CHARACTERISATION OF THE DYNAMICS OF A VARIABLE STAR

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Abstract. Tentatives to detect chaos in variable stars are presented. Constraints on the accuracy on individual measurements and length of the records, necessary to obtain defenitive conclusions, are discussed. Some directions for future improvements are sketched.

1. Introduction

Different facts favor the existence of chaos in pulsating stars:
- many of them have irregular pulsations (see i.e. Perdang 1985),
- laboratory experiment and theoretical works have shown that chaos exist in simple systems,
- extended hydrodynamical stellar models display chaotic radial pulsation in peculiar stages of evolution.

If chaos exists in stellar pulsations, it must manifest itself by a chaotic attractor, characterized by its major features:
- divergence of trajectories responsible for the long term unpredictability, as measured by Lyapounov exponents. Lyapounov exponents are probably the most useful indicators of a chaotic dynamic, but they have never been used (to our knowledge) in variable stars studies and they are not discussed in the following. Measurements of Lyapounov exponents on noisy data and limited time series are difficult as the positive exponents are often small and not easily distinguishable from zero.
- fractal structure at "small" scales, except for self-similar objects. But, the scale at which it appears in the reconstructed state space depends on the method of reconstruction (see §3) and is not known a priori.
- specific routes to chaos, as period doubling or tangent bifurcation. The routes to chaos have been extensively studied, especially in laboratory experiments in which it is possible to vary control parameters. For stars, global parameters, which could play this role, vary on the nuclear time scale, too long to be really useful.

2. The Data: Present and Future

The observables are radial velocity or brightness measurements in broad or narrow bands.

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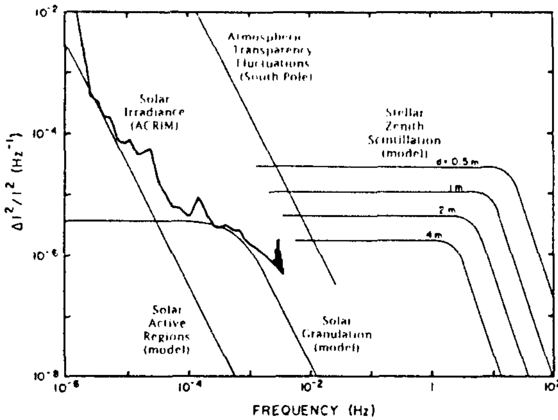

Fig. 1. Schematic Fourier spectra of scintillation and transparency fluctuations.

2.1. Radial Velocity

Radial velocity measurements are almost not affected by the atmospheric noise, at least in a first approximation. The accuracy is quite low $\sim 10^{-2}$. Improvements of the precision faces intrinsic limitations due to stellar atmosphere physics. Measuring velocities on one line is limited for most stars by the low photon flux or time resolution. Multiline measurements, as they are made by a Coravel type device, have to be performed on lines formed at the same level. The accuracy depends also strongly on the rotation and classical methods are limited to rotational velocity smaller than 20 km/s.

2.2. Brightness

Brightness measurements are generally done from the ground, in more or less broad visible bandpass. The three independent sources of errors are photon noise, scintillation and transparency fluctuations. On figure 1 the Fourier spectra of the transparency and scintillation contribution are shown (Harvey 1988). The scintillation model assumes a wind velocity of 30 ms^{-1} at an altitude of 2 km. Let us remind that, to obtain a given accuracy, α on individual measurements, the noise level at all frequencies must be smaller than α. A typical spectrum of real brightness measurements of a constant star is given on figure 2.

2.3. Photometric Techniques and their Accuracies

(For a complete discussion see the paper of Young et al. 1991.) Photometry being the most common technique to study variable stars, we briefly review the various techniques and their accuracy.

- Classical one channel photometer: average accuracy about 10^{-2}.

Differential photometry with such a device, gives a small time resolution

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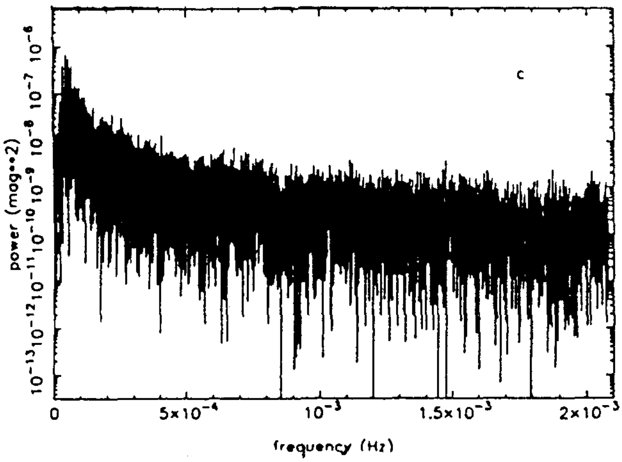

Fig. 2. Fourier spectra of 232 hours of measurements of the assumed constant star HD 213 473, $V = 9.1$ (from Michel et al. 1992).

due to the regular telescope depointing from the target star to the comparison star. As the two stars are not very close, variations due to transparency fluctuations are partly uncorrelated, unless for very good but rare nights.

- Multi channel photometer: average accuracy at most 10^{-3}.

The time resolution is better. The comparison stars are taken in the telescope field of view, improving the correlation of the transparency fluctuations. The sky background can be easily monitored. But, particularly for bright stars, the spectral types of the variable star and of the comparison are often different and some chromatic effects in the atmospheric absorption are present.

- CCD photometry: average accuracy a few 10^{-4}

This method is very promising: applied to a star cluster, several variables, several comparisons and the sky background can be measured at the same time. The limitation on the accuracy comes essentially from scintillation (Frandsen 1992). But it produces images at a high rate and reduction procedures are heavier than for time series obtained with multi channel devices.

- Photometry from space.

It can be limited only by instrumental drift and photon noise because the diameter of the collector remains small. The number of collected photons outside the atmosphere is

$$N = 10^5 10^{-0.4V} d^2 \, \delta\lambda \, \mu$$

where d is the diameter of the entrance pupil, V the visual magnitude, $\delta\lambda$ the spectral bandwidth and μ the quantum efficiency of the photometer. With $\delta\lambda \, \mu = 100$ and an exposure time of 30 s., the variance of the photon noise is $A = 1/\sqrt{N dt}$. Lines of constant A are drawn on figure 3, in an

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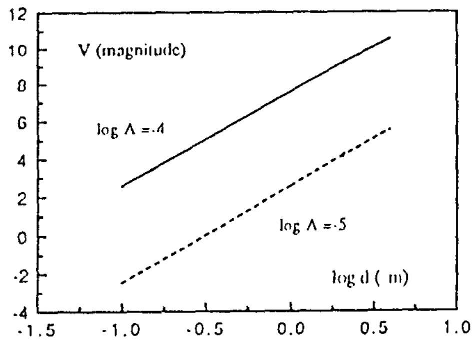

Fig. 3. Lines of constant A as function of the magnitude of the star and of the diameter of the collector.

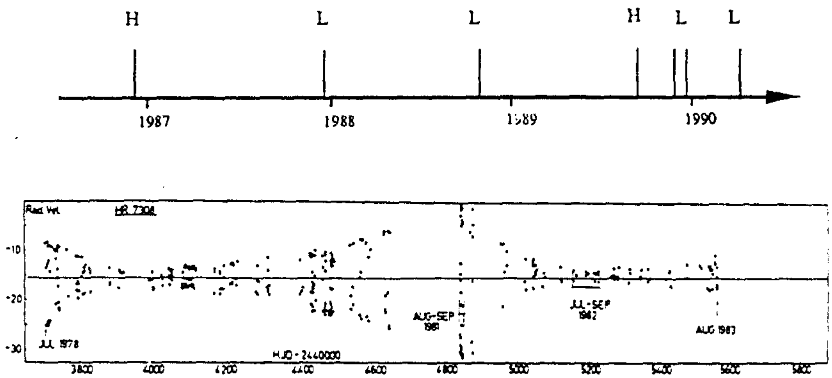

Fig. 4. a: Variations of the amplitude of the white dwarf G191-16 at different epochs from 1987 to 1990; H and L stands for high (0.2 Δm) and low amplitude (0.04 Δm); from Auvergne et al. 1990. b: Velocity curve of HR 7308 from Burki et al. 1982.

apparent magnitude - $\log d$ diagram. As they represent an unreachable limit for ground based observations they show that with a 2 meters telescope a relative precision of 10^{-4} cannot be achieved for a star fainter than 8.

2.4. Time Scales

To obtain a complete description of the dynamic one needs to record a large number of each characteristic time scale; but the long ones are not always known. For instance, very long time scale (several months to one year) exist in objects with short period. Two examples are shown on figures 4a and 4b. Long runs are necessary, favoring multisite campaigns or space missions. One of the best example is the monitoring of the Sun during 160 days by the IPHIR experiment on the Phobos mission.

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3. Procedures to Detect Chaotic Behavior

For dynamical systems known only through a single observable $s(t)$ like stars, several methods exist to construct an attractor in a n dimensional space *equivalent* to the original one.

3.1. Delay Vector

A theorem due to Takens (1981) states that a set (attractor) *equivalent* to the original one is obtained through the following algorithm known as the time delay method. Defining the set of vectors $x(t_i) = \{s(t), s(t+\tau), s(t+2\tau), \ldots, s(t+(n-1)\tau\}$, where n is the dimension of the Euclidian space used for the reconstruction, Takens shows that this process is an embedding i.e. that the original set and the embedded set are related through a diffeomorphism, with the condition $n \geq 2D+1$ where D is the dimension of the original set. This result is independent of the delay $\tau \neq 0$. However if τ is too small, $s(t) \sim s(t+\tau) \sim \ldots \sim s(t+(n-1)\tau)$ and all points are close to the first bisector. Experience shows that a "good" value of τ is of the order of 30% of the characteristic time scale. This value has been related to the first minimum of the autocorrelation function.

3.2. Singular Value Analysis

The method known also as Karhunen-Loeve expansion is a decomposition of the signal on the basis of the eigenvectors of its correlation matrix. This linear transform determines the dominant components of the dynamic, but not necessarily those which govern the nonlinear part. Projection on the subspace generated by the eigenvectors associated with the largest eigenvalues gives also an embedding. Examples are given on figure 5. In some cases the components seem to correspond to physical parameters, temperature, velocity and radius, as in hydrodynamical models.

3.3. Geometrical Characterisations

Once, the attractor is reconstructed, Poincaré map and return maps provide information on its structure, hence on the underlying dynamics. Unfortunately, noise acts as a diffusive process in the embedding space.

-Poincaré map.

It contains a priori the same topological information as the flow and the same stability properties. Figure 6a show the Poincaré map of a simple model. Figure 6b is the map of the same model where one percent white noise, mimicking atmospheric transparency fluctuations, has been added. The fractal structure is partly swapped out as the attractor is broadened and the small scale structures are hidden. A statistical study of the density of points in the plane, allows to retrieve at least a part of the lost structure (Bijaoui 1974).

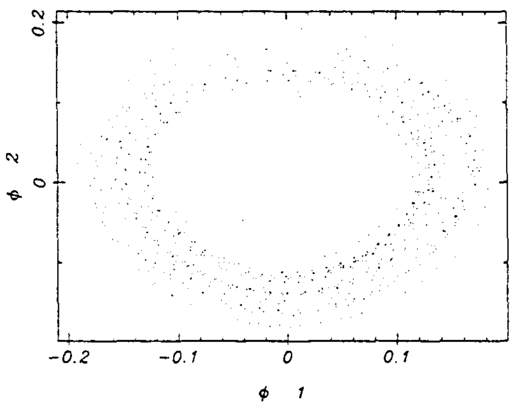

Fig. 5. Projections on the two eigenvectors Φ_1 and Φ_2 corresponding to the two largest eigenvalues. The object is the white dwarf PG 1351+489. (Auvergne 1988)

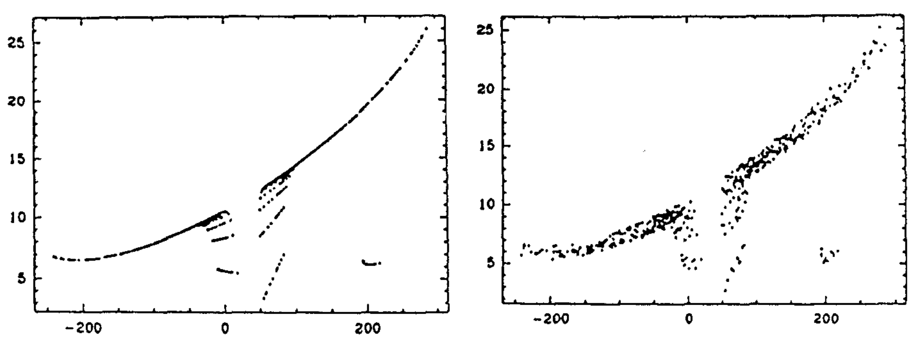

Fig. 6. a: Poincaré map of a chaotic solution of a simple model (Auvergne and Baglin 1985). b: the same plus 1% noise.

-Return map.

A return map contains the same information than the Poincaré section, but it is generally difficult to use on astrophysical data as it seems to be more sensitive to noise than the Poincaré map. A complex first return map can arise from a high dimensional chaos. This is the case of a chaotic dynamic arising from a subcritical bifurcation of a subharmonic limit cycle. In this case a fourth return map displays a simpler one dimensional shape. Figures 7 and 8 display several first return maps from observations and from hydrodynamical calculations.

3.4. Dimension Computations

Roughly speaking, the dimension of a set is related to the amount of information needed to locate a point on it. Several "fractal dimensions" have been defined corresponding to the nature of the information one wants to

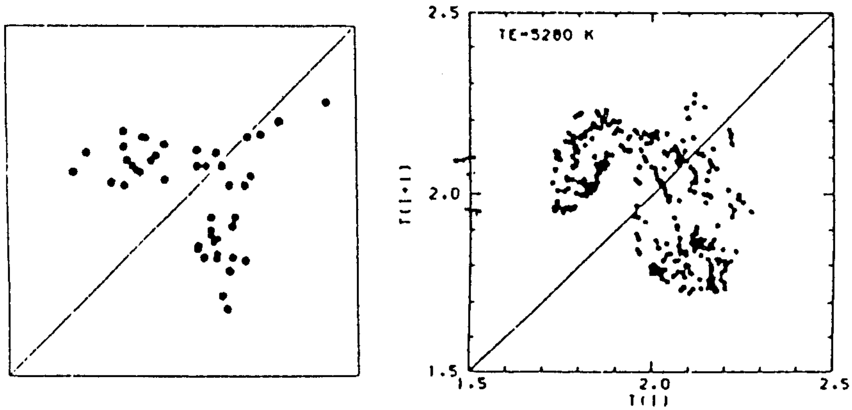

Fig. 7. a: Return map of the semiregular variable S Vul, from Saitou et al. 1989. b: the same for the numerical simulation of a chaotic population II model, from Aikawa 1990.

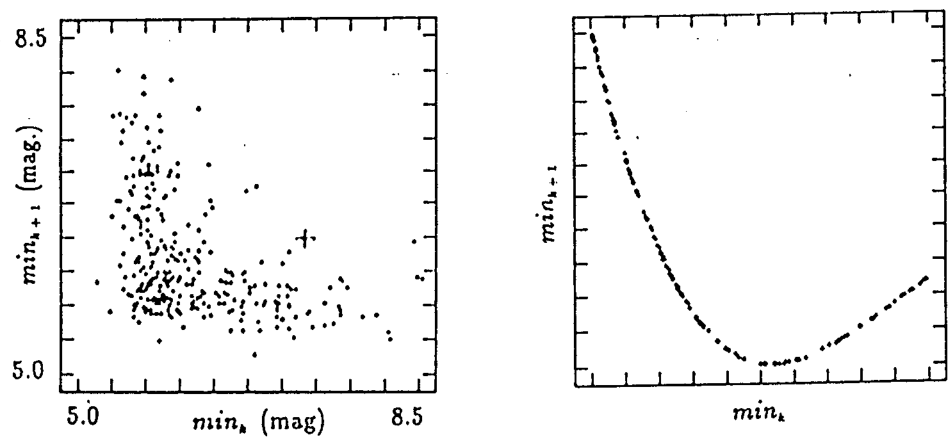

Fig. 8. a: Return map of the RV Tauri star R Scutum, using light minima. b: return map of the Rössler attractor, from Kolláth 1990.

obtain (for a review on dimensions of chaotic attractors see i.e. Farmer et al. 1983). The most popular method to detect fractal structure is the correlation integral, defining:

$$C(l) = \lim_{N \to \infty} \frac{1}{N(N-1)} \sum_{i,j=1}^{N} \theta(l - ||x_i - x_j||)$$

where θ is the Heaviside function (Grassberger and Procaccia 1983). $C(l)$ counts the number of couples of points such that $l < |x_i - x_j|$, and for small values of l one can show that $C(l) \sim l^D$. Since the dimension of the attractor is not known, the dimension of the embedding space is also unknown. Therefore $C(l)$ is computed for increasing values of n. When the

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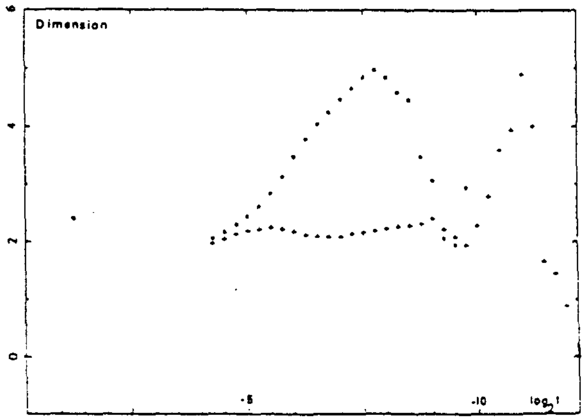

Fig. 9. Slope of the correlation integral of a dimension 2.08 attractor without and with 1% white noise added.

slope of $C(l)$ becomes independent of n one has reached the dimension of the set.

But, as the scale at which the fractal structure appears is unknown (unless for self similar object), we are never sure to reach a noise free significant scale. This remark has not been always well understood and many published results are just artefacts. Figure 9 shows the noise influence on the correlation integral.

Several other important artefacts have been emphasized in the literature: the effect of a coloured noise by Osborne and Provenzale (1989) or the role of a low pass filter which, if the cutoff frequency is too small, increases the value of the computed dimension (Badii et al. 1988). Theiler (1986, 1990) and Smith (1987) have given constraints on the minimum number of points (as a function of the embedding dimension) to ascertain a satisfactory statistic on the attractor.

3.5. Route to Chaos

Some simple dynamical systems, for instance the popular Rössler attractor, display (even in the chaotic state) subharmonic frequencies. This property is due to the phase coherence of such systems. Subharmonics have been found in several very different types of stars, like white dwarfs and Mira variables.

They show up naturally in Fourier spectra or on light curves by the alternation of different amplitude maxima or minima (Vauclair et al. 1989, Goupil et al. 1988, Buchler and Regev 1990). Figures 10 and 11 show power spectra of the Mira variable T UMa and of the white dwarf PG1351+489, with such subharmonics.

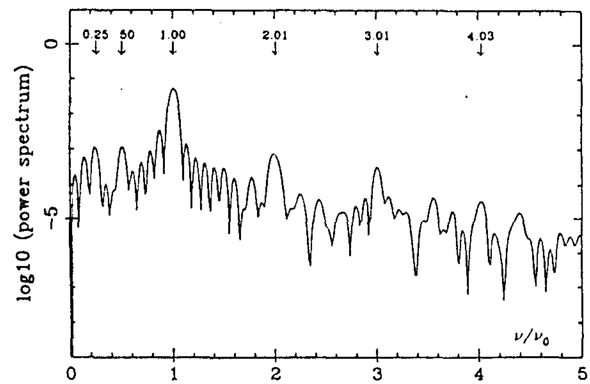

Fig. 10. Power spectra of the light curve of T UMa (data from AAVSO) showing to peaks at $\nu/2$ and $\nu/4$.

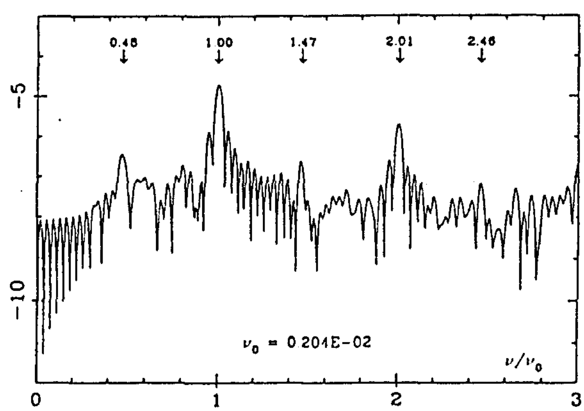

Fig. 11. Power spectrum of PG 1351+459 (Goupil et al. 1988)

4. Conclusions

During the last ten years a lot of efforts have been devoted to the search of chaos in stars. Significant progresses have been made in the theoretical and computational domains (Aikawa 1987, Buchler et al. 1987, Buchler and Regev 1990) as well as in the time series treatment (Casdagli et al. 1990, Scargle 1992).

In real data some encouraging results have been obtained, but definitive conclusions can be derived only with extremely high quality data. In the most common case of photometric measurements from the ground, improvements will come essentially from the correction of transparency fluctuations, as the problem of the scintillation and photons noise is “simpler” to solve by increasing the telescope diameter.

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Observations from space, though very costly, can be designed to be free from these difficulties. Two experiments, EVRIS and PRISMA, devoted to stellar variability detection are in preparation. They will produce, in a near future, low noise and very long time series, adequate for such studies.

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