

# Hypothesis on the Origin of Solar Torsional Waves

Yu. V. Vandakurov

A. F. Ioffe Physicotechnical Institute, Russian Academy of Sciences, St. Petersburg, 194021  
Russia, email: yv.mhd@mail.ioffe.ru

Latitude distribution of the solar convective zone rotation rate is studied by expanding it in a complete system of orthogonal modes (both symmetric and asymmetric relative to the equator). We find the solution to the problem of determination of the rotation rate of using helioseismic data available for the latitudes of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  of one (for instance, the northern) hemisphere. The pole,  $90^\circ$ , will also be included into the above latitude list for the reason to be disclosed below. It is known that any rotation rate can always be represented by an expansion in a complete system of orthogonal vector spherical harmonics with a zero superscript (see, e.g., Varshalovich et al. 1988). If only five modes with the smallest subscripts are left, we obtain, by introducing the relative angular rotation rate  $\Omega/\Omega_0$ ,

$$\Omega/\Omega_0 = u_1 + u_3(7/8)^{1/2}(5\cos^2\vartheta - 1) + u_5[(55)^{1/2}/8](21\cos^4\vartheta - 14\cos^2\vartheta + 1) + 5^{1/2}\cos\vartheta [u_2 + u_4(3/8)^{1/2}(7\cos^2\vartheta - 3)], \quad (0.1)$$

where  $\Omega_0 = \text{const}$ ,  $\vartheta$  - is the polar angle, and  $u_j = u_j(r)$  are radius-dependent numerical coefficients, which for even (odd)  $j$  determine the asymmetric (symmetric) rotation of the medium relative to the equator. Denote by  $\nu_k$  the relative angular rotation rate at the  $k$ -th latitude, where  $k$  for the above latitudes varies from 1 at the equator to  $k = 4$  at  $60^\circ$ , and let  $k = p$  at the pole.

To eliminate the quantity  $u_1$  (and also  $u_3$ ) from the equations of equilibrium at all latitudes in question, consider now the differences  $[\Omega/\Omega_0]_k - [\Omega/\Omega_0]_{k+1}$  (and also the differences of the latters). We see that  $u_1$  and  $u_3$  are excluded if corrections of the order  $10^{-6}$  are not taken into account. Similarly  $u_5$  may be excluded. We finally obtain

$$-0.5472u_2 + 1.3516u_4 = \nu_1 - 3(\nu_2 - \nu_3) - \nu_4, \quad (0.2)$$

$$-0.0520u_2 + 0.2191u_4 = \nu_2 - 3(\nu_3 - \nu_4) - \nu_p, \quad (0.3)$$

Substituting the helioseismic data of Schou et al. (2002) into the right-hand sides of the equations, one can obtain equations for determination of the asymmetry in solar rotation with respect to the equator. It is essential also that the fast growth of the azimuthal components of the vector spherical harmonics entering Eq. (0.1) as coefficients of  $u_j$ , which occurs with increasing  $j$ , creates additional difficulties at the high latitudes. These components scale as  $j^{3/2}$ . This gives rise to the problem of convergence of the series, whose most probable solution lies in postulating the existence of near-polar non-rotating zones, in which case Eq. (0.3) drops out, and the above difficulties may be inessential. This conclusion is consistent with the data of Birch & Kosovichev (1998) and Schou et al. (1998).

Considering now Eq. (0.2), we find that, other conditions being equal, the asymmetry will be the strongest if coefficient  $u_2$  is predominant, in which case viscous effects will probably be significant at low latitudes as well. We assume subsequently that in Eq. (0.2)  $u_2 = 0$ .

To study the situation in a stationary case, we consider a model of the solar convection

zone which consists of 12 layers spread between the relative radii  $0.71r/R_{\odot}$  and  $0.99r/R_{\odot}$ . In the northern hemisphere, we set the initial values of the angular rotation rates at the boundaries of the above layers and at latitudes  $0^{\circ}$ ,  $30^{\circ}$ ,  $45^{\circ}$ , and  $60^{\circ}$  in accordance with the data displayed in Fig. 1 of Schou et al. (2002). Our goal consists in determining the rotation rate at all depths in both hemispheres of the convective zone.

Note that even small inaccuracies in the above-mentioned initial data, which may lie within the error limits specified by Schou et al. (2002), may entail extremely large variations in the rotation asymmetry. In this connection, we introduced certain corrections into these initial data for the northern hemisphere in order to obtain a theoretical model with an as small asymmetry in the rotation rate relative to the equator as possible (see, Vandakurov (2004)). Note, however, that we introduced corrections only into the variations which bring about noticeable changes into the model at each step (otherwise one could obtain the zero rotation asymmetry model).

It turns out that there are some differences between rotation rates for the southern and northern hemispheres, but these rates coincide at all latitudes at  $r/R_{\odot} \approx 0.84$  and  $0.92$ . The latitude at which there is no rotation asymmetry at all depths of the convection zone is approximately  $\pm 41^{\circ}$ . This figure is close to the value (equal, according to Vorontsov et al. (2002) to  $42^{\circ}$ ), which corresponds to the zero velocity of the torsional waves observed in the Sun. There are also large differences between the rotation rates in question, e.g., at  $r/R_{\odot} \approx 0.77$  and  $0.90$ .

Thus, the hypothesis by which excitation of torsional waves is responsible for smoothing of some sharp variations in the rotation rate seen in the figure appears plausible enough. We have in mind that formation of a spatial nonuniformity may be accompanied by its displacement to other layers, as a result of which this nonuniformity will not have time enough to appear in real conditions.

This smoothing of the revealed sharp variations in the rotation rate may be hindered both at comparatively high latitudes and close to the outer boundary of the convection zone (at which some outflow of matter may set in). In this connection, the peak at the latitude of  $-75^{\circ}$  and  $r/R_{\odot} = 0.95$  present in our calculations. The parameters of the peak coincide with those of the rapidly rotating submerged jet detected by Schou et al. (1998). Note also that in this particular case the jet exists in one hemisphere only.

## References

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