

THE MATHEMATICAL ASSOCIATION

The fundamental aim of the Mathematical Association is to promote good methods of mathematical teaching. A member receives each issue of one or more of *The Mathematical Gazette*, *Mathematics in School*, *Primary Mathematics* (according to the class of membership chosen), together with Newsletters. Reports are published from time to time and these are normally available to members at a reduced rate. Those interested in becoming members should contact MA Headquarters for information and application forms. The address of the Association Headquarters is **Charnwood Building, Holywell Park, Loughborough University Science and Enterprise Park, Leicestershire LE11 3AQ, UK (telephone 0116 221 0013)**. The Association should be notified of any change of address. If copies of the Association periodicals fail to reach a member through lack of such notification, duplicate copies can only be supplied at the published price. If change of address is due to a change of appointment, the Association will be glad to be informed. Subscriptions should be submitted to the Treasurer via Headquarters. Correspondence relating to **Teaching Committee** should be addressed to TC Chair. The **Association's Library** is housed in the University Library, Leicester.

Views expressed in the *Mathematical Gazette* by authors or advertisers are not necessarily those of the Association.

THE MATHEMATICAL GAZETTE

Editor:

Dr Gerry Leversha,
15 Maunder Road,
Hanwell,
London W7 3PN
g.leversha@btinternet.com

Problem Corner:

Mr Chris Starr,
c/o Kintail,
Longmorn,
Elgin IV30 8RJ
czqstarr@gmail.com

Production Editor:

Mr Bill Richardson,
Kintail,
Longmorn,
Elgin IV30 8RJ
wpr3145@gmail.com

Reviews Editor:

Mr Owen Toller,
4 Caldwell House,
48 Trinity Church Road,
London SW13 8EJ
owen.toller@btinternet.com

Potential advertisers e-mail Charlotte Dyason at: charlotted@media-shed.co.uk

Material for publication should be sent to the Editor.

Books for review should be sent to the Reviews Editor.

Advice to authors of notes and articles.

Study the format of articles in the *Gazette*. Please note the format for references, which should be listed in their order of appearance in an article. MSS may be submitted electronically, preferably in pdf format, or, if sent by post, should be typed and two copies included. (Mathematical expressions may be hand written.) Please send electronic files by e-mail. This edition of the *Gazette* was produced on an Acorn machine using TechWriter and Draw.

10.1017/mag.2024.102

CONTENTS (continued)

Notes 108.19 to 108.35 (continued)

An amazing quartet of integrals	<i>Paul Levrie</i>	316
The n days of Christmas and other series	<i>Robin V. W. Murphy</i>	321
A recurrence relation derived graphically	<i>Paul Stephenson</i>	324
Covering a triangular number with heptagonal numbers	<i>Günhan Caglayan</i>	325
A pair of interesting inequalities for e^x	<i>Rasul A. Khan</i>	326
PWW: A property of triangular numbers	<i>K. B. Subramaniam, Amarnath Murthy</i>	329
More on the Euler limit for e	<i>Reza Farhadian, Vadim Ponomarenko</i>	329
π is a mean of 2 and 4	<i>Erik Vigren</i>	331
A geometric mean–arithmetic mean ratio limit	<i>Reza Farhadian, Vadim Ponomarenko</i>	334
Nearly isosceles right-angled triangles and square triangular numbers	<i>Terry S. Griggs</i>	336
Generalised Thales intercept theorem	<i>Francesco Laudano</i>	338
Point masses and polygons	<i>A. F. Beardon</i>	342
Some inequalities for a triangle	<i>Des MacHale</i>	345
One sharpening of the Garfunkel-Bankoff inequality and some applications	<i>Martin Lukarevski</i>	348
Analogues of the 12 pentagons theorem for other families of polyhedra	<i>Nick Lord</i>	351
Teaching Notes		
Betting games: higher-order thinking projects for calculus students	<i>Eduardo V. Teixeira, Katiuscia C. B. Teixeira</i>	353
Two quick direct proofs of the irrationality of $\tan 15^\circ$	<i>Nick Lord</i>	356
Infinite sums of reciprocal quadratics	<i>Nick Lord</i>	358
An Appreciation		
Nick Lord	<i>The Editor</i>	362
Problem Corner		
	<i>Chris Starr</i>	364
Student Problems		
	<i>Agnes Bokanyi-Toth</i>	370
Reviews		
		373

CONTENTS

Articles

Why Britain needs a national mathematics discovery centre	<i>Jack Abramsky</i>	193
Equally spaced squares and some impossible identities	<i>G. J. O. Jameson</i>	201
Three discs for the Mittenpunkt	<i>Martin Lukarevski</i>	209
Pythagorean triples using the relativistic velocity addition formula	<i>Howard Sporn</i>	219
A provocative tale of unwinding	<i>Subhranil De</i>	225
Large deviations and fairness for a betting game with a constant ratio of capital	<i>Toshio Nakata</i>	237
Viva 'Vis-viva'	<i>John D. Mahony</i>	248
Dependence of events, revisited	<i>Peter Braza</i>	257
Narayana's integer sequence revisited	<i>Tony Crilly</i>	262
An extension of the curious binomial identity of Simons	<i>Necdet Batir, Sevda Atpinar</i>	270
Sudoku strategies using graph theory	<i>Jeff Brown</i>	275
The asymmetric propeller with squares, and some extensions	<i>Quang Hung Tran</i>	283
More variations on Nagel and Gergonne analogues of the Steiner-Lehmus theorem	<i>Sadi Abu-Saymeh, Mowaffaq Hajja</i>	292

Obituary

Tony Gardiner (17th May 1947 – 22nd January 2024)	<i>Adam McBride</i>	303
---	---------------------	-----

Notes 108.19 to 108.35

Countable lists of rational numbers by removing digits	<i>Richard Kaufman</i>	311
Euler's totient theorem and Fermat's little theorem are generalisations of one another!	<i>O.A.S. Karamzadeh</i>	313

© The Mathematical Association 2024

Typeset by Bill Richardson

CAMBRIDGE
UNIVERSITY PRESS



Printed in Great Britain by Henry Ling Limited,
The Dorset Press, Dorchester, DT1 1HD
ISSN 0025-5572