where $b(m)$ denotes the m-th Fourier coefficient of a cusp form for the full modular group. The two methods were developed independently, but share important features: notably the rôle played by rational values of either $f'(x)$ or $f'(x)$, which echoes the relevance to the discrepancy for a plane region of tangents with rational gradient. The main results (on exponential sums, on the discrepancy of plane regions, on sums with modular form coefficients, on Riemann's zeta-function and on prime numbers) are collected together in Part 5. In Part 6 the author discusses related work (outside the scope of the Bombieri-Iwaniec method), as well as several open problems and ideas for future researchers: the above-mentioned rôle of rational values means that the full modular group comes into the picture, hence (as Bombieri and Iwaniec noticed) one avenue for progress may be through the application of deep results from the theory of non-holomorphic modular forms.

The great wealth of material in the book is appropriately distributed amongst the six parts and has been further subdivided into sections and subsections (sections do not exceed 30 pages). For ease of location, results and important displayed formulae are numbered by section, subsection and place in subsection (e.g. 'Lemma 4.2.1'). The style is not exactly leisurely, but there are frequent substantial passages explaining the ideas behind the bare mathematics.

Apart from some quoted results on Fourier coefficients of modular forms, the treatment of the main topics in this book is self-contained: proofs assuming only a little real and complex analysis (and some algebra known to Newton) are given for each result. References are given where necessary (over 150 in all). There are no exercises.

This book is going to be useful for researchers in the field of exponential sums and their applications. It could be used either for learning the Bombieri-Iwaniec method, or as a reference. Sections 8, 13, 14 and 18 (with some reference to Sections 1, 5 and 7) would be enough to prepare a postgraduate course on the use of the Bombieri-Iwaniec method to bound the discrepancy of a plane region, but to reach the 'state-of-the-art' one would have to include Sections 15 and 16, which are the deepest and most demanding in the book.

N. WATT

KANNAN, R. and KRUEGER, C. K., *Advanced analysis on the real line* (Universitext, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong, 1996) 260 pp., 0 387 94642 X, softcover DM68.

A century ago, according to Kline's *Mathematical thought from ancient to modern times* (Oxford, 1972), mathematicians of the eminence of Poincaré were objecting to the then current research into discontinuous functions, non-differentiable functions and similar topics. They would no doubt be horrified by the mere existence of this book – just as there are few of today's undergraduates who would be happy with the material. However, we should not fall into the trap of equating a popularity poll with a judgement of quality.

This book is exactly what it claims to be: it is an analysis of real functions of real variables – the 'awkward' topics such as absolute continuity, Dini derivatives, bounded variation, singular functions etc. The authors have drawn their material from diverse sources and have made a good job of synthesising it into a coherent whole. Although virtually all of the content is technical (and this applies to the statements of the results as well as the proofs), it is elegantly put together, relying heavily on the Vitali Covering Theorem.

Those who work in the area know (and perhaps love) the content. For most who need to use real analysis beyond dealing with our traditional 'sufficiently well-behaved' functions, this collection will save some time by giving efficient proofs of the odd results we need from time to time. For example, just how much do you *need* to know about f' to be able to conclude that f is increasing?

The book is well written and has a decent set of problems at the end of most chapters. Your reviewer would have welcomed more non-routine problems, but that is a personal view – the supply is adequate. There are, inevitably, a few misprints but, fortunately for such a technical topic, not enough to distract.

The authors presume a lot of the reader's motivation. Perhaps this is realistic, for those who have not discovered that they do need some of this may take some convincing that it is interesting to them. The main shortcoming is editorial: the book badly needs a notation index and an improved main index. The index is far too sketchy for a book of this sort, whose main value will be for those who dip into it from time to time. Authors may be excused for overlooking this, but part of the importance of editors is to ensure that they are reminded of such essentials of publishing a book.

This is intended to be the first of two volumes. I look forward to the second.

D. S. G. STIRLING

BERGGREN, L., BORWEIN, J. and BORWEIN, P. *Pi: A Source Book* (Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong, 1997), xix + 716 pp., 0 387 94924 0 (hardcover), £37.50.

The number π is surely the most widely known of the fundamental mathematical constants: its origins lie in the problems of finding the area and circumference of a circle – indeed, before the general adoption of a symbol for this number it was often referred to as 'the quadrature of the circle' (the radius 1 being understood) – and these are familiar to high school pupils; yet it appears frequently in formulae at the highest levels of mathematics. From early times people have been calculating π with increasing accuracy, using a variety of methods which have opened up new areas of research, and in more recent times there has been much theoretical activity devoted to understanding the nature of π. All this has generated a vast literature, from which the authors have made a personal selection of some seventy items. The Borweins have been at the forefront of recent research both theoretical and computational, so we may be sure that the selection has been made with care and authority.

The authors identify three broad classes into which their selected items fall:

(i) 'the accumulated mathematical research literature of four millennia';

(ii) 'historical studies of pi' and 'writings on the cultural meaning and significance of the number';

(iii) 'treatments of pi that are fanciful, satirical or whimsical, or just wrongheaded'.

The selections are arranged according to the period to which they refer: items 1–15 were written in or refer to the pre-Newtonian period; items 16–24 take us up to Hilbert; and items 28–70 are concerned with twentieth century developments. Item 25 belongs to the authors' third class and relates along with items 26 and 27 to the attempt by the state of Indiana to legislate on the value of π.

The pre-Newtonian items begin with Problem 50 from the Rhind papyrus and contain material on Egyptian, Greek, Chinese, Indian and European contributions. Newton, Euler, Lambert, Shanks, Hermite, Lindemann, Weierstrass and Hilbert are the authors represented in the second period; they deal with computations, irrationality and transcendence. Well over half of the items presented are twentieth century developments, theoretical and computational aspects occurring in equal measure. A detailed outline of this material would take up too much space, so I simply note that Ramanujan, arithmetic-geometric means and the Borweins are well-represented and there are several items which fall into the third class. Some items are devoted to the number e, Euler's constant γ or aspects of the Riemann zeta function, all of which are intimately connected with π.

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