Real analysis and infinity by Hassan Sedaghat, pp 547, £60 (hard), ISBN 978-0-19289-562-2, Oxford University Press (2022)

This attractively produced book covers all of the topics one would expect to find in an introductory text on real analysis. Thus a short scene-setting chapter is followed by a background chapter on sets, functions, logic and countability and then six long chapters on sequences and limits, the real numbers (constructed in detail using $\mathbb{Q}$-Cauchy sequences), infinite series, differentiation and continuity (in that order), Riemann integration (using the Darboux formulation) and infinite series of functions. The pace is leisurely with concepts motivated and described informally (often with historical context as well) before being developed more formally. For example, uniform convergence is preceded by a careful analysis of the behaviour of the familiar geometric series $(1 - x)^{-1} = 1 + x + x^2 + \dots$ as x approaches ± 1. Proofs are given in full and routine exercises (tightly tied to the text) are given at the ends of chapters. One unusual feature is that theorems and exercises are together numbered in sequence—so the book ends with Theorem 387!

The "infinity" in the book's title refers both to the central occurrence of infinite sets in analysis and to the ubiquity of infinite processes. The latter is highlighted by the author's preference for sequence arguments over $\varepsilon - \delta$ ones: for example, the definition of uniform continuity is given as "$\lim_{n\to\infty}(x_n - y_n) = 0 \Rightarrow \lim_{n\to\infty}[f(x_n) - f(y_n)] = 0$". The organisation and selection of material is exemplary with key results given in a form appropriate for a first course rather than in fullest generality. There is a very nice account of the possible behaviour of the Newton-Raphson iterative method by viewing it as a fixed-point iteration, $x = x - \frac{f(x)}{f'(x)}$. And it was good to see room for an introduction to Fourier series (far enough to allow for a solution to the Basel problem via pointwise evaluation of the series for $|x|$), a proof of the everywhere non-differentiability of the Weierstrass function $\sum_{k=0}^{\infty} 2^{-k}\cos(m^k\pi x)$, and the neat constructive proof of the Weierstrass approximation theorem using Bernstein polynomials. One novel feature is the author's frequent use of the "ε-index", the least N_ε that works in the definition of convergence, although I am still musing on whether this creates almost as many pedagogical problems as it solves.

Unfortunately, however, this edition of the book is marred by mistakes. There are many easily corrected typos, there are some numerical slips in worked examples, and several proofs that need adjusting. A few specific examples: Theorem 4 states only one of De Morgan's laws (although both are later cited); multiplying odd and even functions is incorrectly said to work "analogously to multiplying odd and even integers" (p. 31); $\binom{n}{2}$ is rendered as $\frac{1}{2}n(n + 1)$ on pp. 186, 188; the mean value theorem for integrals is mis-stated as $\int_a^b f(x)\,dx = f(c)(x - c)$ (p. 357); "$e \approx 2.7182815$" (pp. 362, 363);

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + 1}dx = \int_{-\infty}^{0} \frac{1}{x^2 + 1}dx + \int_{0}^{\infty} \frac{1}{x^2 + 1}dx = -\frac{\pi}{2} + \frac{\pi}{2} = 0 \text{ (p. 381);}$$

the proof of the root test on p. 185 mistakenly has some lines from the proof of the ratio test on pp. 181-182; 'differentiability implies continuity' is used on pp. 229, 230 before it is proved on p. 247; the proof of the general binomial series for $(1 + x)^p$ goes haywire towards the end of p. 483 with the *non sequitur* $|x|\left(1 + \frac{|p|}{N + 1}\right) \leqslant |x^2|$ with $|x| < 1$.

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In its current state, I really cannot confidently recommend this book for its intended audience of those meeting real analysis for the first time. I read every page closely and felt that there was a useful and attractively readable textbook trying to get out, but that more editorial work is needed to eradicate the bugs in this first edition.

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Fourier analysis (new edition) by T.W. Körner, pp 591, £47.95 (paper), ISBN 978-1-009-23005-6 Cambridge University Press (2023)

Fourier's idea that a periodic function can be represented as a trigonometric polynomial, and his startling generalisation to non-periodic functions via the Fourier transform, are not only mathematically interesting, but also powerful tools for understanding nature.

Körner wrote this wonderful book, first published in 1988 and now reissued in the Cambridge Mathematical Library series, as a shop window displaying some of the best of Fourier analysis. He intended it to be 'accessible to an undergraduate at a British university after two or three years of study', but it will appeal to a much wider readership. The book is long but each of the 110 chapters is short, showcasing an idea, technique, application or elegant result related to Fourier analysis. The chapters are largely independent, and so after assimilating the basics (covered in the first ten pages) the reader can more or less browse the book at will. The book is broad, rather than deep, although Körner adjusts the rigour of the discussion to suit the subjects discussed. Alongside the more rigorous chapters there are many interesting applications of Fourier analysis, some historical essays, and occasional words of advice to the reader. The chapters are divided up into six parts; Fourier Series, Some Differential Equations, Orthogonal Series, Fourier Transforms, Further Developments, and Other Directions.

Much of the material that undergraduate mathematicians need to know (issues of existence, uniqueness and convergence, orthogonal polynomials and so on) is covered in the book. But the real delights are the unexpected and varied discoveries awaiting the browser. Here is a tiny sample:

Chapter 13 on Monte Carlo methods for integrating over an m-dimensional cube reveals that, in contrast to a generalised Simpson's type rule, the accuracy does not depend on the dimension of the cube.

Chapter 35 includes Liouville's own proof that 'Liouville's number' $10^{-1!} + 10^{-2!} + 10^{-3!} \ldots$ is transcendental.

Chapter 90 includes Burgess Davis's remarkable probabilistic proof of Picard's theorem: If $f : \mathbb{C} \to \mathbb{C}$ is analytic and non-constant then the range of f omits at most one point of $\mathbb{C}$.

The proof is based on the mathematical theory of Brownian motion—a topic which links several chapters in the book. How can a probabilistic theory prove such a definite result? Because if you can show something happens with probability one, then it certainly happens *sometimes*, and in this instance that is all that is needed to complete the proof.

Chapter 95, on 'The Diameter of Stars' asks how we can measure these when the blurring due to the Earth's atmosphere is much greater than the diameters we wish to observe. In a comment which dates the book, Körner says "Soon … we will be able to spend our way out of trouble by putting our telescope in orbit above the