The 490 members in England are distributed thus: London 133, Bedfordshire 5, Berkshire 13, Bucks 12, Cambridge 30, Cheshire 6, Cornwall 1, Cumberland 4, Derbyshire 4, Devonshire 10, Dorset 1, Durham 5, Essex 10, Glos. 24, Hants 18, Herts 12, Hereford 2, Isle of Wight 3, Kent 22, Lancashire 24, Leicester 4, Lincs 1, Middlesex 8, Monmouth 1, Norfolk 5, Northants 3, Northumberland 3, Notts 3, Oxford 13, Rutland 4, Shropshire 4, Somerset 6, Staffs 3, Suffolk 1, Surrey 22, Sussex 14, Warwickshire 11, Westmorland 1, Wilts 1, Worcs. 8, Yorkshire 35.

CORRESPONDENCE.

HARROW SCHOOL, Jan. 20th, 1911.

To the Editor of the Mathematical Gazette.

DEAR SIR,

Canon Wilson,* a reformer of forty years' standing, has kindly sent me the enclosed letter with a view to its suggestions being laid before the M.A. Teaching Committee. The letter seems to me of such general interest that I have obtained leave from Canon Wilson to send it to you for publication in the *Gazette.*

Trusting that you will be able to find space for it.

I am, Yours truly, A. W. SIDDONS.

COLLEGE, WORCESTER, Jan. 17th, 1911.

DEAR MR. SIDDONS,

You were good enough to say that I might commit to writing some suggestions that occurred to me in reading the Draft Report of your "Committee on the Teaching of Algebra and Trigonometry," and that you would lay them before the Committee.

My conviction that you have a great opportunity before you, and my strong desire that you should use it to the full, make me venture to address you. I think that what I suggest may give greater unity to your report as a whole, bringing out more clearly a central principle, and showing the relation to it of your analysis of the aims of teaching algebra in § II., and of your recommendations.

I think that you should make it clear *ab initio* that it is a *teaching syllabus for schools* you are outlining, and not a *philosophic basis for algebra.* Few men, and still fewer boys and girls, are capable of what you describe as "scientific interest in form" in the examination of fundamental laws. Historically and educationally algebra is generalised arithmetic.

* The Ven. J. M. Wilson, D.D., was Senior Wrangler in 1859, and the next year was elected to a Fellowship at St. John's Coll., Cam. For twenty years he was Mathematical and Science Master at Rugby. His eleven years as Head Master of Clifton College marked a period of great educational activity, during which his interest in the Association for the Improvement of Geometrical Teaching continued unabated. His *Elementary Geometry* passed through many editions and reprints. This was an able attempt to deal with a problem that cannot yet be said to have received an entirely satisfactory solution. With the "Syllabus of the Association" it received an amusing castigation at the hands of that doughty defender of undiluted Euclid—C. L. Dodgson, better known, perhaps, to the younger generation as the author of *Alice in Wonderland* (*v.* "Euclid and his Modern Rivals," Act II. scene vi. and Act III. scene ii., in which Nostradamus—*nostra*, pl. of *nostrum*, 'a quack remedy,' and *damus*, 'we give'—may possibly be intended for Dr. Wilson himself). It is almost an impertinence to add that the re-appearance in our midst at the annual meeting of one of whom it might almost be said of his connection with the work of the Association—*quorum pars magna fuit*—has been to many of us not merely an awakening of memories of the early labours of nearly half a century ago, but a potent stimulus to renewed effort. [Ed.]

The algebra, therefore, that you here speak of is the algebra for school, arithmetical in origin, practical in its early applications. There is much which is of interest to pure mathematicians, which is an after-thought in the history of algebra, and is out of place in teaching the young.

When a boy is for the first time introduced to algebra, he wishes, or ought to wish, and may be induced to wish, to know what is the use of it; and it is very desirable to gratify his wish, and not only to tell him that he will know some day. This then appears to me to be the right moment for asking him, and helping him to think out, what mathematics are. How would he define mathematics and their use? Then the teacher criticises, corrects, enlarges, retrenches, etc., the boy's proposed definition, in Socratic method, till he finally leads him to discover for himself Comte's definition, *mathematics is the science of indirect measurement of magnitude, and the processes subsidiary thereto.*

That is the best definition of algebra for boys.

Every word in the above definition must be examined and shown to be important,—*science*, *indirect measurement*, *magnitude*,—and thoroughly illustrated. The boy will soon see that there must be a unit—a foot, a ton, a velocity, and so on—in any subject to which mathematics can be applied, and illustrations will be readily found. Physical science furnishes data and relations between them by direct measurement; and mathematics enables us to measure things indirectly which we cannot measure directly. It gives a boy a new sense of the meaning of geometry, if he is taught that every theorem can be represented and used as an indirect measurement of some magnitude; that Euclid i. 4 enables him to measure the distance across a pond or through a haystack, and Euclid i. 26, with a little extension to proportion, shows him how to measure indirectly the distance of an inaccessible object, a buoy at sea, the moon, even the fixed stars. He will believe in the use of geometry as a science of indirect measurement.

But algebra? Of what use is it? I should tell him at once that algebra also is employed for the indirect measurement of magnitudes of all sorts, and that this measurement is generally effected by the *formation and solution of equations* which involve the quantity to be measured. This measurement may be of such varied things as the weight of the sun or moon, the velocity of light, the thickness of a soap-bubble, the horse-power of an engine, the temperature of a furnace, or the mass and velocity of an electron, all incapable of direct measurement. This he can only understand as a generalization from some examples. Take him at once into a problem. "A fish is caught, too heavy for the weights on our scale; it is cut into three parts, head, tail and body; and it is noticed that the body weighs as much as the head and tail; that the head weighs as much as the tail and half the body; and that the tail weighs 9 lbs. What is the weight of the fish?" Can we indirectly measure it?

The boy is led slowly and thoroughly to make his equation, using all the data. Then follows the axioms,—if equals be added to equals, etc., and the equation is solved; and the solution verified. *He has learnt the use of algebra* as a *science of indirect measurement.*

It is quite possible, and highly stimulating, to introduce here problems which lead to simultaneous, or quadratic, or cubic equations, and to leave these equations formed, but unsolved: and show the need of "subsidiary processes."

This gives a boy a map of the whole field he has to traverse, and a clue to his path; it satisfies his natural desire to see the use of what he is doing: it gives him patience in working at the "subsidiary processes."

But it also serves another purpose. It gives a valuable guide both to the order of teaching, and to the extent to which each subsidiary process should be studied and developed. Subsidiary processes should be carried, in the first instance, just so far as they are really subsidiary and no further, not as mere curious exercises. They are subsidiary to the two purposes, of solving equations, and of otherwise effecting indirect measurements.

The latter purpose justifies the retention, at an early stage, of some other subjects besides equations, such as progressions. A boy is told to add $1+2+3$ . . . up to 100. The direct method is of course possible, but tedious; teach him to take them in pairs $1+100$, $2+99$, $3+98$, and he sees that there are 50 pairs of 101 each, and indirectly gets the amount. So the principle justifies the retention of permutations and combinations as indirect ways of counting. Tell a boy to draw a regular decagon and all its diagonals; and then to count how many lines he has drawn. He will succeed probably in counting 45. Now tell him to produce every line in both directions, and then count how many triangles there are on the paper; and that in a fortnight he will be able to say there are exactly 10,000. The use of logarithms is justified to calculate indirectly such results as the number of digits in 2^{100}. Tell him the old story of the regiment of 100 monkeys, the first of whom came to a pile of nuts, and took half the pile and one more; the second took half of what was left and one more; and so on, till the last monkey did the same, and finished the pile. How many nuts were there? And how large a cube would they fill if the diameter of a nut was one-third of an inch?

I need not go into the application of the same principle to trigonometry. The indirect measurement of distances, areas, angles, etc., in teaching this subject takes the same place as it does in algebra, and similarly depends on the formation of equations, and on solving them by means of subsidiary processes.

If this principle is grasped in your report, and recommended for adoption in the construction of syllabuses and text books, it seems to me that more rapid progress will be made by boys into the heart and application of mathematical subjects: they would sooner be ready for mechanics and the calculus; and would gain much from beginning them; and less time will be wasted, or nearly wasted, in teasing boys with what seem to them useless manipulations, clever dodges. The ingenious proofs of identities in algebra and trigonometry, for example, will find a subordinate place outside the course which the whole stream of boys will follow.

The boy who is capable of becoming a philosophic mathematician will not be sacrificed by this order of teaching: and the mass of boys will take more interest in mathematics, and learn far more of the power that mathematical methods give.

If I may make a general remark on the teaching of science I should say that we are in danger of sacrificing the natural *educational* method to what appears a *logical* method. In teaching botany it is an educational blunder to begin with the cell: one should begin with the flower and comparison of flowers and pass to generalizations, ever widening: so botany developed. In mechanics it is an educational blunder to begin with the vector: one begins with the lever. All science teaching should proceed from the known to the unknown, and the history of its development is a good guide. We must avoid this mistake in teaching mathematics.

I have not attempted to work this suggestion into your report or to examine how far your report would need to be modified. This must be for you to consider. But I trust that you will find some truth and value in my suggestions. Very truly yours, JAMES M. WILSON.

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