of Cauchy's theorem, Möbius maps (including their dynamics), normality, conformal maps, and harmonic functions. The remaining 6 chapters look at zeroes of holomorphic functions (and the factorisation theorems of Weierstrass and Hadamard), interpolation and approximation, extension theorems, the great theorems on the ranges of holomorphic functions, and the uniformisation theorem. There are worked examples throughout the text and a magnificent collection of 368 interesting and well-structured exercises at the ends of chapters, some giving alternative proofs of theorems.

Although the book opens with a rather ominous quotation from Thurston about reading mathematics textbooks, "Even one page a day can be quite fast", the author works hard to assist the reader. The writing is clear and unfussy with a strong narrative thrust and constant concern for the needs of an independent reader meeting the material for the first time. But experts will also find much to admire in the author's exemplary organisation of results and choice of proofs: the latter range from the classical "pole-pushing" proof of Runge's theorem to four contrasting approaches to the theorems of Bloch, Schottky, Montel and Picard, complete with a roadmap of interdependencies, and Zalcman's remarkable rescaling argument from the 1970s.

The contextualisation of results is flawless; thus, contrasts with the behaviour of real-valued functions are constantly drawn out, and the marginal photographs and historical remarks about the named contributors to the subject add much to the exposition. Unusual features include the theorems on the boundary behaviour of maps in the Riemann mapping theorem, a generalised Schwarz reflection principle for analytic arcs, a very full account of conformal metrics (including Ahlfor's far-reaching generalisation of Pick's version of Schwarz's lemma), and a detailed treatment of holomorphic branched coverings. I particularly liked the incremental generalisation of results: for example, arriving at Painlevé's vast generalisation of Riemann's theorem on removable singularities which guarantees holomorphic extensions for bounded holomorphic functions on $U - K$ where K is not just a point (as in Riemann) but a compact set with Hausdorff measure 0. And some things which are often skated over are given a full treatment, such as the use of Runge's theorem in constructing sequences of functions with surprising limiting behaviour.

I have reviewed books in the *Gazette* for nearly 40 years and none has given me greater pleasure than this one. Beautifully produced, beautifully written, on an incomparably beautiful area of mathematics, this is an inspirational book that I shall gratefully return to again and again.

10.1017/mag.2023.134 © The Authors, 2023
Published by Cambridge University Press on behalf of The Mathematical Association

NICK LORD
Tonbridge School, Kent TN9 1JP
e-mail: *njl@tonbridge-school.org*

3000 years of analysis by Thomas Sonar, pp. xx+706, £109.99 (hard), ISBN 978-3-030-58221-0, also available as an e-book, Birkhäuser (Springer Nature Switzerland AG) 2021

You will gather from the title of this book that 'analysis' is interpreted in a broad sense; the author quotes from *Encyclopedia Britannica*: 'a branch of mathematics that deals with continuous change and with certain general types of processes that have emerged from the study of continuous processes ...'. This implies that Zeno's paradoxes of motion and any matters connected with indivisibility or discreteness of time or space or numbers (the 'continuum') come within the scope of the book. In fact the scope is much wider than this: it aims to give a summary of the main political events happening alongside scientific developments and to give quite detailed biographies of the main characters in the story, including reproductions of paintings or photographs or, in the case of classical figures, fanciful representations by later artists. The first 154 pages are devoted to pre-Renaissance mathematics. The focus is on Europe and the Middle-East: India and China do not figure in the index, though there are three pages on 'analysis in India' which cover series expansions for the sine, cosine and arctangent, and a formula which could be regarded as an approximation to the indefinite integral of $x^n dx$. There are numerous references to secondary sources and modern editions of original works, the citation list occupying 19 pages. Altogether this is a massive (1.3kg) work of scholarship; it first appeared in German in 2016 and I shall make some remarks about the translation later. The author has published widely in the history of mathematics and in numerical methods—*Mathematical Reviews* lists 80 publications, and 11 of his articles are referenced in the book under review.

Besides setting the scientific developments alongside contemporary world events, a striking feature of this book is its level of mathematical detail. To take one example: about 18 pages are devoted to the invention of logarithms by John Napier and Henry Briggs, and possibly earlier by Jost Bürgi, with very detailed expositions of the calculation and construction of the various systems, including numerical examples. There is also a description of Nicholas Mercator's slightly later system of logarithms and their connection with the area under (quadrature of) a hyperbola. ('Mercator' = merchant, and this one is no relation to Mercator of the map projection.) This work on logarithms is part of more than 100 pages devoted to the precursors of Newton and Leibniz in France, the Netherlands and England which includes the well-known figures of Descartes, Pascal, Fermat, Hudde, Barrow, Wallis and so on, but also lesser-known people such as William Neile and his quadrature of the semi-cubical parabola $y^2 = x^3$. Newton and Leibniz share a whole chapter, concentrating naturally on their contributions to 'analysis', and the exposition does convey the truly breathtaking originality of both men. The great 18th century figures of the numerous Bernoullis, Euler, Lagrange and so on are well represented, of course including Euler's audacious but uncannily correct use of infinite series, including divergent ones. That still leaves about 160 pages for the nineteenth and twentieth centuries—conceptual rigour, functional analysis, set theory and non-standard analysis—and understandably details are not so full for the later developments. The two most recent approaches to nonstandard analysis described in the book date from the end of the twentieth century and are certainly unknown to me. One is due to Edward Nelson and the other, called 'smooth infinitesimal analysis', is based on ideas of Francis Lawvere in category theory.

There is an index of persons and a subject index, but these are not well itemised: for example 'Newton, Isaac' in the first index is followed by a list of 44 page numbers without any further subdivision, and 'Principia' does not occur in the second index, so it is hard to chase up individual topics through the indices. There is evidence of inadequate editing: on page 262 there is a spurious change of typeface to a smaller font which lasts until page 322; there are numerous spelling and language errors, presumably arising from the translation. The translation itself is decidedly uneven, though only rarely descending into unintelligibility: '[Queen Elizabeth I] succeeded in creating a sense of atmosphere of departure in the country'. Sometimes the translation is unintentionally charming: 'Newton is said to have gone to church some time in 1692 and left his doggie in one of his rooms ...' And different traditions in English and German have resulted in rather an overdose of exclamation marks!

So, who is this book for? Advanced school students and well-motivated undergraduates can profitably read it (but note the price), and it is a very useful general reference for the history of substantial parts of mathematics, placed in the context of contemporary social and political events. There are other books specialising in specific periods, of course, and indeed there are substantial books devoted to such major figures as Newton, Leibniz and Euler. But as a readable (with some caveats, as above) and refreshingly detailed account of the whole sweep of 'infinitesimal methods' from antiquity to the 1990s, this book is highly recommended.

10.1017/mag.2023.135 © The Authors, 2023
Published by Cambridge University Press on behalf of The Mathematical Association

PETER GIBLIN
Department of Mathematical Sciences,
The University of Liverpool L69 7ZL
e-mail: *pjgiblin@liv.ac.uk*

Complex analysis by Andrei Bourchtein and Ludmila Bourchtein, pp. 346, £54.99 (paper), ISBN 978-9-91159-221-8, Springer Verlag (2021)

There being no shortage of textbooks on complex analysis, some justification is needed to add another one to what has been a crowded market for many years. The preface of this one states that it "covers all the traditional topics of complex analysis, starting from the very beginning—the definition and properties of complex numbers, ... —and ending with properties of conformal mappings, including the proofs of the fundamental results of conformal transformations." There are only five chapters, with the titles: 1. Introduction; 2. Analytic functions and their properties; 3. Singular points, Laurent series, and residues; 4. Conformal mappings; elementary functions; 5. Fundamental principles of conformal mappings; transformation of polygons.

Since it is meant to start from the beginning, it is peculiar that there is no mention of the triangle inequality for complex numbers at all in the whole book—it is indispensable, of course, but it is just applied throughout without any comment. To paraphrase Littlewood: *Equations are just balancing acts*, but *inequalities are something else, because* … . Some of the familiar terms are defined in unusual ways; for example, on page 12, the notion of a curve is made difficult by not including continuity in the definition, so that it is necessary to say a 'continuous curve', whereas a 'simple and closed curve' is presumed to be continuous. Also, instead of its usual definition, the residue of $f(z)$ at z_0 is defined by the value of the integral

$$\frac{1}{2\pi i}\int_{|z - z_0| = \rho} f(z)\ dz.$$

A lengthy remark is then required on the relevance of the unquantified number ρ, leaving the feeling of having 'the cart before the horse' for most readers.

The usual material associated with the topics in Chapter 3 are dealt with thoroughly, and the properties of elementary functions are covered in some detail in Chapter 4, which begins with the geometric interpretation of the derivative and the notion of a conformal mapping. Thus, linear and fractional linear functions, power and root functions, exponential and logarithm functions, the basic trigonometric functions and the Joukowski function are covered. Analytic continuation and the specification of domains of conformality, and the consideration of inverses, including multi-valued functions and the construction of the corresponding Riemann surfaces, are included in the chapter. Proofs of the main results on conformal mappings are provided in the last chapter. For the Riemann mapping theorem, the authors choose the modern approach based on the calculus of variations, and the proof is essentially that by F. Riesz and L. Fejér using Montel's theory of normal families of functions. This is fine for a reader already familiar with function theory, otherwise some background material, such as that in [1], will be necessary. The chapter also includes Schwarz's lemma, the Schwarz reflection principle, and the Schwarz-Christoffel mapping.