

COMPARING OPTICAL AND RADIO - QUANTUM ISSUES

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Abstract. Once upon a time, there were radio and optical astronomers, each group with its own way of detecting and thinking about radiation. In spite of the shock treatment administered by Hanbury-Brown and Twiss more than thirty five years ago, this dichotomy persists to a surprising extent even today. As new techniques emerge, we clearly need to do better. Coherent and incoherent detection, heterodyning and interferometry are all situations where a full analysis involves quantum behaviour of the radiation. This informal review covers the general approach needed and goes over some points of principle which arise.

1. Introduction and scope

Astronomers are not the only people who confront both the wave and particle aspects of electromagnetic radiation. But this particular group of astronomers, concerned as it is with imaging the sky at all wavelengths by techniques which now cut across conventional boundaries, must feel the discomfort induced by this dual standard more acutely than most others. I have certainly felt it during my own encounters with radio synthesis and optical speckle, and would like to offer some practical suggestions which may alleviate the symptoms, though a full cure will become available only after quantum measurement has been fully sorted out. The kind of issues raised below will largely pertain to idealised thought-experiments. Real observations and techniques are of course well represented at this meeting by the people best qualified to do so. Even with this limitation, one is going over very well trodden ground and I can only claim to have chosen the route and some sights to point out along the way.

2. Modes and quanta

My unlikely patron saints for this journey into quantized electromagnetism are Rayleigh and Jeans, whom we remember for an unsuccessful radiation law but should really thank for teaching us that the electromagnetic field is a collection of harmonic oscillators. The most convenient choice of modes depends on the problem. For our purposes we will use travelling modes having a central frequency ν , a bandwidth $\Delta\nu \ll \nu$, a longitudinal extent $\sim c/\Delta\nu$, a transverse extent $D \times D$ and a solid angle $\theta \times \theta$, with $\theta \sim \lambda/D$. We count this as *one* degree of freedom of the field (per polarisation) and specify its complex amplitude $a = q + ip$, which has information both about the energy and the phase. Alternatively, we can use in-phase and quadrature components p and q and write the electric field as a sum over modes,

$$E(\mathbf{x}, t) = 2 \sum [q \cos(2\pi(\nu t - \mathbf{x}/\lambda)) + p \sin(2\pi(\nu t - \mathbf{x}/\lambda))] \quad (1)$$

or, in terms of the complex amplitude a

$$E(x, t) = \sum [(a) e^{-2\pi i \nu t} e^{\frac{2\pi i x}{\lambda}} + (a^\dagger) e^{+2\pi i \nu t} e^{-\frac{2\pi i x}{\lambda}}]. \quad (2)$$

Our in-phase and quadrature components are happily the momentum p and coordinate q of a harmonic oscillator of frequency ν .

All that is left is to apply quantum mechanics and statistics to these oscillators, converting p and q into operators in the usual way. Note that the wave function is $\psi(q)$ [its Fourier transform $\tilde{\psi}(p)$ if you want it in p space], *i.e.*, it depends on one of the electric field components. It is *not* a function of spatial position x ! Uncertainty relations hold between the in-phase and quadrature components. Of course, a shift in the origin of time would rotate p into q and vice versa. (This is unlike other harmonic oscillators where we usually feel that we can tell position from momentum). The equally spaced energy levels $(n + \frac{1}{2}) h\nu$ of this oscillator represent states of the field with n quanta in the chosen mode. As the time dependence in (2) suggests, the effect of the operator a (for annihilation) is to reduce the energy by $h\nu$, and the effect of $a^\dagger = q - ip$ is to increase it [1].

Why is one being so roundabout and filling space with oscillators? Can't we just treat photons as particles with wave functions $\psi(x)$ handled according to Bose statistics, but otherwise in the same way as electrons in atoms? Unfortunately not! Photons are massless, always relativistic, and freely emitted and absorbed. In fact, the usual process of registering a photon by a photodetector annihilates it - one can't make a second measurement to make sure it's still there! A single particle wave function $\psi(x)$ cannot handle this but in oscillator language there is no problem. One has just demoted one of the modes from n to $n - 1$ which an ordinary first-quantized mortal can handle using $\psi(q)$. The policy here will therefore be to stick to oscillators, leaving a particle description of photons to the twice-quantized. The oscillator point of view is actually quite respectable [2], and in any case no stranger than its counterpart for electrons, the Dirac sea.

Notice that Bose statistics is also automatic in this description. There is no limit to the occupancy n . Further, if two modes are both excited to the $n = 1$ state, it doesn't even make sense to ask which of the two photons is in a given mode. Another bonus is that interference fringes and in general all phenomena contained in the *spatial* dependence of the modes carry over smoothly to the quantum theory.

3. Photon counting

Let us start at the extreme optical end of the spectrum. The quantum oscillator which represents our mode interacts with a detector to which it gives up its energy, producing photocounts. Disregarding questions of efficiency, the ideal detector "cleans up" the travelling mode falling onto it and gives us n counts. Of course, the mode may not be in a state with a definite value of energy in which case one will have a probability distribution for n . A good example is a thermal state where, from elementary statistical mechanics, the probability is given by

$$p_n \propto \exp(-nh\nu/kT) \quad (3)$$

i.e., p_n form a geometric series. In the limiting case of a continuously varying intensity I with a mean I_0 , this gives

$$p(I) dI = \exp\left(-\frac{I}{I_0}\right) \frac{dI}{I_0} \quad (4)$$

This is just Rayleigh statistics and corresponds to a Gaussian distribution in the two components of the electric field which is what the radio people see. Long ago, Bothe [3] observed that one can go in the opposite direction, *i.e.*, from the classical distribution to the quantum one, by the following recipe

1. Construct a *Poisson* distribution p_{nI} for the number of quanta when the mean is $\bar{n} = I \times \Delta t/h\nu$, with intensity I fixed for the moment.

$$p_{nI} = \exp[-(\bar{n})] (\bar{n})^n / n! \quad (5)$$

This would be appropriate for uncorrelated events with a fixed rate $I/h\nu$ per unit time.

2. Now, regard I as a fluctuating quantity with the Rayleigh distribution, and average (5) with (4). The answer is (3). Notice how one has managed to complicate what was a very direct result in the quantum oscillator picture!

Semiclassically, this line of thought culminates in the Mandel photocounting formula [4]. One can calculate the probability of getting n counts by using an appropriate distribution function for the intensity I regarded as a continuous variable. This could be the Rayleigh distribution for thermal light or a delta function for an amplitude stabilised signal like a laser. The second step is the same as before, *viz.*, averaging a Poisson distribution whose mean is proportional to I . This formula is an excellent bridge between classical electric fields and photocounts. It is physically appealing because it separates wave noise, due to fluctuating electric fields, from particle noise (given by Poisson statistics even for a fixed intensity). The physical concept of wave noise manifesting itself as bunching of photons underlies accounts of the celebrated Hanbury-Brown and Twiss experiments [5]. In a practical sense, the Mandel formula means that one can get by in almost all applications with classical electric fields plus using the consequent wave noise to modulate Poisson statistics. Certainly this is true of the thermal excitation of the modes which is what all astronomical sources, barring little green men, are expected to give us. But the formula is basically a derived calculational rule rather than a basic principle. For example, there are minor precautions like leaving out the zero point energy of the oscillator from I . More seriously, one has situations where the variance of the counts is less than the mean \bar{n} , *i.e.*, less than for a Poisson distribution. An obvious example is a mode in a state of definite energy which gives zero variance for the total count. One is not going to explain this by *widening* a Poisson distribution further with a distribution of wave noise! Closer examination of the derivation of the formula reveals some fine print. The function which masquerades as the probability distribution of the intensity is not a genuine probability at all and must go negative in cases where we have sub-Poisson fluctuations! In such subtle ways does the quantum underworld resist attempts to capture it in a classical framework. This general phenomenon *viz.*, classical language with quantum meaning, was uncovered in the context of coherence theory by George Sudarshan and Roy Glauber in the early sixties [6].

4. Electric fields and phase space

Now let us swing over to the radio end of the spectrum, where our telescopes record complex electric fields (amplitudes and phases) in the modes which come in, *i.e.*, they measure q and p instead of $q^2 + p^2$. The uncertainty principle restricts us here – one cannot localise q and p to an area smaller than Planck's constant h in phase space. Fig. 1 shows that if the excitation energy ($\bar{n}h\nu$) is very large compared to the zero point energy, *i.e.*, $\bar{n} \gg 1$ one can pick a circular region of area h which has small fractional uncertainties for both amplitude and phase, *viz.*, $1/\sqrt{\bar{n}}$. This seems a nice way to approach classical behaviour.

But, what does a distribution in phase space mean, since simultaneously defining p and q is not allowed? Wigner was the first to show [7] that it is possible to use such a distribution and his definition would appeal to this gathering. Suppose we want to know the "probability" $W(q, p)$ of getting momentum p at a point q . The prescription is to take a baseline b symmetrically located about q , multiply the wave functions at the two ends and then Fourier transform this "visibility"

$$W(q, p) = \int \psi^*(q - b/2) \psi(q + b/2) \exp(-ipb) db$$

Like the related distribution function on phase space which Sudarshan and Glauber introduced, thirty years later, the Wigner function $W(q, p)$ is real but can go negative. But it does allow us to calculate averages correctly, using reasonable formulae like $\int \int W(q, p) F(q, p) dq dp$, provided one is clever enough (as Hermann Weyl was) to associate the right function F with the quantum operator \hat{F} . For the electric field components q and p , this is no problem. In this language, the ground state is a Gaussian in *phase space*, not just in real or momentum space alone. It is tempting to displace this rigidly to any desired point on the qp plane and obtain a state with electric fields defined as closely as the uncertainty principle allows (Fig.1). This goes by the name of a *coherent state* and was invented by Schrodinger though not in phase space language. These states have been thoroughly exploited in quantum optics [6,8] because of many beautiful mathematical properties, but here are two interesting physical ones.

1. A quantum oscillator excited from its ground state by a classical source (*e.g.*, a very heavy charge moving on a well defined trajectory with small quantum uncertainty) attains precisely this Gaussian distribution in phase space, which has minimum uncertainty. The centroid $z = q(t) + ip(t)$ moves around on a circle just like a classical oscillator. The wave packet does not spread. *Classical sources radiate coherent states.*
2. Let a field of this kind in a given mode transfer its excitation to two other modes, say by a beam splitter. Then (i) each of these daughter modes is itself in a coherent state and, (ii) experiments do not tell us whether these arose from a single source or two independent sources. This "cloning" is a characteristic property of classical signals [9] (the ultimate example being digital audio!). In contrast, a thermal state fed into a beam splitter shows positive correlations between counting fluctuations in the two output channels, a la Hanbury-Brown

and Twiss. A fixed number state would show *anti* correlation since any excess in one channel would have to be compensated by a deficiency in the other.

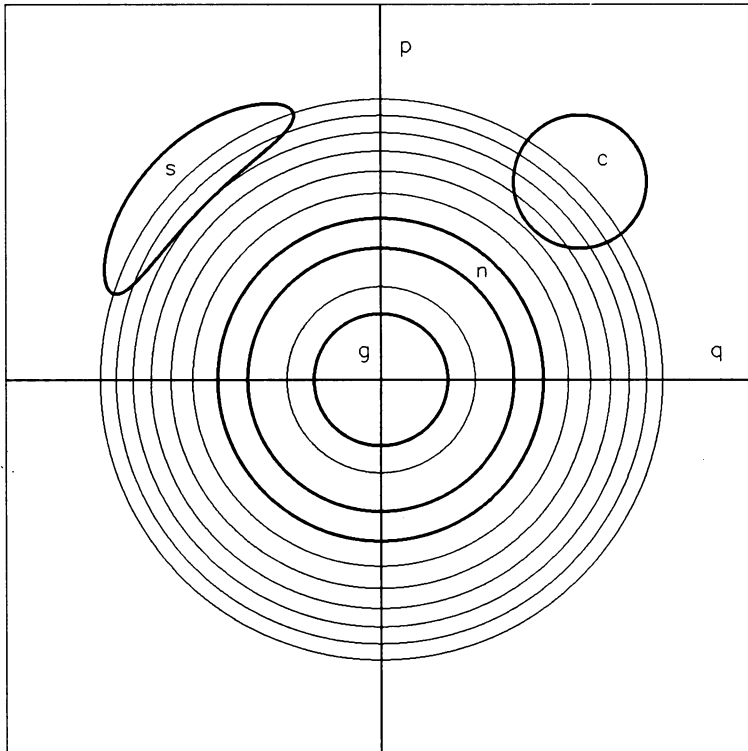


Figure 1: Phase space description of a quantum oscillator. The rings in thin lines correspond to classical orbits with action in multiples of h . The areas surrounded by thicker lines schematically denote the distributions corresponding to different states. The annulus marked n is the $n = 3$ energy eigenstate. The circle marked g is the ground state, the radius being the *rms* uncertainty. c is a coherent state (displaced ground state) with $\bar{n} = 9$, $\Delta n = 3$. s is a squeezed state with the same value of \bar{n} as c , but a reduced Δn and an increased $\Delta\phi$ (phase uncertainty).

It may seem surprising that a single mode laser field, idealised by a coherent state, exhibits Poisson counting statistics. One associates a laser with stimulated emission, hence with Bose statistics, and thereby with bunching. But, there is one case when Bose statistics *does not* imply bunching, *viz.*, when all the bosons are in the same state! For non-relativistic bosons, one would write a wave function like $\psi(x_1)\psi(x_2)\cdots\psi(x_N)$ which is already symmetric in $x_1\cdots x_N$, and at the same time factorized, implying uncorrelated particles. Although I have discouraged you from thinking of photons as bosons with wave functions $\psi(x)$, the parallel works in this case.

The coherent state, with $\Delta n \approx \sqrt{n}$ is not the only minimum uncertainty state with phase space area h . One can conceive of squeezing the circular distribution in $q-p$ space to reduce the variance in number and increase the variance in phase (Fig.1). This is an example of the squeezed states which have become so popular recently. As the phase space picture suggests, they show sub-Poisson intensity fluctuations. Futuristic gravitational wave detection concepts embody the use of squeezed light to reduce noise in interferometric configurations for detecting small movements in mirrors attached to the suspended masses (see the review by Yamamoto et al. [10]).

One of the common methods of measuring an electric field is to add a strong local oscillator field and detect the total by a square law detector. This superposition of electromagnetic fields is *not* the same as superposition of wave functions. The mode amplitude is not the wave function but the independent variable on which the wave function depends. In terms of the phase space distribution function, the result, when worked out, is a convolution – what one would expect for independent classical variables. Notice that translating the signal distribution to higher amplitudes by beating it with a local oscillator would increase the uncertainty in the number of quanta in the mode (as measured by the number of rings intersecting the distribution in Fig.1). Although, one has gained in the size of the effect caused by the signal, one has not gained in terms of detectability against quantum fluctuations, which is reassuring. In fact, an l.o. in a coherent state will add its own intrinsic fluctuations to those of the signal. The phase space picture shows that this could be reduced by squeezing the l.o. in the amplitude direction. Of course, all this gives us only one of the two quadrature components of the input signal. To get both, one has to split the original signal, degrading the signal to noise ratio in the process.

5. The radio-optical comparison

The example just given of measuring the complex electric field has brought the discussion to the radio domain, being nothing but a schematic quantum-limited heterodyne receiver. As a general point, electric field measurement is quite distinct from photon counting. Formally, the operator (2) for the electric field contains separate a and a^\dagger terms, which means that in the process of measuring it photons can either be emitted or absorbed. More physically, imagine the input field acting on a heavy charge, whose movement can be monitored. The output from such a measurement would be a signal going up and down in some way, not one steadily

building up. Clearly there is no net energy transfer to the field measuring device, consistent with the earlier argument.

There is no basic principle which forbids reaching the quantum limit in the radio domain and indeed, Rydberg atoms have been discussed as possible photon detectors at radio frequencies. However, real radio receivers are currently not quantum limited since their noise temperatures are by and large greater than $h\nu/k$ (about 5°K at 100 GHz).

Once the receiver output is dominated by classical noise equivalent to many quanta, copying, amplification and recording are possible, without *further* degradation (in principle). Notice that all these modern conveniences which radio astronomers enjoy in contrast to their optical cousins are really a consequence of their having thoroughly ruined the signal in the first place with thermal noise [11]. Having a photon rich *input* signal is not the central issue for two reasons. Firstly, even in this regime, the photon counting techniques of the optical interferometrists would give better signal to noise if they could be implemented. Secondly, one is not particularly photon rich with sub-kelvin signals and millimetre (or shorter) waves.

From the quantum view point, the Michelson two-element interferometer is really a different animal at optical and radio wavelengths. In the optical case, one is directly measuring the occupation of a Young-fringe like mode on the sky defined by two coherently combined apertures on the ground. Unless one uses beam splitters (and degrades S/N), a given aperture is committed to a single baseline. In the radio case, one is trying to measure electric fields at all the apertures simultaneously. The interference occurs between the classical signals or computer tapes which result. It should be regarded as a statistical correlation between the results of two quantum measurements. The optical example is a single measurement of the correlation of two quantum fields.

Finally, it is amusing to try and decide just where things become truly "measured" or "classical" - is it the current in the diode or (as some observatory directors would maintain) the print in the journal which collapses the wave function? Such questions are the Achilles heel of quantum theory [12] but fortunately, practical predictions do not seem to depend on how we answer them [13].

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References

- 1 The quantum mechanics of harmonic oscillators is reviewed in standard texts, e.g., L.I.Schiff, Quantum Mechanics Mc Graw Hill (1968). This will also supply the normalisation and units which have been ignored in equations (1) and (2)
- 2 E.Fermi, *Rev. Mod. Phys.*, 4, 125 (1932)
- 3 W.Bothe, *Z. Physik*, 41, 345 (1927)
- 4 L.Mandel, *Proc. Phys. Soc.*, 74, 233 (1959)
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- 6 E.C.G.Sudarshan, *Phys. Rev. Lett.*, **10**, 277 (1963)
R.J.Glauber, *Phys. Rev.*, **131**, 2766 (1963)
- 7 E.P.Wigner, *Phys. Rev.*, **40**, 749 (1932)
- 8 'Coherence and Fluctuations of Light'- L.Mandel and E.Wolf, Eds. Dover (1970) is an excellent reprint collection with many of the early and basic papers on coherence and quantum optics upto 1966 including refs. [5], [6] and [9]. In particular, one can find here a *third* distribution on phase space, given by Husimi and Kano, which has the virtue of not going negative.
- 9 Y.Aharonov et al. *Annals of Physics*, **34**, 498 (1966) Notice however that the model of a beam splitter used by these authors has no "fourth port" and this, at least in my view, makes their characterisation of coherent states more mathematical than physical.
- 10 Y.Yamamoto et al. *Progress in Optics*, **28**, 89, Ed. E.Wolf, North Holland (1990) - is a fascinating review article on the brave new world of quantum - limited optical measurements, squeezed states, etc., including actual experimental realisations. I cannot resist passing on a piece of Zen wisdom from this source which is relevant to reference [9] above. When one taps a waveguide to let some signal out, one inevitably lets the vacuum (*i.e.*, mode ground state) in through the fourth port !
- 11 R.D.Ekers in 'Optical Telescopes of the future' F.Pacini, W.Richter, R.N.Wilson eds., p.387, ESO Conference Proceedings, (1978). An early, brief comparison and contrast of optical and radio techniques.
- 12 J.S.Bell, "Speakable and Unsayable in Quantum Mechanics", Cambridge, 1987
- 13 R.E.Peierls, "More surprises in theoretical physics", p.6, Princeton, 1991

Discussion:

Baldwin:

Are there any circumstances in which the use of amplifiers would be useful, assuming they reach the theoretical limits of performance? I have in mind systems in which a beam has to be split and there are also losses.

Nityananda:

Even a quantum limited amplifier would add its fluctuations to those of the signal. Theoretically, one could therefore measure the signal and then share that information over many baselines, rather than amplify it. However, this statement does not take into account considerations of bandwidth and computation.