partial fractions arising from general linear and quadratic denominators, and a treatment of first and second order differential equations that addresses the existence and uniqueness of solutions. In line with recent pedagogic thinking, the mean value theorem makes its late appearance in Chapter 6, with applications to the various versions of l'Hôpital's rule and the remainder formula for Taylor series approximations, as well as rigorous proofs (using tagged Riemann sums) of the familiar integration formulae for arc length and the volume and surface area of solids of revolution. The chapter ends with a look at the error formulae for the common methods of numerical integration.

Chapter 7 deals with sequences and series; leaving this topic late enables full advantage to be taken of the connections with limits of functions and l'Hôpital's rule. Finally, Chapter 8 starts by looking at power series in order to complete the work on Taylor expansions, before contrasting these with Fourier series (where the theory is taken far enough to show pointwise convergence for piecewise differentiable functions). The book ends with a quick account of complex numbers and how they clarify the foregoing results on power and Fourier series.

Throughout the text are 'tasks'—quick-fire checks on understanding—and there are some 400 exercises at the ends of sections of chapters. These are a carefully graded mixture of routine and more challenging examples, with full answers provided for the odd-numbered ones. (A nice touch was to include ten examples on l'Hôpital's rule from Euler's treatise on differential calculus!) There are also longer 'thematic exercises' dealing with extension topics such as cardinality, polynomial interpolation, Stirling's formula, the gamma and beta functions, and uniform convergence. I noticed just a handful of typos, including slips in the formulae for the first two derivatives of $(x - x^3)^{1/3}$ on p. 167, and in the integrals for the arc length of an ellipse on p. 244; there is also a missing minus sign in the error term for Simpson's rule on p. 257.

I thoroughly enjoyed reading this book. For students, it provides a review of familiar topics treated rigorously and an exposure to new and powerful ideas in real analysis. The material is very skilfully woven together with just the right amount of supporting detail and motivation and apposite examples (and counterexamples). And for lecturers, there is much food for thought in the author's innovative approach to what is ostensibly very standard fare, as well as some excellent and well thought-through collections of exercises.

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Visual differential geometry and forms: a mathematical drama in five acts by Tristan Needham, pp 501, £40 (paper), ISBN 978-0-69-120370-6, Princeton University Press (2021)

Visual differential geometry and forms is written in the same inimitable, highly visual style as Needham's widely acclaimed *Visual complex analysis* (VCA): indeed, there is some inevitable overlap of content, but also some differences. Among the 235 diagrams, relatively more are photos depicting the results of experiments with practical objects (often fruit and vegetables) and, in the text, fuller explicit use is made of the concept of "ultimate equality" applied geometrically in limiting arguments, as Newton did in the *Principia*. As in VCA, the author writes beautifully in a conversational manner which speaks directly to the reader. In the Prologue, he writes that, "I have made no attempt to write this book as a classroom textbook", but it is certainly a lucid and authoritative guidebook to the central concerns of differential geometry, written by an enthusiastic guide eager to share his insights in an enjoyable and engaging way, leavened with frequent touches of gentle humour.

The 34 chapters are organised into five "Acts" dealing thematically with: I: The nature of space, II: The metric, III: Curvature, IV: Parallel transport, and V: Forms. There are numerous fully worked examples and apposite footnotes, and each Act ends with an eclectic and interesting set of exercises. There is also a meticulously detailed index and a fully annotated biography with suggested pathways for further reading—would that more authors would share their libraries in this way!

Acts I–III comprise a compelling account of the differential geometry of curves and surfaces with crystal-clear explanations, often featuring multiple approaches, of the role of geodesics and the difference between intrinsic and extrinsic geometrical properties. The standard surfaces of constant curvature are discussed with an emphasis on the part played by Möbius transformations in representing their respective symmetries. The shape operator is used to streamline the development of the theory, and mathematical highlights include three proofs of the Gauss-Bonnet theorem relating the total curvature of a closed, orientable surface, S, to its Euler characteristic, $\chi(S)$, where $\chi(S)$ can be further related to the total angular excess and the sum of the indices of a vector field on S.

Although I sensed a step-up in difficulty in Act IV, the concepts of parallel transport and holonomy are the key to a spectacular sequence of chapters featuring an elegant holonomic proof of Gauss's *Theorema Egregium* (with a second proof using a neat geometrical derivation of the formula for curvature intrinsically in terms of the metric) and a fourth proof of the Gauss-Bonnet theorem, this time done intrinsically. Act IV ends with the introduction of the Riemann curvature tensor and the Ricci tensor as the prelude to a superbly motivated, Penrose-inspired derivation of the Einstein field equation in general relativity.

Act V has a very different flavour. The first five chapters constitute a self-contained introduction to forms done concretely without heavy multilinear algebra and with the ancillary goal of placing the div, grad and curl of vector calculus in a unified context. This is all carefully done in order to bolster confidence in calculating with forms, including differentiation and integration, with the formulation of Maxwell's equations being a memorable highlight. The sixth (and final) chapter of Act V is magnificent! It uses Cartan's method of moving frames to derive first his two eponymous structural equations and then the six fundamental form equations of a surface, *en route* to a fresh forms-based proof of the *Theorema Egregium*. This work (in three dimensions) is extended to curvature 2-forms and their use in efficiently calculating the entries of the Riemann curvature tensor (in n dimensions). The book ends with a verification that the Schwarzschild metric really is a solution of Einstein's vacuum field equation. Incredibly, Schwarzschild found his solution working in a World War 1 trench, just a month after Einstein announced his equation!

Throughout the book, Needham alerts the reader to the different notational conventions that plague the literature in this area, and he works hard to avoid the plethora of sub- and superscripts that can deluge calculations. I also applaud some of his suggested renaming of theorems—especially "Fundamental theorem of exterior calculus" for the generalised Stokes's theorem. Derivatives are frequently interpreted as velocities and accelerations to give a dynamical gloss on results such as Clairaut's theorem (skilfully related to Kepler's second law) and the Jacobi equation of geodesic deviation for a sphere (which is related to the equation for simple harmonic motion). Comprehensive historical background is given for all the concepts covered, including an extract from Riemann's private notes showing his remarkable anticipation of the Bianchi identity.

I thoroughly enjoyed reading this book. The author's constant concern not to lose his readers keeps you going, and I am sure that even seasoned geometers will find fresh insights, new visualisations, and much inspiration from the way the topics are shaped and organised. Sadly, Needham acknowledges, "This is my second book, and it is also my last book." From the very personal account of the effort involved, they both represent extended labours of love exemplifying his overarching philosophy, that (p. 158) "… direct, geometric reasoning frequently allows us to completely bypass symbolic manipulation to obtain an intuitive, visual grasp of mathematical reality".

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Introduction to differential geometry by Joel W. Robbin and Dietmar A. Salamon, pp 418, £54.99 (paper), ISBN 978-3-662-64339-6, also available as e-book, Springer Studium Mathematik (Master) (2022)

As the series title suggests, this is a graduate level introduction to differential geometry, assuming a sound knowledge of calculus of several variables and linear algebra as well as a hefty dose of mathematical maturity. Alternatively it could be a 'second course' in differential geometry, following on from a more gentle undergraduate course based on, say, Barrett O'Neill's famous *Elementary Differential Geometry* (Academic Press, second edition 1997). The book is based on one-semester courses at ETH in Switzerland and at the University of Wisconsin-Madison in the USA and these will have been quite demanding courses. There is a fair amount of optional material, mainly in the area of intrinsic differential geometry (not assuming surfaces and other manifolds are embedded in Euclidean space), but even so there is a lot of very detailed work here. The final chapter is also optional and covers some very interesting topics which can be understood at this stage, such as the Morse index, isometries of compact Lie groups and semisimple Lie algebras.

The inclusion, for those interested, of intrinsic geometry alongside extrinsic—that is, where it is assumed manifolds are embedded in Euclidean space—is a significant virtue. The book is also thorough, providing background material, results and proofs as well as a steady development of the main material. This is fairly standard: submanifolds of Euclidean space, tangent spaces, vector fields, Lie groups and diffeomorphisms, vector bundles, connections, geodesics and curvature. The presentation is succinct (it is definitely a Master's level book) and there are very few pictures! This does not preclude helpful visual examples, such as sliding and rolling of train wheels, and a sphere rolling on a plane. (Neither sliding, nor rolling nor the next topics, twisting and wobbling, are listed as such in the Index, though you can find three of them under 'Motion'.) So, as an advanced introduction or second pass, as a reference resource, and as a prelude to further more abstract study, this is a fine addition to the differential geometry literature.

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