for symbolic series [Risso (2015)] indicates that it is significant with a statistic of 9119.86 and a Chi-squared (63) at 95% of 82.53, so the hypothesis of no causality. The statistic for the mean of the simulations is 0.01, well below the critical value mentioned above.

From Table 2, the figures below are obtained, which show the evolution of the probability of being at low levels of poverty at time *t* for increasing regimes of incomplete education (Figure 8a) and the same probability given increasing regimes of per-capita GDP (Figure 8b). We observe that if at *t*−1, a municipality was in a low incomplete-education level (E1) and high-income region (Y4), the probability of being in the low-poverty region P1(*t*) at *t* is 90%. Conversely, if a municipality is at higher levels of incomplete education at *t*−1, even if it is at a high-income level (Y4),
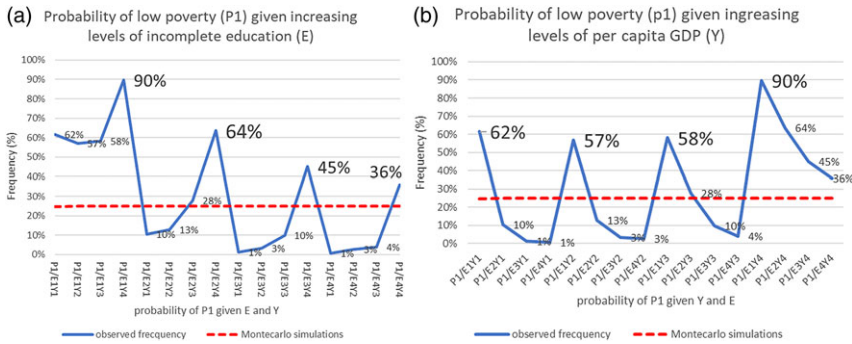
Figure 8. (a) Probability of low poverty (P1) given increasing level of incomplete education (E). (b) Probability of low poverty (P1) given increasing level of per capita GDP (Y). Source: own elaboration.
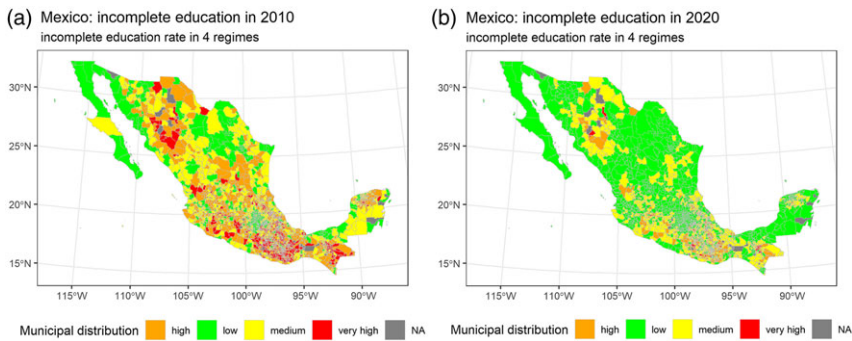

Figure 9. (a) Mexico: incomplete education in 2010. (b) Mexico: incomplete education in 2020. Source: own elaboration.

the probability that it will be in the low-poverty region decreases to 64% if the municipality is in incomplete education E2, to 45% if it is in the E3 region, and to 36% if it is in the E4 region. Hence, this shows the importance of reducing the values of incomplete education (illiteracy or educational backwardness) since even being in regions of higher average income at time $t-1$, there is only a 36% probability of being in the low-poverty region P1.

Figure 8b reveals a growing probability of low poverty as the per-capita GDP increases. However, for a low level of incomplete education E1, given Y1, the probability of P1 is 62%, and for a low incomplete education E1, given Y2, the probability of P1 is 57%. With a low level of incomplete education E1, we have to reach the very high regime of income (Y4) to observe a probability of 90% of being in a low-poverty regime P1.

Maps (Figure 9a and b) show that between 2010 and 2020, the geographical areas with the lowest rate of incomplete education shrunk. In particular, more green areas are seen in the north of the country and part of the Yucatan peninsula.

Regarding income (Figure 10a and b), it can be noticed that the highest is also found in the north of the country and that in 2020, the places with the highest income expanded, together with the Yucatan peninsula.

3.2. Econometric results

To complement the results obtained by the symbolic analysis, we first perform a panel data causality test. Then, we use random effect (RE) panel data as shown in equation (2), as suggested by Bell and Jones (2015), who remarked that RE can provide everything that fixed effects promises and more. As a second model, we consider the Arellano–Bover/Blundell–Bond method proposed by

Table 3. Panel VAR-Granger causality Wald test

Null hypothesis	Chi-sqaure	Prob.
L(Pob) does not homogeneously cause L(Edu Incomp.)	0.674	0.412
L(Edu Incomp.) does not homogeneously cause L(Pov)	97.532	0.000*
L(Pob) does not homogeneously cause L(Y/L)	7.701	0.053**
L(Y/L) does not homogeneously cause L(Pov)	157.151	0.000*

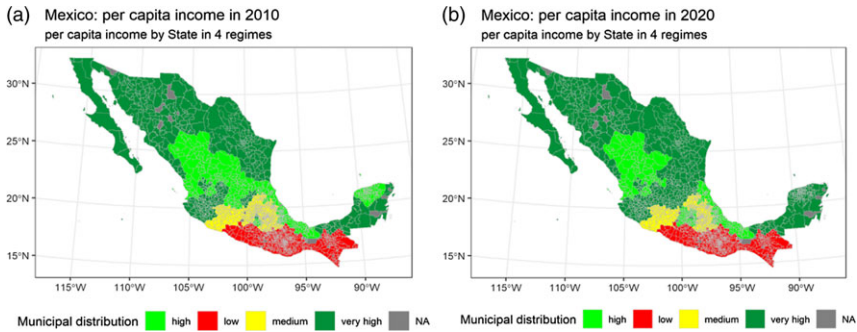*Source*: Own elaboration. *rejection of the null hypothesis at 0.01.

Figure 10. (a) Mexico: per-capita income in 2010. (b) Mexico: per capita income in 2020. Source: own elaboration.

Arellano and Bover (1995) and Blundell and Bond (1998), as expressed in equation (3).

$$\text{Ln}(p)_{i,t} = \alpha_1 + \alpha_2 \text{Ln(EI)}_{i,t} + \alpha_3 \text{Ln(YL)}_{i,t} + \upsilon_I \tag{2}$$

$$\Delta(\text{Ln}(p))_{i,t} = \beta_1 \Delta(\text{Ln}(p))_{i,t-1} + \beta_2 \Delta(\text{Ln(EI)})_{i,t} + \beta_3 \Delta(\text{Ln(YL)})_{i,t} + \Delta \epsilon_{i,t} \tag{3}$$

where $\text{Ln}(p)_{i,t}$, $\text{Ln(IE)}_{i,t}$, and $\text{Ln(YL)}_{i,t,}$ respectively, define the logarithm of poverty, incomplete education, and per capita GDP with municipality $I = 1, 2, \ldots, n$ and year $t = 2000$, 2005, 2010, 2015, 2020, and the variables in the second model are in differences, that is, $\text{D(Ln}(p))_{i,t-1} = \text{Ln}(p)_{i,t} - \text{Ln}(p)_{i,t-1}$.

In the present analysis, the determinants of the poverty rate of the municipalities of Mexico are evaluated using panel data ($N = 2430$ and $T = 5$, with 12,150 observations). The variables are the poverty rate of the municipalities of Mexico, the rate of incomplete education (educational backwardness, illiteracy) in the municipalities of Mexico, and the GDP per capita (income) of the federal states to which the municipalities belong.

Table 3 shows the results of the Granger causality test for panels. We observe that both the share of the population with incomplete education and the GDP per capita of the federal state to which the municipality belongs cause the poverty rate in the sense of Granger (i.e., they currently predict it).

Table 4 presents the results of the RE and Arellano–Bover/Blundell–Bond models. The coefficients are statistically significant, with the expected signs. That is, the incomplete education rate has an elasticity between 0.23 and 0.34, which means that an increase of 100% in the rate of incomplete education results in a 23% increase in the poverty rate. The long-run elasticity is 0.28.

The GDP per capita of the state also harms the poverty rate. As can be seen, in the Arellano Bond model, the short-term elasticity is −0.44, and the long-term elasticity is 0.55. Thus, an increase of 100% in the GDP per capita reduces the poverty rate by 55%. In addition, the COVID-19 pandemic worsened poverty, with a long-run elasticity of 0.03.

Table 4. Econometric model results

		ln(Pov)$_{i,t}$
	Random effects	Arellano–Bover/Blundell–bond
ln(Pov)$_{i,t-1}$		0.19
		(6.46)***
ln(Ed. Incomp)$_{i,t}$	0.34	0.23
	(29.80)***	(14.53)***
ln(Y/L)$_{i,t}$	−0.42	−0.44
	(−15.45)***	(−9.78)***
covid.ln(Y/L)$_{i,t}$	0.01	0.02
	(9.35)***	(1.70)*
c	7.52	7.35
	(22.71)***	(13.42)***
Specification	1996.09 [0.00]	405.13 [0.00]
No. of observations	12,150	9720

Source: Own elaboration. *significance at 0.1, **significance at 0.05, and ***significance at 0.01.

Table 5. Elasticity of incomplete education and per capita GDP on poverty

Year	Incomplete education	Per capita GDP	Const.
2000	1.02* (25.50)	−0.38* (−10.52)	4.22* (7.97)
2005	0.79* (17.84)	−0.34* (−9.86)	4.74* (9.77)
2010	0.83* (35.63)	−0.31* (−11.66)	4.28* (12.02)
2015	0.72* (35.18)	−0.46* (−14.93)	6.57* (16.22)
2020	0.56* (25.15)	−0.46* (−16.50)	7.10* (19.31)

Source: Own elaboration. *significance at 0.01.

3.2.1. Time elasticities

To study the evolutionary dynamics of the elasticities of incomplete education and income over the poverty rate, a cross-section model is applied for each year, and the changes over the years are examined.

Table 5 shows that the elasticity of incomplete education fell from 1.02 in 2000 to 0.56 in 2020, indicating a decrease in the impact of this variable on poverty reduction. This means that reducing incomplete education still reduces poverty but less strongly, and increasing incomplete education increases poverty but less strongly.

In the case of state GDP per capita, there is an increase (in absolute value) in its impact on poverty from −0.38 in 2000 to −0.46 in 2015. That is, increasing per-capita income is constantly positive in the magnitude of its impact on the reduction of poverty. Nevertheless, it can also be seen that between 2015 and 2020, the elasticity of income remained constant. In short, the goals of improving the levels of education and increasing the income of Mexicans have not been ambitious enough to escape the Mexican poverty traps. It is time to act more harshly, steadily, and lastingly to increase the levels of quality education together with income for all Mexicans, with a view to overcoming the poverty traps.

We also performed Arellano–Bover/Blundell–Bond modeling in a 4-year window to check for the evolution of the elasticities. The results are summarized in the following table, which reveals that, according to the model, the impact of incomplete education has increased from 0.10 to 0.18 between 2000–2015 and 2005–2020. Likewise, the impact of GDP per capita rose from 0.19 to 0.33 in the same period.

Table 6. Elasticities of incomplete education and per capita GDP considering two temporal windows and the Arellano–Bover/Blundell–Bond

Period	pob(t-1)	Incomplete education	Per capita GDP	Covid	Constant
2000–2015	−0.07	0.10**	−0.19***	N/A	6.14***
	(−1.27)	(2.06)	(−2.99)		(7.53)
2005–2020	0.39***	0.18***	−0.33***	−0.004***	5.65***
	(10.34)	(3.57)***	(−5.73)	(−3.39)	(6.01)

Source: Own elaboration. **significance at 0.05 and ***significance at 0.01.

The implication of the growing impact of incomplete education in the last 15 years is that education levels in Mexico have not risen at all, quite the opposite; therefore, poverty continues to be present as a stable and stationary state. Thus, in the last 15 years, increases in income have been key but insufficient to get out of the poverty traps. In other words, as previously stated, better levels of education and higher levels of income are sustained and lasting and are the key to overcoming poverty traps.

4. The "big push" policy

Considering the estimated econometric model, we project scenarios to analyze the evolution of poverty regimes, which are overcome according to possible variations in per-capita income and the rate of incomplete education.

Per-capita income increased on average by 9% in 2010–2015 and decreased by 9% in 2015–2020, while incomplete education fell on average by 12% in 2010–2015 and by 14% in 2015–2020. Therefore, the first scenario assumes that the per-capita income remains the same (0%) in 2020–2025 and 2025–2030, with a decrease of 15% in incomplete education in each quinquennial (2020–2025 and 2025–2030). Table 7 presents the results of this scenario and others. In scenario 1, the percentage of municipalities in regime P4 (very high poverty) would be 0.9% in 2025 and 0.2% in 2030, and most of the municipalities fall under regime 1 (low poverty), going from 33.5% in 2020 to a projected 51.8% in 2030. This is an important result; however, it should be noted that the Mexican low-poverty regime corresponds to a poverty rate of 51.43% or lower, which remains high in the international context.

The results of the scenarios indicate that for 0% of the municipalities to be in poverty regime 4 (high poverty, in which more than 84% of the citizens are poor) in 2030, a 10% decrease is necessary during the two 5-year periods 2020–2025 and 2025–2030 as well as a 10% increase in per-capita income in the same periods. This means that between 2020 and 2025, each municipality should lower its rate of incomplete education by 10% and increase its per-capita income by 10%, and then again between 2025 and 2030.

Therefore, as shown by the evolutionary game of poverty traps developed above, the income/wages (or marginal income) obtained when the levels of education are completed, bW_i, are so high that they cover the costs of education and any negative effects or synergies between peers facing citizens with an incomplete education. Therefore, completing education is economically profitable, which makes the economy fully converge toward complete-education status for its high per-capita income citizens. Thus, poverty-trap regimes are repulsive, with the high equilibrium being the unique asymptotically stable equilibrium.

5. Concluding remarks

The economic and social situation in Mexico over the last three decades has been characterized by ups and downs. Although the country's economy has achieved macroeconomic stability in terms

Table 7. Distribution of municipalities by poverty regimes for different scenarios

Year	2020	2025	2030
Scenario 1:		(DIE = −15% and DYL = 0%)	(DIE = −15% and DYL = 0%)
P1	33.5	43.0	51.8
P2	25.0	35.0	35.2
P3	21.1	21.0	12.8
P4	20.4	0.9	0.2
Scenario 2:		(DIE = −5% and DYL = 0%)	(DIE = −5% and DYL = 0%)
P1	33.5	39.5	44.1
P2	25.0	36.5	33.4
P3	21.1	22.5	21.6
P4	20.4	1.5	0.8
Scenario 3:		(DIE = 0% and DYL = 5%)	(DIE = 0% and DYL = 5%)
P1	33.5	40.9	47.1
P2	25.0	36.0	33.1
P3	21.1	21.9	19.1
P4	20.4	1.3	0.6
Scenario 4:		(DIE = −5% and DYL = 5%)	(DIE = −5% and DYL = 5%)
P1	33.5	42.6	50.6
P2	25.0	35.3	34.7
P3	21.1	21.1	14.5
P4	20.4	1.1	0.2
Scenario 5:		(DIE = −10% and DYL = 10%)	(DIE = −10% and DYL = 10%)
P1	33.5	47.3	63.0
P2	25.0	33.8	33.2
P3	21.1	18.4	3.8
P4	20.4	0.5	0.0

Source: Own elaboration.

of prices, exchange rates, and interest rates—that is, monetary and exchange rate stability—it has also registered low levels of economic growth, with persistently high levels of poverty and inequality compared to other countries with similar development in the world.[18] In this research paper, we used evolutionary game theory, nonparametric empirical techniques, and panel data analysis to demonstrate the existence of poverty traps in Mexico. We showed that incomplete education and low per-capita income are the driving force behind poverty-trap regimes in Mexico, which are evolutionarily stable. Thus, we revealed the existence of regimes of poverty traps in Mexico and identified their roots or conventional causes as the poor educational attainment of Mexican citizens and the low levels of per-capita income. Therefore, the scarce amount of human capital is what prevents the Mexican economy from achieving sustained economic development. Human capital is defined as the number of school years completed by household members. Human capital is then defined as how many people in the household were currently attending school. However, the price of acquiring human capital is so high that Mexicans seem to strategically decide to drop out of school and thus converge toward equilibria/regimes characterized by incomplete education and low per-capita income.

The solution to these poverty traps is simply to implement big push policies aimed at achieving complete basic education for all Mexicans and increasing their per-capita income. It is well known that advanced economies are characterized by educated people and higher income and not trapped in social evils such as poverty. To really transform Mexican

society deeply, the Mexican government must focus on these two aspects: education and the distribution of wealth for all Mexicans. The so-called Fourth Mexican Transformation (https://en.wikipedia.org/wiki/Fourth_Transformation) led by the current President of Mexico, Andrés Manuel López Obrador, is taking important steps toward more inclusive social programs (https://www.imf.org/en/News/Articles/2022/01/24/cf-fostering-inclusion-in-mexico), for example:

i. Recognizing the challenge of poverty, the government introduced important changes to social programs (social assistance and active labor-market programs) in 2019, focusing on universal coverage and social (non-contributory) pensions, as well as support for indigenous people, the elderly, and people with special needs. These changes resulted in an increase in overall spending on social assistance from 1.8% of GDP in 2018 to 2.1% of GDP in 2019. In particular, social pensions related to the Pension for People's Well-being program for older adults more than tripled between 2018 and 2020, motivated by the prevalence of poverty in old age and limited pension coverage.

ii. In education, there are now three programs according to the different educational levels. One of them, the Benito Juárez Basic Education for Welfare Scholarship Program, concerns families living in poverty and with children under 15 years of age. Unlike Prospera, the discontinued conditional cash transfer program that used to pay per student, payment is now per family and not conditional on regular student health checks. The impact of this change should be monitored to ensure that dropout rates do not increase and that student health and nutrition are not negatively affected.

Future research may study, for example, the roots/causes of agents' preference for incomplete education. These preferences are likely based on opportunity costs—for instance, what is the point of studying a scientific career if the probability of being unemployed after study completion is high? Faced with this dilemma, the best option is to forgo studying and immediately enter the labor market, which is characterized by demand for workers with low educational levels. Once again, a labor market-oriented economic policy program focused on high human capital and high wages is the key to breaking out of poverty traps.

Notes

1 For example, typical social safety nets (SSNs), designed to build short-term resilience against uninsured risk, can have long-term positive effects on household well-being if they lift a poor household out of the poverty trap. Alternatively, a short-lived negative health or climate shock, such as a drought, such as a pandemic, that is expected to have a temporary impact, can leave a non-poor economy permanently destitute, thus driving it into poverty [Jalan and Ravallion (2004); Bischi et al. (2022)].

2 Mexico is a culturally and naturally very rich country, with great natural and socioeconomic diversity among its states and regions. The country is comprised of 32 federal entities (31 states and the capital, Mexico City) and a total of approximately 2480 municipalities (https://en.wikipedia.org/wiki/Mexico).

3 In addition, the levels of poverty in rural areas are critical and affect the indigenous or afro-descendant population to a greater extent than the non-indigenous population [Mckinley and Alarcon (1995); World Bank (2005)]. See: https://news.azpm.org/s/88596-poverty-in-mexico-rose-to-nearly-half-the-country-in-2020/, https://mexiconewsdaily.com/news/poorest-municipalities-oaxaca-chiapas-guerrero/

4 For example, Agüero and Beleche (2013) showed that 90% of public and private schools correspond to a poverty index estimated by Mexico's National Council of Population (CONAPO), and so there are critics' heterogeneous effects. This poverty index is a function of measures of education (percentage of the population that is illiterate or without primary education), housing (percentage of houses without water, sewage, electricity, and non-dirt floors), and access to goods (having a refrigerator) in a given locality. The results show that increasing the number of school days benefits students who attend schools in more affluent localities (those with very low or low-poverty rates) more than their counterparts who attend schools in areas with higher-poverty rates. In other words, the level of poverty in Mexican localities or municipalities is a crucial variable determined by the education level of its citizens.

5 See: https://www.kff.org/global-health-policy/issue-brief/economic-impact-of-covid-19-on-pepfar-countries/, https://sgp.fas.org/crs/row/R46270.pdf, https://cepr.org/voxeu/columns/lasting-impact-covid-crisis-economic-potential

6 According to ECLAC (2020), the COVID-19 pandemic will cause a steady increase in the number of people living in poverty and extreme poverty in Mexico (i.e., who cannot meet several of their basic needs such as food, drinking water, shelter, health, and education).

7 Samuel Bowles (2006) highlighted the notion of institutional poverty traps, which indicates that an institution is composed of one of several possible conventions. At equilibria at which members of a population typically act in ways that maximize payoffs given the actions taken by others and agents' beliefs about what others will do, these factors support continued adherence to conventional actions. A convention is thus an outcome in which it is in the interest of agents to adhere to the convention as long as they believe that most others will do the same. Because a convention is one of many possible mutual best responses, institutions are not environmentally determined but humanly constructed (although not necessarily of deliberate design). Conventions are self-enforcing, that is, they are evolutionarily stable as a result of the positive feedback associated with members' conforming to a common strategy.

8 These calculations are based on subtracting the number of primary school-age children enrolled in primary or secondary school from the total population of the official primary school-age children (World Development Indicators: https://databank.worldbank.org/source/world-development-indicators). The percentage of lower secondary school-age adolescents who are out of school in Mexico was reported at 7.6487% in 2019, while the percentage of primary school-age children out of school was reported at 0.79265 % in 2019, amounting to 106,131 children according to the World Bank collection of development indicators.

9 https://www.coneval.org.mx/Medicion/MP/Documents/MMP_2018_2020/Notas_pobreza_2020/Nota_tecnica_sobre_el_rezago%20educativo_2018_2020.pdf; https://www.coneval.org.mx/Medicion/Paginas/Medición/Avances-Rezago-educativo.aspx

10 This endogeneity is important from a political perspective and calls for a critical examination of our thought patterns, institutions, and disciplinary models: What values and preferences do they capture, support, and help maintain? Replacing our paradigmatic image of human behavior, *homo oeconomicus*, with a more cooperative human image such as *homo cooperativus* or *homo sustinens* could help anchor a different orientation in human decision-making (Heinz and Koessler, 2021). Agents may exhibit other-regarding preferences, for example, in the form of altruism, inequality/ inequity aversion, or reciprocity (Fehr and Schmidt, 1999; Cooper and Kagel, 2016). Those who follow other-regarding actions may approve institutional arrangements that improve the conditions for other humans or pro-environmental policies even when they come at a cost for themselves (Heinz and Koessler, 2021).

11 The game-theory model of educational options proposed here may generate skepticism because it does not consider crucial dimensions such as subsistence consumption levels, the opportunity cost of education, and the existence of liquidity or indebtedness restrictions that individuals or households consider when making such decisions. However, the game proposal achieves the objective of explaining the strategic action of behavioral decision of the agents/players/citizens regarding whether to complete education or not.

12 This relates to the notion of neighborhood effects or the membership theory of poverty traps [Durlauf (2003); Accinelli and Sanchez Carrera (2012)], which is a theory of incentives for the segregation of people—for example, the economic segregation of some richer families into homogeneous neighborhoods so that interaction with poor members of the economy is costly; moreover, the dynamics of these combinations of interactions between non-homogeneous members explain the persistent income inequality. It is precisely the concept of neighborhood effects that allows Durlauf to explain why poverty traps exist and persist.

13 Some empirical evidence can be found at: https://www.oecd.org/education/school/50293148.pdf https://www.oecd.org/education/innovation-education/35750159.pdf https://gpseducation.oecd.org/revieweducationpolicies/#!node=41761&filter=all

14 The replicator dynamics thereby assumes that a comparison of the payoff of strategy i when facing the current population and compared to the average payoff of the population determines the speed and direction of selection. The replicator dynamics is of the form $\dot{x}_i = x_i[\overline{\pi}_i(\cdot) - \hat{\pi}_i(\cdot)]$, where $x_i \in (0, 1)$ and $x_i + x_j = 1$ for all pairs $i \neq j$ of any player-population. It ensures that the trajectory $x(t) = \{(x_i(t), x_j(t)), t_0 \leq t\}$ is bounded in the state space of the unit square $\mathbb{C} = [0, 1] \times [0, 1]$. In other words, the growth rate $\frac{\dot{x}_i}{x_i}$ of the associated population share is equal to its excess payoff, $\overline{\pi}_i(\cdot) - \hat{\pi}_i(\cdot)$, over the average payoff in its player-population.

15 See: https://www.coneval.org.mx/Medicion/Paginas/Pobreza-municipio-2010-2020.aspx

16 Basic education in Mexico comprises pre-school (*preescolar*) (age category 5–6 years), primary school (*educación primaria*) with a nominal duration of 6 years (age category 8–12 years), and junior secondary school (*secundaria*) with a nominal duration of 3 years (age category 14–15 years).

17 See: https://www.inegi.org.mx/app/tabulados/default.aspx?pr=17&vr=6&in=2&tp=20&wr=1

18 Mexico, as an emerging middle-income economy, has experienced a process of economic and financial opening since the end of the 1980s, with a growing integration of the country into the regional and global economy, particularly that of the United States. Between 1990 and 2020, economic growth reached an annual average of 2.1%, a relatively modest rate for a country with the economic and social characteristics of Mexico. In terms of gross domestic product (GDP) per inhabitant, growth was around 1.0% on average in the same period. In addition, investment (both public and private) has been around 20% of GDP, below 25%, which may be a desirable threshold to boost economic activity and job creation (Brida et al., 2021).

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APPENDIX A: EXISTENCE AND STABILITY OF THE EQUILIBRIA AND BIFURCATION ANALYSIS

In Section 2, the RD dynamical system is

$$\dot{x}_E = \frac{dx}{dt} = x_E (1 - x_E) \left[bW_1 - 2\beta \left(1 - (1 - y_E) y_E \right) - \sigma \right]$$

$$\dot{y}_E = \frac{dy}{dt} = y (1 - y_E) \left[bW_2 - 2\beta \left(1 - (1 - x_E) x_E \right) - \sigma \right] \tag{A1}$$

where all the parameter are non-negative. The trivial equilibria of system (A1) are

1. The equilibrium $(0, 0)$ and the eigenvalues of the linearized vector field of (A1) in the vicinity of this equilibrium are $bW_1 - 2\beta - \sigma$ and $bW_2 - 2\beta - \sigma$.

2. The equilibrium $(0, 1)$ and the eigenvalues of the linearized vector field of (A1) in the vicinity of this equilibrium are $bW_1 - 2\beta - \sigma$ and $-(bW_2 - 2\beta - \sigma)$.

3. The equilibrium $(1, 0)$ and the eigenvalues of the linearized vector field of (A1) in the vicinity of this equilibrium are $-(bW_1 - 2\beta - \sigma)$ and $bW_2 - 2\beta - \sigma$.

4. The equilibrium $(1, 1)$ and the eigenvalues of the linearized vector field of (A1) in the vicinity of this equilibrium are $-(bW_1 - 2\beta - \sigma)$ and $-(bW_2 - 2\beta - \sigma)$.

The nontrivial equilibria of system (A1) are $(\hat{x}_E, \hat{y}_E) \in [0, 1]$ which the $x-$component satisfies

$$2\beta x^2 - 2\beta x - (bW_2 - 2\beta - \sigma) = 0 \tag{A2}$$

and the $y-$component satisfies

$$2\beta y^2 - 2\beta y - (bW_1 - 2\beta - \sigma) = 0 \tag{A3}$$

The discriminant of (A2) and (A3) is $4\beta(2bW_2 - 3\beta - 2\sigma)$ and $4\beta(2bW_1 - 3\beta - 2\sigma)$, it implies that the nontrivial equilibria exist if

$$W_1, W_2 > \frac{3\beta + 2\sigma}{2b} \tag{A4}$$

Fold or saddle-node bifurcation occurs when $W_1 = W_2 = \frac{3\beta + 2\sigma}{2b}$ in which two equilibria collide and annihilate each other.

Suppose x_2 and x_3 are solution of (A2) and y_2 and y_3 are solution of (A3). By applying Vieta's formula, we get

$$x_2 + x_3 = y_2 + y_3 = \frac{1}{2} \tag{A5}$$

$$x_2, x_3 = \frac{-(bW_2 - 2\beta - \sigma)}{4\beta} \tag{A6}$$

$$y_2, y_3 = \frac{-(bW_1 - 2\beta - \sigma)}{4\beta} \tag{A7}$$

In order to get $x_2, x_3, y_2, y_3 > 0$, we get

$$W_1, W_2 < \frac{2\beta + \sigma}{b} \tag{A8}$$

Thus, we will have four positive nontrivial equilibria when condition (A4) and (A8) satisfied, that is,

$$\frac{3\beta + 2\sigma}{2b} < W_1, W_2 < \frac{2\beta + \sigma}{b} \tag{A9}$$

The nontrivial equilibria of system (1) are

$$\hat{x}_E^1 = \frac{\beta - \sqrt{\beta(2bW_2 - 3\beta - 2\sigma)}}{2\beta}, \hat{y}_E^1 = \frac{\beta - \sqrt{\beta(2bW_1 - 3\beta - 2\sigma)}}{2\beta}$$

$$\hat{x}_E^2 = \frac{\beta - \sqrt{\beta(2bW_2 - 3\beta - 2\sigma)}}{2\beta}, \hat{y}_E^2 = \frac{\beta + \sqrt{\beta(2bW_1 - 3\beta - 2\sigma)}}{2\beta}$$

$$\hat{x}_E^3 = \frac{\beta + \sqrt{\beta(2bW_2 - 3\beta - 2\sigma)}}{2\beta}, \hat{y}_E^3 = \frac{\beta - \sqrt{\beta(2bW_1 - 3\beta - 2\sigma)}}{2\beta}$$

$$\hat{x}_E^4 = \frac{\beta + \sqrt{\beta(2bW_2 - 3\beta - 2\sigma)}}{2\beta}, \hat{y}_E^4 = \frac{\beta + \sqrt{\beta(2bW_1 - 3\beta - 2\sigma)}}{2\beta}$$

The eigenvalues of the linearized vector field of (1) in the vicinity of $(\hat{x}_E^1, \hat{y}_E^1)$ or $(\hat{x}_E^4, \hat{y}_E^4)$ are

$$\pm \frac{1}{\beta} \sqrt{\beta \left(bW_1 - 2\beta - \sigma\right) \left(bW_2 - 2\beta - \sigma\right) \sqrt{2bW_1 - 3\beta - 2\sigma} \sqrt{2bW_2 - 3\beta - 2\sigma}}$$

The eigenvalues of the linearized vector field of (A1) in the vicinity of $(\hat{x}_E^2, \hat{y}_E^2)$ or $(\hat{x}_E^3, \hat{y}_E^3)$ are

$$\pm \frac{1}{\beta} \sqrt{-\beta \left(bW_1 - 2\beta - \sigma\right) \left(bW_2 - 2\beta - \sigma\right) \sqrt{2bW_1 - 3\beta - 2\sigma} \sqrt{2bW_2 - 3\beta - 2\sigma}}$$

Transcritical bifurcation occurs when $W_1 = W_2 = \frac{2\beta + \sigma}{b}$ in which the trivial and nontrivial equilibria collide.

Condition (A9) implies the following:

Equilibrium	Eigenvalues 1	Eigenvalues 2	Stability
$(0, 0)$	$-$	$-$	Stable node
$(0, 1)$	$+$	$-$	Saddle point
$(1, 0)$	$-$	$+$	Saddle point
$(1, 1)$	$+$	$-$	Unstable node
$(\hat{x}_E^1, \hat{y}_E^1)$	$+$	$-$	Saddle point
$(\hat{x}_E^4, \hat{y}_E^4)$	$+$	$-$	Saddle point
$(\hat{x}_E^2, \hat{y}_E^2)$	ci	$-ci$	Center
$(\hat{x}_E^3, \hat{y}_E^3)$	ci	$-ci$	Center

where $c = \frac{1}{\beta} \sqrt{\beta \left(bW_1 - 2\beta - \sigma\right) \left(bW_2 - 2\beta - \sigma\right) \sqrt{2bW_1 - 3\beta - 2\sigma} \sqrt{2bW_2 - 3\beta - 2\sigma}}$