THE FUTURE OF ARTIFICIAL SATELLITE THEORIES

Hybrid Ephemeris Compression Model

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Abstract. Since the time of Newton, astrodynamics has focused on the analytical solution of orbital problems. This was driven by the desire to obtain a theoretical understanding of the motion and the practical desire to be able to produce a computational result. Only with the advent of the computer did numerical integration become a practical consideration for solving dynamical problems. Although computer technology is not yet to the point of being able to provide numerical integration support for all satellite orbits, we are in a transition period which is being driven by the unprecedented increase in computational power. This transition will affect the future of analytical, semi-analytical and numerical artificial satellite theories in a dramatic way. In fact, the role for semi-analytical theories may disappear. During the time of transition, a central site may have the capacity to maintain the orbits using numerical integration, but the user may not have such a capacity or may need results in a more timely manner. One way to provide for this transition need is through the use of some type of satellite ephemeris compression. Through the combined use of a power series and a Fourier series, good quality ephemeris compression has been achieved for 7 day periods. The ephemeris compression requires less than 40 terms and is valid for all eccentricities and inclinations.

1. Introduction

Prediction of the motion of objects in space can be traced to ancient astronomers and their desire to understand the motion of the planets. The works of Kepler and Newton were seminal accomplishments in that they changed the way in which we tried to understand the motion of objects in space. This change was from one of observation reduction and pattern recognition to derivation of solutions based on the laws of physics. The launch of Sputnik, the first artificial Earth satellite, in 1957 created the need for specialization of orbital mechanics to the region near a planet.

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The solution of the main problem by Brouwer (1959) was a milestone in the history of the development of artificial satellite theories. Most analytical theories today can still be traced to origins in the work of Brouwer.

From the earliest days of artificial Earth satellites, the theory was a means to the end of computation of actual numbers for prediction of future position of the satellites. The catalog of orbital elements was initially accomplished using graph paper, hand calculations, and limited computer resources (Fitzpatrick and Findley, 1960). This method sufficed through the early 1960s when there were still less than 50 satellites in Earth orbit.

With the introduction of large scale computers came the possibility of automatically maintaining orbital elements for a large number of satellites. The total catalog workload is the number of satellites in the catalog multiplied by the average number of computer operations required to maintain that satellite each day. Early computers were limited in their computational power and could not accomplish the catalog workload dictated by using the entire Brouwer theory. At the central satellite cataloging center for the United States, the decision was made to implement only the main terms of Brouwer's work. Ironically, at that time in history the quality of the analytical models exceeded the computational power available to use them.

The Brouwer model is known as a General Perturbations (GP) satellite theory. It contains explicit analytical equations for prediction of satellite motion. Because it is analytical in nature, it is relatively fast running on the computer, and the computational speed is independent of the length of the prediction interval. However, its accuracy is limited by the shrewdness and persistence of the developer, and adding new perturbations requires additional analytical development. Perhaps the most notable characteristic of analytical theories is the insight which they give into the character of the motion due to various perturbations.

In contrast to the GP theories is the use of numerical integration to solve the equations of motion. This method is called a Special Perturbations (SP) satellite theory. Because it uses numerical integration, it is relatively slow running on the computer, and the computational speed is proportional to the length of the prediction interval. However, its accuracy can be quite good with a proper choice of integration method and suitably small integration step size. Including new perturbations requires little additional development. In contrast to the GP theories, SP theories give no direct insight into the character of the motion due to various perturbations.

2. Evolution of Satellite Cataloging

The number of satellites in the catalog has followed an approximate linear growth to a level of more than 7,000 satellites currently. Fortunately, the power of computers has grown much faster than a linear rate. In recent years, it seems than the growth in power is more of an exponential curve. Even in the early days of computers, there were a small number of satellites which required a higher accuracy than could be obtained with GP theories. This need was met by numerical integration (SP theories) running on dedicated computers. Today an example of such a specialized system is the Global Positioning System constellation of satellites.

The application of SP theories to small numbers of satellites created a disparity between the accuracy of these few orbital element sets and the accuracy of the large number of orbital element sets in the catalog maintained by GP theories. This void was filled with the introduction of Semianalytical satellite theories. Such a theory provides a compromise between desires for higher accuracy and computational resource limitations. It achieves this by a transformation of variables which removes periodics having the satellite period. The transformed differential equations have a much slower variation and can be numerically integrated using a much larger step size than an SP theory would use. The character of the Semianalytical theory is a mix of GP in that it contains analytical equations for the periodics and SP in that it utilizes numerical integration. It provides an increase in accuracy over GP theories with only a modest growth in computer run time. Inclusion of additional perturbations still requires some development but the level of accuracy can be improved with choice of a suitable integration method and reduction of integration step size.

3. Future Direction

The total catalog workload should be viewed in the context of the computational power available. If the entire catalog is maintained with GP theories, then the total catalog workload represents some fraction of the computational power available. If the entire catalog is maintained with Semianalytical theories, then the total catalog workload is a larger fraction of the computational power available. Finally, maintaining the entire catalog with SP theories may cause the fraction to be larger than unity. Since the computational power is growing at a much faster rate than the number of objects in the catalog, the fraction of available computational power required is getting smaller for all three methods. At some point in time, the fraction for an all SP catalog becomes smaller than unity and an all SP catalog becomes realizable.

Today there is an unprecedented growth in the power of individual computers as well as the technology for rapid communications between computers. The Naval Space Command has recently installed a new center for satellite catalog processing. This center is based around an architecture with centralized data management and distributed processing through a cluster of work stations. Ongoing research at the Naval Research Laboratory is focusing on parallel processing of astrodynamics problems through both massively parallel machines as well as parallel virtual machines. Such innovations in hardware, software and creative application are foreshadowing the dawning of the day when it will be possible to maintain the entire catalog of Earth orbiting objects with SP theories.

As that day approaches, consideration should be given to the future roles of SP, GP and Semianalytical theories. The accuracy of SP theories already exceeds the majority of devices used for tracking satellites. To enjoy the complete benefits of an SP catalog, research should be directed toward improvement of satellite measurement accuracy. The greatest shortcoming in the SP theories of today is the physical modeling of atmospheric density. Emphasis should be applied to the improvement of modeling in this regime as well as improved measurement of the environmental parameters which influence the atmosphere.

Even when a complete SP catalog becomes a reality, GP theories will still have an important role. As discussed above, one cannot overstate the value which comes from the insight into the functional character of various perturbation effects. Such insight can best be gained by actually examining the functional form of the terms in an analytical theory. It would have been much harder to have discovered the secret to creating a sun synchronous orbit without the benefit of Brouwer's elegant perturbation model. Finally, the challenge of developing analytical solutions will always beckon the astrodynamicist in the same way that the great mountains beckon the climbers – "Because it is there."

The achievement of a totally SP catalog will signify the end of the role for Semianalytical theories. The ultimate achievement of SP accuracy for all satellites using readily available computer resources will virtually eliminate any need for a theory which attempts to achieve SP accuracy while saving computer run time. If one has more computer power than needed for SP, why sacrifice accuracy to save run time? Finally, Semianalytical theories do not provide any functional insight like GP theories do. Semianalytical theories filled an important niche in the history of artificial satellites theories when accuracy needs were greater than computational capabilities. But because computational power is far outdistancing catalog workload, the role of Semianalytical theories is coming to the end of an era.

4. Transition Period

Today an all SP catalog is only in the experimental stages. Even when such a catalog is achieved, there will be a transition period where the end users of orbital element sets do not have the computational power necessary to propagate SP state vectors. During this transition period, users will need a way to rapidly predict satellite locations with good accuracy without the associated run time penalty of use of the full SP theory. During this transition there will be a temporary role for such a model just as there was a temporary role for Semianalytical theories.

Ephemeris compression is one possible option for providing a user with a fast running model which provides a good imitation of the SP accuracy. Ephemeris compression is a means of achieving a compact representation which approximates a time series of satellite positions. Typically the time series of satellite positions is first generated with a precise numerical integration of the equations of motion. Such methods are commonly used for representing planetary ephemerides (The Astronomical Almanac, 1987). Some research has been done to apply ephemeris compression to satellite orbits. Representative among these is the paper by Deprit *et al.* (1979). Most recently a paper by Coffey *et al.* (1996) provides the most extensive examination to date of the application of ephemeris compression to a large catalog of Earth satellites. Most of the previous approaches used a single set of basis functions for the ephemeris compression. They generally calibrated the coefficients of these functions by directly fitting inertial space coordinate data. As a result, the applicability was limited to only those satellites with small eccentricity. Most of these approaches required hundreds of terms to achieve a good imitation of the SP theory for an extended period of time.

5. Hybrid Ephemeris Compression Model

The rectangular coordinates of a satellite ephemeris experience large variations each revolution primarily due to the main two body motion of the satellite. Any attempt to model this variation with a set of basis functions will require a great number of terms just to model the two body motion. Then additional terms will be required to model the perturbations superimposed on the two body motion. The approach taken here is to retain the two body equations as an integral part of the ephemeris compression model. Since Kepler's equation is contained in the two body motion, this feature also eliminates much of the dependence on eccentricity thereby allowing development of an ephemeris compression model which is valid for all eccentricities. This method of ephemeris compression can be considered a hybrid method. It provides a unique marriage of GP and SP methods. The functional form of GP theories provides insight into the optimum selection of basis functions for the ephemeris compression model while the SP theory provides the reference orbit which is imitated by the ephemeris compression model.

The Hybrid Ephemeris Compression Model (HECM) approach is to first select a type of function which will successfully model the secular motion of the satellite. Based on the solution of Brouwer as well as other works (Hoots and France, 1987) which have added drag to the Brouwer solution, a time series is chosen to model the secular motion of the satellite. Let the secular values of the orbital elements at time t be denoted by single-primed variables. Then the model is

$$\begin{aligned} n' &= n_0 + n_1 t + n_2 t^2 + n_3 t^3 \\ e' &= e_0 + e_1 t + e_2 t^2 \\ i' &= i_0 + i_1 t \\ \Omega' &= \Omega_0 + \Omega_1 t + \Omega_2 t^2 \\ \omega' &= \omega_0 + \omega_1 t + \omega_2 t^2 \\ M' &= M_0 + M_1 t + \frac{1}{2} n_1 t^2 + \frac{1}{3} n_2 t^3 + \frac{1}{4} n_3 t^4 \end{aligned}$$

where

n = mean motion
e = eccentricity
i = inclination
Ω = right ascension of ascending node
ω = argument of perigee
M = mean anomaly
t = time since epoch and subscript 0 denotes a value at epoch time.

Single-primed position is computed by

$$\begin{aligned} x' &= r'(-\sin\Omega' \cos i' \sin u' + \cos\Omega' \cos u') \\ y' &= r'(\cos\Omega' \cos i' \sin u' + \sin\Omega' \cos u') \\ z' &= r' \sin i' \sin u' \end{aligned}$$

where u' is the single-primed true argument of latitude corresponding to the single-primed mean anomaly M' and argument of perigee ω'. The computation of u' is where the key use of Kepler's equation enters the method.

The model contains 17 parameters which must be computed based on the reference ephemeris. A Gauss least squares is used to find values of the parameters which create a best fit of the secular portion of the hybrid model to the reference ephemeris. This fit is done for the entire length of the reference ephemeris. Once the 17 parameters have been computed, most, if not all, of the secular character of the reference ephemeris will have been captured in the HECM parameters. All that remains to be modeled are periodic variations. Returning again to knowledge of the Brouwer solution, these variations are known to have period equal to the period of the satellite. A natural choice of basis functions to model this is trigonometric functions. The model chosen for the periodic variation is the first few terms of a Fourier series.

$$\begin{aligned}
\Delta x &= a_{x0} + \sum a_{xk}\cos(ku') + \sum b_{xk}\sin(ku') \\
\Delta y &= a_{y0} + \sum a_{yk}\cos(ku') + \sum b_{yk}\sin(ku') \\
\Delta z &= a_{z0} + \sum a_{zk}\cos(ku') + \sum b_{zk}\sin(ku')
\end{aligned}$$

where $k = 1$ to 3 in all sums.

The Fourier coefficients are obtained by first creating from the reference ephemeris a series of points equally spaced in true argument of latitude over the interval $[-\pi, \pi]$. It is only necessary to create this set of points for the first revolution of the reference ephemeris. The reason is that the Fourier series is assumed to be periodic with period equal to the satellite period. If the calibration of the 17 parameters successfully removed all secular effects, then the period associated with the predicted u' will changed secularly so that the 2π periodicity will hold true throughout the time span of the reference ephemeris. Let

$$u_1, u_2, u_3, \ldots, u_q$$

be the set of q points equally spaced in true argument of latitude and covering the interval $[-\pi, \pi]$. The secular portion of the HECM can be used to provide a prediction of the single-primed position at each of these sample points. Let

$$\begin{aligned}
&\delta x(u_1), \delta x(u_2), \delta x(u_3), \ldots, \delta x(u_q) \\
&\delta y(u_1), \delta y(u_2), \delta y(u_3), \ldots, \delta y(u_q) \\
&\delta z(u_1), \delta z(u_2), \delta z(u_3), \ldots, \delta z(u_q)
\end{aligned}$$

denote the differences between the reference ephemeris and the single primed positions predicted with the secular portion of the HECM. The Fourier coefficients can be calculated from

$$\begin{aligned}
a_{x0} &= \frac{1}{q}[\delta x(u_1) + \delta x(u_2) + \ldots + \delta x(u_q)] \\
a_{xk} &= \frac{2}{q}[\delta x(u_1)\cos(ku_1) + \delta x(u_2)\cos(ku_2) + \ldots + \delta x(u_q)\cos(ku_q)] \\
b_{xk} &= \frac{2}{q}[\delta x(u_1)\sin(ku_1) + \delta x(u_2)\sin(ku_2) + \ldots + \delta x(u_q)\sin(ku_q)]
\end{aligned}$$

$$\begin{aligned}
a_{y0} &= \frac{1}{q}[\delta y(u_1) + \delta y(u_2) + \ldots + \delta y(u_q)] \\
a_{yk} &= \frac{2}{q}[\delta y(u_1)\cos(ku_1) + \delta y(u_2)\cos(ku_2) + \ldots + \delta y(u_q)\cos(ku_q)] \\
b_{yk} &= \frac{2}{q}[\delta y(u_1)\sin(ku_1) + \delta y(u_2)\sin(ku_2) + \ldots + \delta y(u_q)\sin(ku_q)] \\
a_{z0} &= \frac{1}{q}[\delta z(u_1) + \delta z(u_2) + \ldots + \delta z(u_q)] \\
a_{zk} &= \frac{2}{q}[\delta z(u_1)\cos(ku_1) + \delta z(u_2)\cos(ku_2) + \ldots + \delta z(u_q)\cos(ku_q)] \\
b_{zk} &= \frac{2}{q}[\delta z(u_1)\sin(ku_1) + \delta z(u_2)\sin(ku_2) + \ldots + \delta z(u_q)\sin(ku_q)].
\end{aligned}$$

Once the Fourier coefficients are computed, the osculating position can be obtained from

$$\begin{aligned}
x &= x' + \Delta x \\
y &= y' + \Delta y \\
z &= z' + \Delta z.
\end{aligned}$$

Since k is chosen to be 3, there will be a total of 21 Fourier coefficients. Adding this to the 17 secular coefficients gives a total of 38 parameters needed for the HECM.

6. Model Testing

In order to assess the capability of the HECM, a series of tests was performed. First, a set of element sets was selected which provided a reasonable sample over a wide range of eccentricities, inclinations, and mean motions. The set of sample element sets is given in Table 1.

For each of the sample orbital element sets, a reference ephemeris was generated with points saved every one minute. The reference ephemeris was generated using a Gauss-Jackson 8^{th} order numerical integrator. The force model selected was a 12^{th} order geopotential and a Jacchia 1970 atmospheric density model. The reference ephemeris for each satellite was generated for a total span of 14 days.

In each case the fit span selected for the HECM was the first 7 days of the reference ephemeris. The calibration of the 38 parameters for each case was accomplished as described earlier. A prediction with HECM was then performed and compared with the reference ephemeris for the 7 days which were fit. The quality of the comparison is described as the RMS of the differences between the HECM prediction and the reference ephemeris. The results for each case are presented in Table 2.

TABLE 1. Test element set characteristics.

Case #	Perigee Height [km]	Apogee Height [km]	Eccentricity	Inclination [deg]	Mean motion [revs/day]
1	550	3060	0.15	32.9	11.7
2	200	11050	0.45	23.7	6.6
3	870	990	0.008	66.8	13.9
4	535	1100	0.039	56.0	14.2
5	410	2850	0.15	144.3	12.1
6	1130	2260	0.070	59.4	12.0
7	1150	5910	0.24	85.3	8.8
8	305	40010	0.75	63.0	2.0

TABLE 2. Test case results.

Case #	Perigee Height [km]	Apogee Height [km]	Fit RMS [km]
1	550	3060	0.508
2	200	11050	0.827
3	870	990	0.590
4	535	1100	0.612
5	410	2850	0.567
6	1130	2260	0.452
7	1150	5910	0.378
8	305	40010	1.902

The results show that the HECM provides a good match to the reference ephemeris and appears to do so for a wide variety of eccentricities, inclinations, and mean motions. The RMS of the 7 day fit is typically about 500 meters with only the very large eccentricity case ($e = 0.75$) degrading to about 2 km. The 38 parameters required to produce these results is an order of magnitude smaller than the number of parameters used by Coffey and others while the results provide a closer fit to the reference. Additionally, unlike other ephemeris compression methods, this method applies to all eccentricities.

Since the functional form of HECM is based on the general character of physical GP theories, the long term behavior of the HECM continues to approximate the true motion even when used at time points outside the fit interval. This is a sharp contrast to ephemeris compression methods using Chebychev polynomials which vary wildly beyond their normal interval.

To assess this claim, each test case was also predicted for 7 days beyond the 7 day fit interval. The predictions were compared with the reference ephemeris and an RMS was computed. All cases had a graceful degradation with an RMS of tens of kilometers or less for the 7 days beyond the fit interval. Such a quality is extremely important for users in the event that a new set of ephemeris compression parameters is not received in a timely fashion.

7. Conclusions

Use of a combination of time polynomials and trigonometric series appears to be very effective for satellite ephemeris compression. The inclusion of the two body equations as an integral part of the ephemeris compression model allows a significant economization of the number of terms required to achieve a given accuracy. It also removes any significant dependence on the eccentricity of the orbit.

The HECM has only been compared to a limited but varied sample of orbit types. Thus, the results should be considered as preliminary. Current research is continuing by comparing with a much wider variety of orbits. Also, a study is underway to see the effect of including additional trigonometric terms. Finally, the applicability of HECM to orbits perturbed by lunar and solar gravity and geopotential resonance needs to be studied. It is believed that the HECM will also work for these cases since the underlying character of the perturbations will still have a periodicity related to the satellite period.

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