

SESSION 5

EVOLUTIONARY STATUS OF WOLF-RAYET STARS

Chairman: H. ANDRILLAT

Introductory speakers: C. CHIOSI  
C. DE LOORE

1. G. BERTELLI and C. CHIOSI: Gradients in supergiant and WR stars across the galactic plane.
2. D. VANBEVEREN and C. DE LOORE: On the expected frequency of WR + compact star systems.
3. A. MAEDER: Properties of the scenario for the formation of WR stars as post-red supergiants.
4. A. TUTUKOV and L. YUNGELSON: Origin and evolution of Wolf-Rayet stars.
5. G. GIURCICIN and F. MARDIROSSIAN: The massive eclipsing binary RY Scuti.
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7. G. BISIACCHI, J. LOPEZ and C. FIRMANI: The ON-stars: A wider spectroscopic definition.
8. P. HELTINGS, F. VANSINA, W. PACKET, C. DOOM, J.P. DE GREVE and C. DE LOORE: On the structure and evolution of the OB-companions in Wolf-Rayet binaries.
9. C. DOOM and J.P. DE GREVE: The occurrence of different Wolf-Rayet phases in massive close binaries.
10. A. MAEDER: The various scenarios leading to WR stars: their relative importance and the role of mixing.

# THE EVOLUTION OF MASSIVE STARS: THE PRODUCTION OF "SINGLE" WR STARS

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## ABSTRACT

In this paper we summarize the basic observational properties of WR stars, and the current status of the evolution of massive stars, giving particular emphasis to those physical mechanisms (mass loss, overshooting, turbulent diffusion...) that may lead O type stars to the WR stage. We consider only those scenarios for WR formation and evolution that have been developed during the last few years and that appear to be sufficiently substantiated by observational and theoretical arguments. Finally, the theoretical results are compared with the observations, pointing out several still unsettled aspects of the problem.

## INTRODUCTION

During the past few years considerable progress has been made in attempting to understand the nature of Wolf-Rayet stars, on the base of improved observational data and theoretical models.

Population I Wolf Rayet stars, whose visible spectra show strong emission lines, are in general separated into two spectral sequences: WN and WC. WN star spectra are dominated by transitions of He and N ions with little evidence of C, whilst those of WC stars show predominantly lines of He and C, with little evidence of N.

Whether the apparent lack of C and N in WN and WC stars respectively, reflects real underabundances of those elements, or it is the result of unusual excitation conditions in their atmospheres is still controversial. Furthermore, the analysis of optical spectra of WN and WC stars seems to indicate little evidence of atmospheric H (Smith, 1973). This feature however does not hold for all WN stars, because they are commonly separated into two subgroups according to whether in their spectra there is little or no evidence of H, or some significant H can be seen. Broadly speaking, this separation corresponds to that between early (WNE) and late (WNL) WN stars. Nevertheless, few exceptions are known to exist in both cases, which somehow weaken the above schematization. It is worth emphasizing that arguments similar to those invoked to explain the apparent lack of C and N can be brought to account for the

apparent underabundance of H.

Detailed analyses of WR spectra favouring the chemical anomaly hypothesis are by Smith (1973), Nugis (1975), Willis and Wilson (1978), Smith and Willis (1981), whereas arguments in favour of excitation effects and normal chemical abundances are given by Underhill (1980, 1981).

Many of the WR emission lines exhibit P Cygni profiles with developed violet absorption component, suggesting the existence of more extensive winds than in all other early type stars.

Mass loss rates for WR stars appear to be much higher than for all OB type stars (Conti, 1978). In fact mean values of  $2.7 \cdot 10^{-5} M_{\odot}/\text{yr}$  and  $4 \cdot 10^{-5} M_{\odot}/\text{yr}$  are estimated for WN and WC stars respectively (Barlow et al., 1981).

One of the more important questions concerning WR stars is whether all of them are members of binary systems or truly single objects may also exist. Summarizing recent work on the subject by several authors (Massey, 1980, 1981; Niemela et al., 1980; Massey and Conti, 1980; Massey et al., 1981; Vanbeveren and Conti, 1980) we may conclude that many of the WR stars with absorption lines appear to be truly single, and that the percentage of visible close WR+O binaries seems to be at most 25%. If this percentage is doubled to account for WR stars with collapsed companions, as suggested by van den Heuvel (1976), the maximum percentage of binary WR's increases to at most 50%. The previous larger frequency (73%) of binary WR stars (Kuhi, 1973), which favoured the binary nature hypothesis for all of them, is thus lowered to the typical value of OB binary stars (Conti, 1976).

The above observational facts (chemical anomalies, high rates of mass loss and membership in binary systems) have strongly supported the view that WR are evolved, both chemically and evolutionary, objects in which the products of interior nuclear burning (H- and/or He-burning) are exposed at the surface, through the removal of the outer layers. Following this idea, Paczynski (1973) suggested that WN stars expose at their surface material which has been processed by CNO cycle in a convective core, whereas WC stars correspond to those stages in which material belonging to the He-burning convective core is exposed, via further mass loss. Although this scheme has been proposed to explain binary WR stars, in which mass is removed by mass transfer, it has been adopted for single WR stars as well.

Models of massive early type binary systems, incorporating mass loss via Roche lobe overflow and possibly mass loss by stellar wind, possess main characteristics that resemble those of binary WR stars. The current status of the subject has been reviewed by de Loore (1980, 1981). On the contrary, how single massive stars may evolve into WR objects is still matter of debate. Several alternatives have been suggested, which share the common idea that mass loss by stellar wind plays the dominant role although other mechanisms (mixing, overshooting and turbulent diffusion) may be equally important.

In this paper we will be only concerned with the problem of formation and evolution of single WR stars.

## 1. FUNDAMENTAL PROPERTIES OF WR STARS

Luminosities and effective temperatures of WR stars are very uncertain. What is reasonably well established is the visual absolute magnitude  $M_V$  for a limited number of objects for which the distance modulus has been determined independently. Effective temperatures of WR stars obtained by various methods differ greatly. Recent analyses of the subject can be found in Conti (1978), Willis (1980), Underhill (1980, 1981) and van der Hucht et al. (1981). Broadly speaking, effective temperatures are in the range 25000°K to 50000°K, although values around 30000°K are likely to be preferred (Willis, 1980). With this effective temperature, angular diameters for individual stars and known distances, Underhill (1980, 1981) gives the following values for the luminosity:  $L_g L/L_\odot = 5.6$  for WN7 and WN8 stars, and  $L_g L/L_\odot = 4.9$  for WN3 through WN6, and all of the WC subtypes. Different values for the effective temperatures are indicated by Conti (1978), who discusses two alternatives according to whether temperatures near 45000°K are adopted for WNL stars (WN7,8,9) and temperatures near 30000°K are used for WN3,4,5 or cool (~30000°K) and hot (45000°K to 53000°K) temperatures are assumed for WNL and WNE stars, respectively. Similar range of effective temperatures is suggested for WC stars. Different values for the luminosity follow from the adoption of the above effective temperatures. It is evident that the location of WR stars in the HR diagram is not as well known, even though one may reasonably assume that they lie near the upper zero age main sequence, in the same region of the brightest O type stars.

The WNE and WNL stars seem to form distinct groupings: fainter and hotter (or cooler) and brighter (or brighter and hotter), respectively. It is not probably a coincidence that WNL stars, that appear to be brighter than other WR stars and have the same luminosity of Of stars, are also known to clearly show evidence of H at the surface.

The mass of WR stars is very poorly known. Mass determinations, within the usual factor  $\sin^3 i$ , for the few WR stars member of binary systems yield an average value of about 20  $M_\odot$ , ranging from 10 to 50  $M_\odot$  (Massey, 1981). Attempts to establish a correlation between the WR subtype and the mass ratio  $M_{WR}/M_{OB}$  have yielded controversial results (Moffat, 1981; Massey, 1981). According to Niemela (1981), estimates of the minimum mass of binary WR stars reveal that WNL's are more massive than WNE's, being about 38  $M_\odot$  and 7  $M_\odot$  respectively, whereas late WC's can be considered as having masses in excess of 15  $M_\odot$ . No useful information exists for binary WC's of early type. Single WR stars elude direct mass determinations.

The most recent study of He, C and N abundances in WN and WC stars is by Smith and Willis (1981), utilizing new ultraviolet spectroscopic data obtained with the IUE satellite, and non LTE escape probability technique. The mean values for C/He, N/He, C/N ratios for WNL, WNE and WC stars as taken from Smith and Willis (1981) are summarized in Table 1. These new abundance determinations greatly improve upon the previous results of Willis and Wilson (1978), putting more constraints on stellar evolution scenarios for producing both binary and single WR stars. According to Smith and Willis (1981), the agreement between deduced

Table 1 (WR chemical abundances)

	WNL	WNE	WC
C/He	2.2(-4)-1.7(-3)	1.3(-4)-4.4(-3)	5.8(-2)
N/He	2 (-2)	2.2(-3)	< 2(-2)
C/N	6 (-3)- 4 (-2)	2. (-2)-6. (-2)	>60.

and theoretically expected abundance ratios strongly favours those scenarios in which WR stars are evolved objects, mass removal (mass exchange and/or mass loss) playing a dominant role.

The distribution of WR stars in the galactic plane has been the subject of a great deal of work during the last twenty years. Amongst others we recall Smith (1973), Stenholm (1975), Moffat and Isserstedt (1980), Gomez et al. (1981). With the aid of the recent catalogue of galactic WR stars by van der Hucht et al. (1981), Hidayat et al. (1981) have re-examined the problem of WR galactic distribution.

The main results of this study can be summarized as follows: the spiral structure is found to be more clearly pronounced than in earlier studies, there is an asymmetric distribution of WR stars in the galactic plane, and there appears to be an asymmetry in the Z distribution of single WR stars. As already noted in earlier studies, WR stars are more abundant toward the galactic centre. Furthermore, WC stars are more frequent than WN stars in the inner galactic regions, and there seems to be a relationship between spectral subclasses and location on the galactic plane (Gomez et al., 1981; Hidayat et al., 1981; Firmani, 1981), which may be of particular significance for theories of WR production and evolution. Number densities of WR stars are given by Maeder et al. (1980), Bertelli and Chiosi (1981) and Firmani (1981), and are compared with those of stars of the same luminosity and different spectral type (O-B, A-G, K-M) in order to assess the frequency of the WR phenomenon. The surface density (number of stars/Kpc<sup>2</sup>) of WR and red supergiant stars increases and decreases respectively toward the galactic centre, whereas the ratio of the number of red to blue supergiants increases with galactocentric distance (Humphreys, 1978). Maeder et al. (1980) find that the ratio of the number of red supergiants to the number of WR stars varies very strongly with galactocentric distance, whilst the ratio of the sum of the number of red supergiants and WR stars to that of blue supergiants remains almost constant. This observational result is however questioned by Bertelli and Chiosi (1981), who argue that the strong variation of  $N_{WR}/N_R$  ratio is mostly due to the different luminosity limits for WR and red supergiant stars adopted by Maeder et al. (1980). The location of WR stars in the HR diagram, their relatively high mass (assessed for binary and suspected for single objects), their anomalous surface chemical composition (if this is the correct interpretation of their spectral characteristics), their distribution on the galactic plane and association with the young population strongly suggest that luminous O type stars are the most natural progenitors of WR's. This fact, originally pointed out by Smith (1973) served as guide line for more so-

phisticated evolutionary scenarios (Conti, 1976; Chiosi et al., 1978, 1979a; Maeder, 1980, 1981a,b; Noels and Gabriel, 1981). Another important question one would like to clarify before fitting WR stars into any evolutionary scheme, is the correlation between the spectral classification and evolutionary stage. In other words, are late WN's and WC's the progenitors of the early ones and do all WN's precede all WC's in the evolutionary history? Or more complicated schemes have to be devised, perhaps governed by the initial mass and average rate of mass loss? Unfortunately, the nowadays observational information is unable to cast light on the point, which is left to mere speculation.

## 2. SUMMARY PROPERTIES OF MASSIVE STAR EVOLUTION

It is easy to understand that if massive O type stars are progenitors of WR's, the formation and evolution of WR stars cannot be studied separately from the evolution of massive stars. It has been known for many years that the observational HR diagram of supergiant stars in the solar vicinity (Humphreys, 1978) cannot be interpreted by means of standard mass conservative models of massive stars (Chiosi, 1978). In particular, the existence of very luminous O type stars and lack of supergiants of the same luminosity but later spectral type (from B3 to M), the ratios of the number of blue to yellow and red supergiants at lower luminosities, and finally the crowding of blue supergiants in the spectral range B0 to B2 are at variance with current theoretical predictions, as illustrated in Fig. 1a.

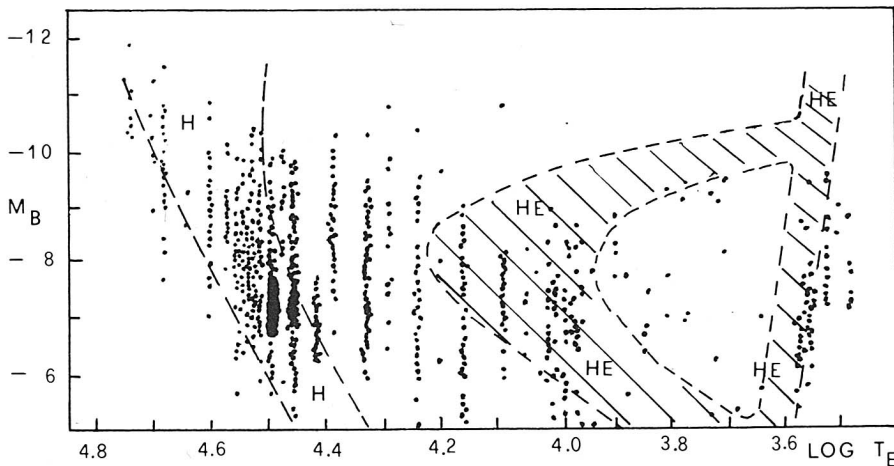


Fig. 1a - Observational (dots) and theoretical (constant mass) HR diagrams.

Although several causes have been invoked to explain the discrepancies between theory and observations, mass loss by stellar wind during the star lifetime appeared to be the most appealing one, as the existence of winds with mass-loss rates significant for the evolution of massive stars is strongly indicated by current observations.



Recent reviews on the mass-loss rates for early and late type stars can be found in Cassinelli (1979), Conti (1981) and Dupree (1981). The theoretical understanding of the mass loss phenomenon is however still unsettled (Hearn, 1981; Linsky, 1981). Moreover there is no general consensus on parametrizations of the mass-loss rates in terms of basic stellar parameters (luminosity, mass, radius, chemical composition...). Amongst others, we recall here Andriesse (1981), Lamers (1981), Chiosi (1981a), Chiosi and Olson (1981) for early type stars, and Reimers (1975) for late type stars. All model computations in which mass loss is taken into account, are based on semiempirical parametrizations which reflect both observational and theoretical uncertainties.

### Evolution with mass loss

Evolutionary models of massive stars through the major phases of nuclear burning and in presence of mass loss by stellar wind have been the subject of a great deal of theoretical work during the last few years. The recent reviews by de Loore (1979, 1980) and Chiosi (1981b) widely summarize the significant results of these computations.

The most relevant features as a consequence of mass loss are:

i) the main sequence band widens at lower mass ranges and shrinks at the highest range for sufficiently high rates of mass loss.

ii) semiconvection and intermediate full convection, the latter known to set in during the shell H-burning phase, are drastically affected by the occurrence of mass loss. Semiconvection is absent even for very small rates of mass loss, whilst intermediate fully convective zone is in general much less extended, the reduction being proportional to the mass-loss rate (Chiosi et al., 1978, 1979b). For sufficiently high rates of mass loss the intermediate fully convective zone never sets in (Noels and Gabriel, 1981; Maeder, 1980, 1981a,b). The development of the H-burning shell favours a redward motion in the HR diagram, which is somehow driven by the extent of the intermediate convective zone that in turn depends on the mass-loss rate (Chiosi et al., 1978; Stothers and Chin, 1979; Falk and Mitalas, 1981; Chiosi et al., 1980; Maeder, 1981a).

This effect is contrasted by the increase of the He-core, which is known to start a blue loop when larger than a critical value (Lauterborn et al., 1971; Chiosi et al., 1978; Chiosi et al., 1980). The mass of the He-core and intermediate convective zone are deeply related, because a well developed convective layer prevents a significant increase of the He-core. The opposite occurs when the convective layer is small or absent. This explains why the location of core He-burning models in the HR diagram is found to sensitively depend on the mean rate of mass loss in both core H- and He-burning phases (Chiosi et al., 1978, 1980; Chiosi, 1981b,c; Maeder, 1980, 1981b; Noels and Gabriel, 1981). In addition to this, it makes also understandable why different results have been found by different authors, giving the impression of an apparently erratic behaviour of He-burning models. In fact, small differences in input physics (opacity, nuclear reaction rates, stability criteria against convection, mass loss rates, ...), and computational techniques may

easily account for the different results obtained by different authors (see Chiosi (1981c) for a thorough discussion of the point). The dependence of the location of core He-burning models in the HR diagram on the mean mass-loss rate led Chiosi (1981b,c) to suggest that core He-burning band merges the main sequence in the region of most luminous O-type stars for suitable values of the mass-loss rate, as illustrated in Fig. 1b.

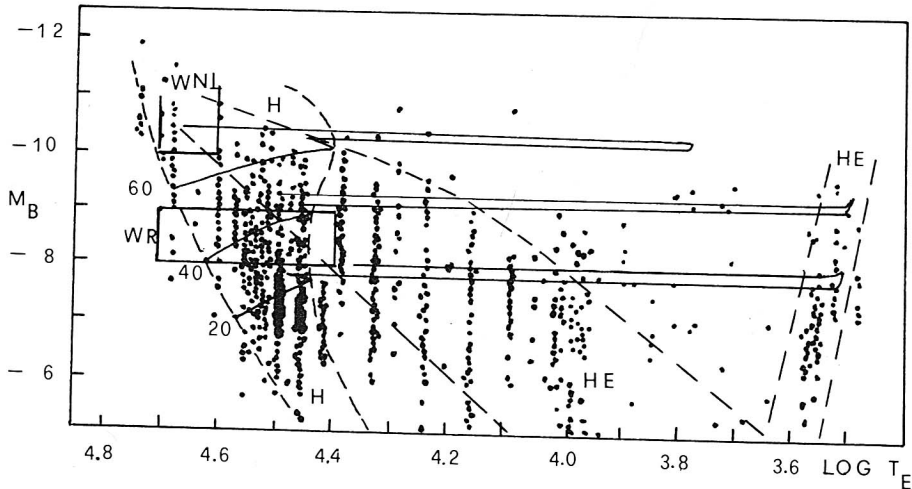


Fig. 1b - Observational (dots) and theoretical (with mass loss) HR diagrams. The location of WR stars is also shown.

iii) Mass loss by stellar wind during both core H- and He-burning phases yields a variety of stellar remnants, which hopefully should be located near the zero age main sequence, and which should be observed as H-poor, He-rich, and N-rich or C-rich stars. For sufficiently high rates of mass loss, the most massive stars expose CNO processed material even at the latest stages of core H-burning, whilst this occur for lower mass stars only during late stages of central He-burning. The appearance of  $3\alpha$ -processed material is possible only for those models that suffered sufficiently high rate of mass loss in both core H- and He-burning phases.

For lack of an unique prescription for the rate of mass loss as a function of basic stellar parameters, and well defined observational constraints, the stage at which anomalous chemical abundances start appearing at the surface is uncertain.

As an obvious consequence of this fact, the gross features of the stellar remnants, mass, lifetime, luminosity, surface abundances... are uncertain, and the various scenarios for producing WR stars on the basis of those remnants are subject to criticism.

Although unadequate, those evolutionary models incorporating the effect of mass loss, explained several features of the HR diagram that could not be matched by constant mass models.

The different shape of the core H-burning band (wider at lower initial masses and narrower at higher masses, the mass boundary being around 50



to  $60 M_{\odot}$ ) was thought to account for the observed upper boundary in the luminosity of early type stars (Humphreys and Davidson, 1979), and the absence of the gap in the HR diagram predicted by the very short duration of the shell H-burning phase. As for the latter, it was suggested that the Kelvin-Helmoltz gap could be populated by stars in the latest stages of central H-burning, early and late stages of core He-burning, for suitable values of mass-loss rates. However simple statistical considerations show that the relative percentages of blue, yellow and red supergiants are at variance with model predictions, because too many B0-B2 stars are observed for being in those rapid stages of evolution. Significant changes in model structure and a revision of the initial mass function for massive stars were suspected to be necessary to explain this observational fact (Bressan et al., 1981; Bertelli and Chiosi, 1981). The lack of very luminous red supergiants was interpreted as due to the fact that stars initially more massive than 50 to  $60 M_{\odot}$  are expected either to spend an extremely short time at low effective temperatures or to live forever as blue objects, for a suitable combination of mass loss during core H- and He-burning phases (Chiosi et al., 1978; Chiosi, 1981b,c). On the contrary, stars of lower initial mass are expected to spend part of their core He-burning phase as red supergiants.

#### Evolution with overshooting and mass loss

The extent of overshooting by convective elements from unstable layers in stellar interiors is not very well known. Recent work on the subject (Shaviv and Salpeter, 1973; Maeder, 1975; Cogan, 1975; Maeder and Mermillod, 1981; Cloutman, 1978; Cloutman and Whitaker, 1980; Roxburgh, 1978) has pointed out that contrary to what believed in the past the scale length of this phenomenon may be a non negligible quantity. Following this line of thought Bressan et al. (1981) have included this effect in the calculation of convective cores in massive stars, extending up to this range of mass a similar analysis performed by Maeder and Mermillod (1981) for low and intermediate mass stars. Bressan et al. (1981) discussed the effect of overshooting on the core H-burning phase of models evolved both at constant mass and with mass-loss rate as formulated by Castor et al. (1975). Models with overshooting and a more recent formulation of the mass-loss rate for OB stars (Chiosi and Olson, 1981), and a suitable parametrization of the rate for late type stars, based on Bernat's (1977) and Bernat's et al. (1979) data, have been calculated by Bertelli et al. (1981) up to the stage of core He-exhaustion. The main characteristics of those numerical computations are summarized in the following:

- i) semiconvection and/or full intermediate convection does never occur as a result of the increased size of the convective core. This feature holds also for models evolved at constant mass, thus ruling out the long lasting uncertainty in massive stars models, due to the adoption of one of the two possible stability criteria against intermediate convection, see Chiosi (1978) for details.
- ii) Models evolve at much higher luminosity for both constant and decreasing mass.
- iii) Mass loss and overshooting contribute to significantly increase the

lifetime of the core H-burning phase.

iv) The main sequence band at lower mass range is much wider than ever before due to the combined effect of mass loss and overshooting. More precisely, the extension of the main sequence band somewhat depends on the mixing length parameter  $\lambda$  that is adopted in the evaluation of the overshooting region. At higher mass range constant mass models would spread across the whole HR diagram. On the contrary, with the rates of mass loss suggested by current observations, the core H-burning band shrinks toward the zero age main sequence, as in previous computations without overshooting. The new location of the main sequence band in the HR diagram, shown in Fig. 1c, is of particular interest as it may extend up to the spectral type B1. The models spend in the spectral range B0-B1

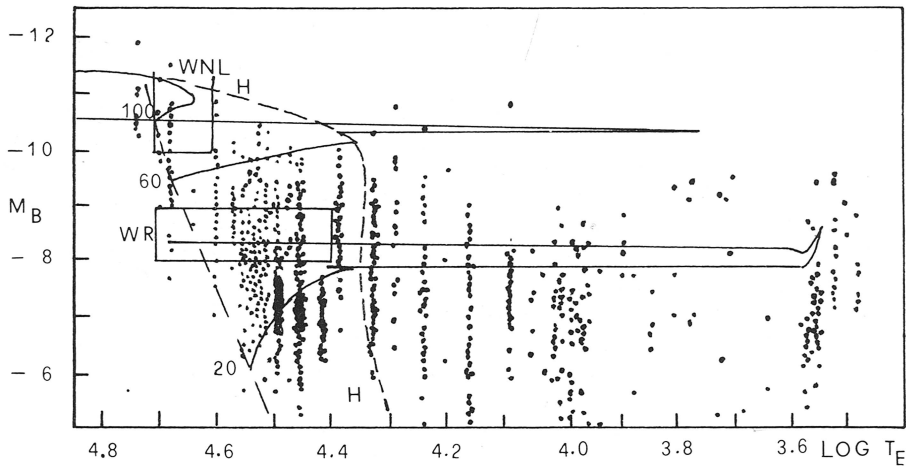


Fig. 1c - Observational (dots) and theoretical (with mass loss and overshooting) HR diagrams. The location of WR stars is also shown.

about 20% of the total core H-burning lifetime, which roughly amounts to the total He-burning lifetime.

According to this, we expect as many blue stars in this spectral range as all other stars (blue, yellow and red supergiants) beyond the spectral type B1.

v) The core He-burning phase takes place partly in the red supergiant regions and partly in the domain of blue stars, for initial masses in the range 20 to 50-60  $M_{\odot}$ . About one third of core He-burning lifetime is spent as red supergiant by a typical initial 20  $M_{\odot}$  star with  $\lambda=1$  (Bertelli et al., 1981). This fraction however depends on  $\lambda$  and rate of mass loss, as the duration of the red phase is known to last as long as the fractionary mass  $q_c$  of He core has become larger than some critical value, which in turn depends on the initial mass (Chiosi et al., 1978; Maeder, 1981a). The closer is  $q_c$  to the critical value at the beginning of the red phase, the shorter is the lifetime there. Two factors contribute to increase  $q_c$ , namely  $\lambda$  and the rate of mass loss. In fact the mass of the He core increases with  $\lambda$ , whereas the fractionary mass of it increases as the total mass decreases by mass loss.

Therefore the higher is the mean mass-loss rate, the shorter is the lifetime of the red supergiant phase. On the contrary, stars of initial mass greater than 50 to 60  $M_{\odot}$  spend their whole core He-burning phase as blue objects located on, or at the left of, the zero age main sequence.

vi) The same explanation for the lack of luminous red supergiants that was suggested on the basis of the old models, still holds with the new ones.

vii) Again a variety of remnant masses is produced, which presents more favourable features for scenarios of WR star formation and evolution.

### 3. SCENARIOS FOR WR FORMATION

Various suggestions exist in the literature aiming at understanding the evolutionary status and past history of WR stars. They differ in the physical mechanism leading to the formation of luminous chemically anomalous objects with relatively high effective temperatures and high rates of mass loss, in the evolutionary stage during which the envisaged process is efficient, and the transition from a normal star to a WR object takes place. This latter point constitutes one of the major uncertainties in all existing scenarios, as often rather arbitrary assumptions have been made in assigning the WR status (for instance the much higher mass-loss rate) to a given evolutionary stage.

The simplest scenario was suggested by Conti (1976) in which a sequential scheme leads normal O stars, as a result of mass loss by stellar wind (radiation pressure, or some other mechanism) to Of and to WNL stars, which in turn evolve into WNE and finally into WC stars, if further mass loss were to occur. The most recent studies of this scenario are by Noels and Gabriel (1981) and Bressan et al. (1981), former studies were by de Loore et al. (1977) and Chiosi et al. (1978). There is some debate as to whether WNL stars, which represent about 20% of the total WR population, are in H- or He-burning phase. Since the basic requirement is that CNO processed material with appreciable H content ( $N(H)/N(He) \approx 2$ ) is exposed to the surface, a suitable combination of two factors is necessary, namely the mass size of the main sequence convective core and the rate of mass loss. In the models of Chiosi et al. (1978) and de Loore et al. (1977), based on the classical prescription for the extension of the convective core and rate of mass loss predicted by the radiation pressure theory of Castor et al. (1975), in spite of the huge mass removal during core H-burning phase ( $\sim 40\%$  of the total mass) the desired condition was met only in the very late stages of central H-burning of the initially most massive stars. The situation was even worst when empirical rates suggested by the observations (Conti and Garmany, 1980) were used (Chiosi, 1981a; Lamers, 1981), as very little mass was expected to be lost during the main sequence phase (less than 10%). These facts led Chiosi et al. (1978, 1979b) and Chiosi (1981c) to suggest that WNL stars are burning He in the core. Noels and Gabriel (1981) improved upon the duration of the WNL phase, as being core H-burning, by suitably and somehow arbitrarily adjusting the rate of mass loss in model computations. On the contrary, models in which overshooting from the convective core is taken into account, no matter of the

adopted mass-loss rate, yield much more favourable conditions for WNL stars being in core H-burning, due to the much longer lifetime spent with typical CNO processed material exposed at the surface (Bressan et al., 1981; Bertelli et al., 1981).

As to whether WNL can evolve to WNE stars, assumed to show no H at all at the surface, and then to the WC type, it is a matter of lifetime and rate of mass loss. Since the total duration of the WNE and WC phases in the most favourable case corresponds to the total core He-burning lifetime, it is the mean rate of mass loss that plays the dominant role. Chiosi et al. (1978) were unable to completely remove the H-rich envelope at the top of the He-core of initially most massive stars, reaching the conclusion that stars in this range of mass may appear only as WNL. On the contrary, Noels and Gabriel (1981) and Maeder (1981a,b), with the aid of higher mass-loss rates, successfully peeled off the stars down to the layers belonging to the He-core and He-burning core, where He is converted into C and O. Bertelli et al. (1981) who adopted lower rates of mass loss were unable to bring  $3\alpha$  material to the surface.

According to Maeder (1981b) the use of models with larger convective cores (overshooting and/or turbulent diffusion) and Lequeux's (1979) initial mass function seems to reproduce the percentage of WNL stars. On the contrary Bertelli et al. (1981) found that using Lequeux's (1979) initial mass function, the whole core He-burning lifetime and up to 0.15-0.25 of the core H-burning lifetime are necessary to reproduce the observed frequency of WNL's for progenitors more massive than  $60 M_{\odot}$ . Their numerical computations however do not support such high fraction of core H-burning spent with low surface H content, because of the lower mass-loss rates predicted by Chiosi and Olson's (1981) relation. From the above numerical computations it turns out that Conti's suggestion might perhaps apply to O type stars with initial mass in excess of about  $60 M_{\odot}$ . In any case, the percentage of WNE and WC stars cannot be reproduced by means of Conti's (1976) scenario alone and other schemes must be devised. Chiosi et al. (1978) distinguished two different ranges of initial mass: an upper range ( $60$  to  $100 M_{\odot}$ ), where Conti's scheme applies, which is mostly responsible of the WNL stars; a lower range ( $20$  to  $60 M_{\odot}$ ), where O stars evolve during the core He-burning phase through blue-yellow and red supergiant stages, and then into WNE and WC stars. Mass loss at the red supergiant stage played the major role, compensating for the lack of sufficient mass removal during core H-burning with the current rates of mass loss in this range of initial masses. Although preliminary, Chiosi's et al. (1978) computations clearly pointed out that the post red supergiant scheme may provide another site of WR star formation. Detailed numerical analyses of the post red supergiant scheme have been recently performed by Maeder (1981a,b) greatly improving upon the original suggestion. With the aid of Bertelli's et al. (1981) models and initial mass function of Lequeux (1979) we derive the number densities of WR stars for Chiosi's et al. (1978) scenario, and compare them with the observational value derived from van der Hucht's et al. (1981) catalogue. Theoretical and observational estimates are summarized in Table 2 where the number density of OBA and red supergiants, derived from Humphreys' (1978) catalogue, are also reported for purposes of comparison. Observational entries of Table 2 are the numbers of stars/Kpc<sup>2</sup> for the solar vicinity,

which is defined as the circular region of 2 Kpc radius centered on the Sun.

Table 2 N(Stars)/Kpc<sup>2</sup>

OBA	MK	WR	WNL	WNE	WC	WNE+WC
Observed						
20.5	0.93	1.43	0.08	0.40	0.95	1.35
Predicted						
31	0.8	0.55	0.04			0.50

Several comments on the star densities of Table 2 are necessary for the sake of completeness. Densities of supergiants are derived by counting only stars more luminous than  $M_p = -7$  in order to have consistency with the densities of WR stars, which appear to be brighter than  $M_p = -7$ . In fact, as massive stars are known to evolve at about constant luminosity, it seems unlikely that WR stars may have progenitors of much lower luminosity on the main sequence. Likely the opposite may be true.

The densities of WR stars are derived from the catalogue of van der Hucht et al. (1981) excluding binary and suspected binary objects. The predicted density of red supergiants may be overestimated because it is derived from models with overshooting and mixing length parameter  $\lambda = 1$ . Larger values of  $\lambda$  (for instance  $\lambda = 2$ ) would significantly decrease the lifetime spent as red supergiant (Bertelli et al., 1981). Finally, initial masses for WNL and WNE+WC stars are tentatively assumed in the range  $M > 60 M_\odot$  and  $20 M_\odot < M < 60 M_\odot$ , respectively.

The comparison of empirical with theoretical values shows that theories of WR formation still poorly agree with the observations. Unless a large fraction of the so called single WR stars are in reality binaries, other yet unsuspected effects must be invoked. Since lifetimes of the WR stage cannot be much longer than those used in the above estimates, we suspect that the reason for the discrepancy lies in the initial mass function. In fact Bertelli and Chiosi (1981), Firmani (1981) and Garmany (1981), independently reached the conclusion that the initial mass function of Lequeux (1979) underestimates the true number of O type stars, and consequently of all later types, by a factor ranging from 2 to 3, due to the incompleteness of Humphreys' (1978) catalogue. If this is the case, all theoretical entries of Table 2 should be accordingly modified reaching a much better agreement, at least as far as WR stars are concerned. Blue and red supergiants do not raise particular difficulties, as for the former the empirical value is underestimated whilst for the latter the theoretical value may be overestimated, thus allowing for shortening in favour of the WR stages.

Within the above scheme, the fact that WNL stars may evolve into WNE's and WC's, thus contributing to increase the frequency of the latter types, is not very important, because of the relative frequency of progenitors under any reasonable initial mass function. Much more intriguing is to foresee whether WNE stars may evolve into WC stars. In any case, as already mentioned in the course of this paper, this possibility depends



on the mass-loss rate for WR stars and the evolutionary stage (sometime during core He-burning) at which those much higher rates set in ( $\dot{M}(\text{WN})=2.7 \cdot 10^{-5} M_{\odot}/\text{yr}$ , and  $\dot{M}(\text{WC})=4.1 \cdot 10^{-5} M_{\odot}/\text{yr}$ , Barlow et al., 1981). If we apply those rates to models in the blue part of core He-burning, which represents the most favourable situation, we get the indicative values for the mass lost during the WR phase, reported in Table 3 ( $\Delta M_a$ ).

Table 3 ( $\tau_{\text{He,B}}$  is in units of  $10^6$  yr; masses are in units of  $M_{\odot}$ )

$M_i$	$\tau_{\text{He,B}}$	$\Delta M_a$	$\Delta M_b$	$M^*$	$M_{\text{He-C-O}}$
20	0.41	13.9	4.1	12	7.6
60	0.33	11.2	3.3	39	32.3
100	0.28	9.5	2.8	62	55.8

For purposes of comparison, we report the same quantity but evaluated with the mean mass-loss rate of  $10^{-5} M_{\odot}/\text{yr}$  ( $\Delta M_b$ ). Both of them have to be compared with the mass  $M^*$  at the beginning of the so called WR phase, and the maximum mass  $M_{\text{He-C-O}}$  of the He-C-O core (taken from Bertelli et al., 1981). All existing evolutionary computations are more or less encompassed by the values of Table 3. Unfortunately, the situation is still so unsettled that no firm conclusions can be advanced. With the highest rates of mass loss CO material can be easily exposed at the surface, but the relative duration of WC with respect of WNE phase is still uncertain, and some difficulties may be encountered in accounting for the mean mass of WR stars, close to  $20 M_{\odot}$  according to Massey (1981). Were the true rates of mass loss of WR stars slightly lower than those commonly quoted in the literature, because of the many uncertainties still existing in mass-loss rate determinations (Barlow et al., 1981), it would appear likely that only those stars that suffered mass loss in the red supergiant phase may evolve up and through WC stages. In conclusion the following sequential scheme for WR production is advanced

Progenitor Mass	Evolutionary Sequence
$M < M_1$	O - BSG - RSG
$M_1 < M < M_2$	O - BSG - RSG - BSG - WNE - WNC
$M > M_2$	O - Of - WNL - (WNE?) - (WNC?)

where  $M_1$  and  $M_2$  are tentatively assumed  $M_1=20 M_{\odot}$ ,  $M_2=60 M_{\odot}$ . In addition to the fact that the above scheme seems to successfully reproduce the frequency of WR stars, it may also account for their anomalous chemical abundances. This aspect of the problem has been studied by Gabriel and Noels (1981), Noels and Gabriel (1981) and Maeder (1981a), who performed detailed nucleosynthesis calculations, and by Smith and Willis (1981), who compared theoretical predictions with abundance determinations. In this picture, WC stars are expected to be totally N deprived, whereas up to now the observations seem to indicate that some N is present in WC stars. The results of Smith and Willis (1981) are not conclusive, as their upper limit for the abundance ratio  $\langle N/C \rangle \approx 0.02$  is still too high compared with the theoretical expectation of  $10^{-6}-10^{-7}$  (Gabriel and Noels, 1981).



In favour of the above scheme is the gradient of the ratio of red supergiant to WR stars ( $N_R/N_{WR}$ ) found to vary strongly with galactocentric distance (Maeder et al., 1980). Those authors pointed out in fact that  $N_R/N_{WR}$  decreases by about a factor of 100 passing from 13 Kpc to 7 Kpc whereas over the same distance the ratio of the sum ( $N_R+N_{WR}$ ) of the number of red supergiant and WR stars to the number  $N_B$  of blue supergiants remains virtually constant. Maeder et al. (1980) interpret this observational fact as due to a strong effect of metallicity on the mean rate of mass loss and in turn on the duration of the red supergiant and WR phases. Although these conclusions have been somewhat weakened by the analysis of Bertelli and Chiosi (1981), the galactic distribution of densities of red, blue supergiant and WR stars may somehow reflect effects of gradients of heavy elements on mass-loss rates and on the duration of red supergiant and WR phases, and finally be consistent with the above scheme.

However the observed distribution of WR subclasses across the galactic plane may raise some difficulties, perhaps leading to a revision or un-plementation of the above scheme. In fact Firmani (1981) has pointed out that WNE stars distribute on the galactic plane differently from all other WN types. Similar behaviour is found for the WC6 subclass which distributes differently from all other WC types. Finally, the distributions of WNL and WC stars (WC6 excluded), in addition to the well known concentration toward the galactic centre, do not present particular asymmetries. Such different spatial distribution between WNE's and WNL's, and between WNE's and WC's (WC6 stars excluded) might perhaps suggest that WNE stars are not progenitors of WC stars. Although the above observational hint is not based on a large statistics, we feel that some deep meaning is hidden. There is the possibility that some physical mechanism either drastically shortens or suppresses the WNE phase of post red supergiant objects (and if necessary of the WNL descendants). It has been shown that when star models have reached the so called WNE configuration, (the whole H-rich envelope expelled and CNO processed layers left at the top of He-burning core), and the mass size of the He-burning core has reached its maximum extension, vibrational instability may set in if the remnant mass is greater than some critical limit (Gabriel and Noels, 1981). This minimum mass is estimated to be around  $16 M_{\odot}$  for pure He stars (Noels and Masereel, 1981). The e-folding time of this instability is much shorter ( $10^3$  yr) than the evolutionary time scale. Although the consequences of this instability are difficult to evaluate, Noels and Gabriel (1981) argued that it causes a rapid strong enhancement of the mass-loss rate. By arbitrarily increasing the rate by a factor from 10 to 250, they rapidly removed the CNO layers, showing that when  $3\alpha$  material appears at the surface stability is restored. The duration of the WNE phase is therefore much shorter. Instead of the mass loss enhancement, other effects would have been equally possible, extended internal mixing for instance, which would act in the same sense, probably on a much shorter time scale than mass loss alone. As a consequence of this, the WNE phase would be missing for all remnant masses greater than the critical limit. It is an amazing coincidence that  $M^*$  for the initial  $20 M_{\odot}$  star of Table 3, roughly corresponding to the mass at the beginning of the WNE stage, is rather close to the stability limit. Furthermore, somewhat supporting this

view is the result of Niemela (1981), who estimates that WNE stars have masses not much higher than  $7 M_{\odot}$ , whilst WC stars have masses in excess of  $15 M_{\odot}$ . However since the state of art of theories of WR formation and evolution is rather uncertain, no conclusions can be drawn. The subject is therefore left open to further investigation.

Several other alternatives for the formation and evolution of WR stars exist in the literature (see for instance the bibliography by Simon and Stothers (1969), the exhaustive discussion by Maeder (1981b) and the suggestion by Underhill (1981)). However they will not be described here, as we prefer to present only those scenarios to which have been paid more attention over the last few years, because they are sufficiently substantiated by observations and systematic theoretical analyses.

#### 4. WR STARS IN THE MAGELLANIC CLOUDS

The HR diagrams of supergiant stars and the global properties of WR stars in LMC and SMC show similar, though systematically different, features when compared to their galactic counterpart. They differ in fact in the properties of stellar winds (Hutchings, 1980), suggesting that OB type stars are losing mass at a lower rate; in the colour of red supergiants, which are bluer than galactic ones; in the highest luminosity limit for early to intermediate spectral type supergiant stars, which lowers passing from SMC to Galaxy (Chiosi et al., 1980); in the ratio between core H-burning and core He-burning supergiant stars (Bisiacchi and Firmani, 1980); finally in the relative frequency of single and binary WR stars as a function of the spectral type (Vanbeveren and Conti, 1980).

As for the latter point, while galactic WR's seem to equally populate each subtype, late WC's are absent in LMC, and an almost complete lack of WC stars is observed in SMC. In addition to this, the frequency of binary with respect to single WR's decreases from SMC to LMC and Galaxy. Finally as pointed out by Maeder et al. (1980) the ratio  $N_R/N_{WR}$  varies among the three galaxies. Since the three galaxies are known to differ in the mean metallicity, which systematically increases from SMC to LMC and Galaxy, the most plausible interpretation of the above observational facts is sought in terms of the different metal content. It has been suggested by various authors that the effects of heavy element abundance on the rate of mass loss may be the key for understanding the above differences. A recent review on the subject is by Chiosi (1981c), whilst an exhaustive discussion of the effect of metallicity on scenarios for WR formation is by Maeder (1981b).

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## DISCUSSION FOLLOWING CHIOSI

Vanbeveren: About the uncertainties I have a question. Usually from evolutionary computations is determined the mass loss rate. One may also use the mass-luminosity relation. This has been used for X-ray binaries. If we want to explain the overluminosity of the optical component of X-ray binaries by stellar winds, we need a large stellar wind. The problem is, that if this is true all the optical components in X-ray binaries should have a N/C ratio which approaches the equilibrium abundances for the CNO cycle, i.e. a N/C abundance for WN stars. The optical components have been studied carefully but these ratios have not been found. According to your scenario you should expect that the mass of the WC stars should be lower than that of the WN stars. Is this observed?

Chiosi: The transition from WN to WC stars does not require a very large mass removal because the two zones are separated in the layer. The mass difference is of the order of 0.5 to 1  $M_{\odot}$  and probably this cannot be observed.

Underhill: Spectroscopically you see something; to explain it in terms of abundances requires a good physical model in terms of electron temperature densities; you have not got it. Abundance analyses assume the same representative point for every set of ions. Any knowledge of how ions behave in different plasmas suggest that this is a very rough approximation that cannot be justified.

Maeder: Concerning the scenario for the formation of WR stars I think that one should always be aware that due to the large change through the Galaxy, or the LMC one should mention that the balance between the various possible scenarios is strongly dependent on the galactic location and initial chemical composition. This means that when you make a comparison it should be specified for what region it is used.

Chiosi: This is done for the solar vicinity.

Maeder: Concerning the overshooting it is clear that the effect should be included. I wonder if it is sufficient to explain the main sequence widening. In your computations the effect of  $\mu$ -gradients is included and this stops the overshooting according to your expression. I doubt that this occurs. The  $\mu$ -gradient will not stop the overshooting as the turn-over time of convection is extremely fast and even if you only penetrate by a few 100 m at each time, you will eat the  $\mu$ -gradient very rapidly.

Chiosi: The overshooting models could probably overestimate the effect because a frictional term is not included.