Canadian Mathematical Bulletin
doi:10.4153/CMB-2011-056-5
© Canadian Mathematical Society 2011

Projectively Flat Fourth Root Finsler Metrics

Benling Li and Zhongmin Shen

Abstract. In this paper, we study locally projectively flat fourth root Finsler metrics and their generalized metrics. We prove that if they are irreducible, then they must be locally Minkowskian.

1 Introduction

An important problem in Finsler geometry is the study of the geometric properties of locally projectively flat Finsler manifolds. Locally projectively flat metrics are of scalar flag curvature, namely, the flag curvature is a scalar function of tangent vectors, independent of the tangent planes containing the tangent vector.

A Finsler metric on an open domain in R^n is said to be *projectively flat* if its geodesics are straight lines. Hilbert's Fourth Problem in the regular case is the study and characterization of projectively flat metrics on a convex open domain in R^n. Projectively flat Riemannian metrics are those of constant sectional curvature $\mathbf{K} = \mu$, which can be expressed in the following form on an appropriate ball in R^n:

$$F = \frac{\sqrt{|y|^2 + \mu(|x|^2|y|^2 - \langle x, y\rangle^2)}}{1 + \mu|x|^2}.$$

There are many non-Riemannian projectively flat Finsler metrics. For example, the well-known Funk metric and Hilbert metric on a strongly convex domain R^n are projectively flat. These metrics even have constant flag curvature. (See [6, 11] for more information on projectively flat Finsler metrics of constant flag curvature.) In [12], we studied and characterized projectively flat (α, β)-metrics which are defined by a Riemannian metric $\alpha = \sqrt{a_{ij}(x)y^i y^j}$ and a 1-form $\beta = b_i(x)y^i$. Two special examples are given as follows:

$$F = \frac{\sqrt{|y|^2 - (|x|^2|y|^2 - \langle x, y\rangle^2)}}{1 - |x|^2} + \frac{\langle x, y\rangle}{1 - |x|^2}, \tag{1.1}$$

$$F = \frac{\left(\sqrt{|y|^2 - (|x|^2|y|^2 - \langle x, y\rangle^2)} + \langle x, y\rangle\right)^2}{(1 - |x|^2)^2\sqrt{|y|^2 - (|x|^2|y|^2 - \langle x, y\rangle^2)}}. \tag{1.2}$$

Received by the editors January 8, 2009; revised April 29, 2009.
Published electronically March 31, 2011.
The first author was supported by the NNSFC (10801080), Science Foundation of Ningbo (2008A610014), Foundation 008-011147 and K. C. Wong Magna Fund in Ningbo University. The second author was supported by the National Natural Science Foundation of China (10671214) and the National Science Foundation (DMS-0810159)
AMS subject classification: **53B40**.
Keywords: projectively flat, Finsler metric, fourth root Finsler metric.

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press

The metric in (1.1) is the Funk metric in a unit ball $B^n(1) \subset R^n$ and the metric in (1.2) was constructed by L. Berwald [3].

There are two important classes of reversible Finsler metrics.

$$F = A^{1/4}, \tag{1.3}$$

$$F = (A^{1/2} + B)^{1/2}, \tag{1.4}$$

where $A = a_{ijkl}(x)y^i y^j y^k y^l$ and $B = b_{ij}(x)y^i y^j$. A Finsler metric in the form (1.3) is called a *fourth root metric* and a Finsler metric in the form (1.4) is called a *generalized fourth root metric*. When A is a perfect square of a quadratic form, $F = A^{1/4}$ is just a Riemannian metric. Thus (generalized) fourth root metrics include Riemannian. The fourth root metrics are special m-th root metrics defined in the form $F = \{a_{i_1 \ldots i_m}(x)y^{i_1} \cdots y^{i_m}\}^{1/m}$. These metrics were first studied by M. Matsumoto, K. Okubo, and H. Shimada [7, 8, 13]. In four dimensions, the special fourth root metric in the form $F = \sqrt[4]{y^1 y^2 y^3 y^4}$ is called the *Berwald–Moore metric*. This metric is singular in y and not positive definite. Recently, physicists have been interested in fourth-root of metrics [10]. Thus, it is important to study the geometric properties of fourth root metrics.

A Finsler metric $F = F(x, y)$ is said to be *locally Minkowskian* if at every point there is a local coordinate domain in which the metric $F = F(y)$ is independent of its position x. In this case, all geodesics are linear lines $x^i(t) = ta^i + b^i$. A Finsler metric $F = F(x, y)$ is said to be *locally projectively flat* if at every point there is a local coordinate domain in which the geodesics are straight lines, namely, $x^i(t) = f(t)a^i + b^i$. The main purpose of this paper is to study locally projectively flat fourth root metrics and their generalized metrics. We prove the following.

Theorem 1.1 *Let $F = A^{1/4}$ be a fourth root metric on a manifold of dimension $n \geq 3$. Assume that A is irreducible. If F is locally projectively flat, then it is locally Minkowskian.*

If $A = (a_{ij}(x)y^i y^j)^2$ is the square of a Riemannian metric of constant sectional curvature $\mathbf{K} = \mu$, then $F = A^{1/4} = \sqrt{a_{ij}(x)y^i y^j}$ is locally projectively flat. But it is not locally Minkowskian when $\mu \neq 0$. Thus the condition on the irreducibility condition of A cannot be removed, although it might be slightly weakenned.

Theorem 1.2 *Let $F = (A^{1/2} + B)^{1/2}$ be a generalized fourth root metric on a manifold of dimension $n \geq 3$. Assume that A and $A - B^2$ are both irreducible and $B \neq 0$. If F is locally projectively flat, then it is locally Minkowskian.*

The following example shows that the irreducibility condition on $A - B^2$ cannot be dropped, although it might be slightly weakened.

Example 1.3 Let $F = (A^{1/2} + B)^{1/2}$ be a fourth root metric on $B^n \subset R^n$ defined by

$$A := \frac{|y|^4 + (|x|^2|y|^2 - \langle x, y\rangle^2)^2}{4(1 + |x|^4)^2} \qquad B := \frac{(1 + |x|^4)|x|^2|y|^2 + (1 - |x|^4)\langle x, y\rangle^2}{2(1 + |x|^4)^2}.$$

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press

Note that

$$A - B^2 = \Big[\frac{-2|x|^2\langle x, y\rangle^2 + (1 + |x|^4)|y|^2}{2(1 + |x|^4)^2} \Big]^2.$$

Thus, $A - B^2$ is reducible; F is projectively flat, but not locally Minkowskian.

The classification of projectively flat generalized fourth root metrics without assumption on the irreducibility has not been done yet. Example 1.3 is a non-trivial solution. Thus we believe that there is a rich class of locally projectively flat generalized fourth root metrics. Fourth root metrics are special m-th root metrics when $m = 4$. In a recent work N. Brinzei [1]derived some equations to characterize projectively flat m-th root metrics. These equations are still too complicated to solve. Thus, no explicit examples have been found via these equations. Further study will reveal some more geometric properties of m-th root metrics.

2 Preliminaries

Let F be a Finsler metric on a manifold M. We always assume that F is positive definite (or strongly convex), namely, the matrix $g_{ij} = g_{ij}(x, y)$ is positive definite, where

$$g_{ij}(x, y) := \frac{1}{2}[F^2]_{y^iy^j}(x, y), \quad (y \neq 0).$$

The geodesics of F are characterized by a system of equations:

$$\frac{d^2x^i}{dt^2} + 2G^i\Big(x, \frac{dx}{dt}\Big) = 0,$$

where $G^i = \frac{1}{4}g^{il}\{[F^2]_{x^my^l}y^m - [F^2]_{x^l}\}$. Clearly, if F is Riemannian, then $G^i = G^i(x, y)$ are quadratic in y. F is called a *Berwald metric* if $G^i = G^i(x, y)$ are quadratic in y. It is called a *Landsberg metric* if $F_{y^i}[G^i]_{y^jy^ky^l} = 0$. Thus every Riemannian metric is a Berwald metric and every Berwald metric is a Landsberg metric.

For a Finsler metric, the Riemann curvature $\mathbf{R}_y\colon T_xM \to T_xM$ is defined by $\mathbf{R}_y(u) = R^i{}_k(x, y)u^k\frac{\partial}{\partial x^i}{}_x$, $u = u^k\frac{\partial}{\partial x^k}|_x$, where

$$R^i{}_k(x, y) := 2\frac{\partial G^i}{\partial x^k} - y^j\frac{\partial^2 G^i}{\partial x^j \partial y^k} + 2G^j\frac{\partial^2 G^i}{\partial y^j \partial y^k} - \frac{\partial G^i}{\partial y^j}\frac{\partial G^j}{\partial y^k}.$$

For each tangent plane $\Pi \subset T_xM$ and $y \in P$, the *flag curvature* of (Π, y) is defined by

$$\mathbf{K}(\Pi, y) := \frac{g_{im}(x, y)R^i{}_k(x, y)u^ku^m}{F(x, y)^2g_{ij}(x, y)u^iu^j - [g_{ij}(x, y)y^iu^j]^2},$$

where $u \in \Pi$ such that $\Pi = \operatorname{span}\{y, u\}$. There is a large class of Finsler metrics whose flag curvature $\mathbf{K}(\Pi, y) = K(x, y)$ is independent of tangent planes Π containing $y \in T_xM$. Such metrics are said to be *of scalar flag curvature*. When the metric is Riemannian, the flag curvature $\mathbf{K}(\Pi, y) = K(\Pi)$ is independent of $y \in T_xM$. Thus it is of scalar flag curvature $\mathbf{K} = K(x, y)$ if and only if it is of isotropic sectional curvature $\mathbf{K} = K(x)$ (constant if dimension ≥ 3). We have the following important theorem.

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press

Theorem 2.1 (Numata [9]) *Let F be a Landsberg metric of scalar flag curvature on a manifold of dimension $n \geq 3$. If the flag curvature $\mathbf{K} \neq 0$, then it is Riemannian.*

On the other hand, we have the following well-known theorem.

Theorem 2.2 ([2]) *Every Berwald metric with $\mathbf{K} = 0$ is locally Minkowskian.*

A simple fact is that a Finsler metric $F = F(x, y)$ on an open subset $\mathcal{U} \subset R^n$ is projectively flat if and only if the spray coefficients are in the form $G^i = Py^i$. This is equivalent to the following Hamel equation

$$F_{x^m y^k} y^m = F_{x^k}. \tag{2.1}$$

In this case, $P = F_{x^m} y^m/(2F)$ and the metric is of scalar flag curvature given by

$$\mathbf{K} = \frac{P^2 - P_{x^m} y^m}{F^2}.$$

Thus, locally projectively flat Finsler metrics are of scalar flag curvature.

Let us consider the special case when $G^i = Py^i$ where $P = P_i(x)y^i$ is a local 1-form. Assume that the dimension $n \geq 3$. Let

$$U := \{x \in M \mid \mathbf{K}(x, y) \neq 0 \text{ for some } y \in T_xM\}.$$

Assume that $U \neq \varnothing$. By Theorem 2.1, F is Riemannian on U with $\mathbf{K} = \mathit{constant} \neq 0$. By continuity, one can easily conclude that $U = M$, namely, F is Riemannian on the whole manifold. Assume that $U = \varnothing$, *i.e.*, $\mathbf{K} = 0$ on M. Since F is a Berwald metric, it must be locally Minkowskian.

The two-dimensional case was solved by L. Berwald.

Theorem 2.3 (Berwald [4]) *Let F be a locally projectively flat Landsberg metric on a surface M. Then it is either Riemannian with non-zero constant Gauss curvature or locally Minkowskian.*

If one allows singular metrics in Theorem 2.3, then besides the above two cases, the metric might be a Berwald metric which can be locally expressed in the following form: $F = (y^1 + f(x^1, x^2)y^2)^4/(y^2)^2$, where $f = f(x^1, x^2)$ is a function satisfying $x^1 + x^2 f = \psi(f)$ for some function ψ with $\psi'' \neq 0$.

We now consider fourth root metrics. Let $A(x, y) := a_{ijkl}(x)y^i y^j y^k y^l$ be a homogeneous polynomial of degree four on tangent spaces. Assume that $A = A(x, y) > 0$ for any $y \neq 0$. Then the Hessian $g_{ij} := \frac{1}{2}[F^2]_{y^i y^j}$ is given by

$$g_{ij} = \frac{1}{8} A^{-3/2}\{2AA_{ij} - A_i A_j\},$$

where

$$A_i := \frac{\partial A}{\partial y^i}, \quad A_{ij} := \frac{\partial^2 A}{\partial y^i \partial y^j}.$$

Thus if $2AA_{ij} - A_i A_j$ is positive definite, then F is a positive definite Finsler metric.

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press

Example 2.4 Let $A := \sum_{i=1}^{n}(y^i)^4$. For some directions, say, $y = (1, 0, \dots, 0)$, $\det(g_{ij}) = 0$. Thus $F = A^{1/4}$ is not positive definite.

Example 2.5 Let A be a two-dimensional homogenous polynomial of degree four in the following form: $A = (y^1)^4 + 2c(y^1)^2(y^2)^2 + (y^2)^4$. It is easy to show that $F = A^{1/4}$ is positive definite if and only if $1 < c < 3$.

Example 2.6 Let $\alpha := \sqrt{a_{ij}y^iy^j}$ be a Euclidean norm and $\beta := b_iy^i$ be a 1-form on R^n with $b := \sqrt{a^{ij}b_ib_j} < b_o$. Then $A := \alpha^4 + \varepsilon\alpha^2\beta^2 + \beta^4$ is strongly convex if

$$1 + \varepsilon s^2 + s^4 > 0 \quad (|s| < b_o)$$

$$4 + 2\varepsilon\rho^2 + (4\varepsilon + 12\rho^2 - \rho^2\varepsilon^2)s^2 + (2\rho^2\varepsilon - 8 + 3\varepsilon^2)s^4 > 0 \quad (|s| \le \rho < b_o).$$

In this case, the fourth root metric $F = A^{1/4}$ is a special (α, β)-metric. Kim and Park [5] characterized locally Minkowskian fourth root metrics in the form $F = \sqrt{c_1\alpha^4 + c_2\alpha^2\beta^2 + c_3\beta^4}$. In [12], the second author classified all projectively flat (α, β)-metrics.

3 Projectively Flat Fourth Root Metrics

In this section, we will discuss projectively flat fourth root metrics $F = A^{1/4}$ on an open subset $\mathcal{U} \subset R^n$. For simplicity, we let $A_0 := A_{x^m}y^m$, $A_{00} := A_{x^px^q}y^py^q$. First we have the following.

Lemma 3.1 *Let $F = A^{1/4}$ be a fourth root metric on an open subset $\mathcal{U} \subset R^n$. It is projectively flat if and only if*

$$4A(A_{x^my^k}y^m - A_{x^k}) = 3A_0A_{y^k}. \tag{3.1}$$

Proof (3.1) follows from (2.1) immediately. ■

Proof of Theorem 1.1 Assume that F is projectively flat. Since A is irreducible and $\deg(A_{y^k}) = 3$, by (3.1), one can conclude that A_0 is divisible by A, that is, there is a 1-form η such that $A_0 = 8\eta A$. Thus, the spray coefficients $G^i = Py^i$ are given by

$$P = \frac{A_0}{8A} = \eta.$$

We see that $G^i = \eta y^i$ are quadratic in y. Therefore F is a Berwald metric.

Assume that $n \ge 3$. By Theorem 2.1, if $\mathbf{K} \ne 0$, then F is Riemannian. Thus A is a perfect square of a Riemannian metric. This contradicts our assumption. Thus $\mathbf{K} = 0$. That is, F is a Berwald metric with $\mathbf{K} = 0$. Therefore, F is locally Minkowskian. ■

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press

4 Generalized Fourth Root Metrics

In this section, we shall consider generalized fourth root metrics $F = (A^{1/2} + B)^{1/2}$, where $A = a_{ijkl}(x)y^i y^j y^k y^l$ and $B = b_{ij}(x)y^i y^j$. We denote A_0 and A_{00} as above and let $B_0 := B_{x^m} y^m$ and $B_{00} = B_{x^p x^q} y^p y^q$.

Let

$$X_k := \frac{BA_{y^k} - 2AB_{y^k}}{2(B^2 - A)}, \quad Y_k := \frac{2BB_{y^k} - A_{y^k}}{4(B^2 - A)}.$$

Lemma 4.1 *Let $F = (A^{1/2} + B)^{1/2}$ be a generalized fourth root metric on a domain in R^n. Assume that $A = a_{ijkl}(x)y^i y^j y^k y^l$ is irreducible. A generalized fourth root metric $F = (A^{1/2} + B)^{1/2}$ is projectively flat on an open domain in R^n if and only if*

$$A_{x^m y^k} y^m - A_{x^k} = \frac{A_{y^k}}{2A} A_0 + Y_k A_0 + X_k B_0 \tag{4.1}$$

$$B_{x^m y^k} y^m - B_{x^k} = \frac{1}{4A} X_k A_0 + Y_k B_0. \tag{4.2}$$

Proof By a direct computation, we get

$$F_{x^l} = \frac{1}{4}(A^{1/2} + B)^{-1/2} A^{-1/2}(A_{x^l} + 2A^{1/2} B_{x^l})$$

$$F_{x^k y^l} y^k = -\frac{1}{16}(A^{1/2} + B)^{-3/2} A^{-1}(A_{y^l} + 2A^{1/2} B_{y^l})(A_0 + 2A^{1/2} B_0) + \frac{1}{8}(A^{1/2} + B)^{-1/2} A^{-3/2}(-A_{y^l} A_0 + 2AA_{x^m y^l} y^m + 4A^{3/2} B_{x^m y^l} y^m).$$

Then using the fact that A is not a perfect square of a quadratic form and the equation $F_{x^k y^l} y^k - F_{x^l} = 0$, we obtain (4.1) and (4.2). ∎

Proposition 4.2 *Assume that A is irreducible and $B \neq 0$. Then $F = (A^{1/2} + B)^{1/2}$ is projectively flat if and only if there is a 1-form η such that*

$$4(A - B^2)\{B_{x^m y^k} y^k - B_{x^k} - 2\eta B_{y^k}\} = (B_0 - 4\eta B)(A - B^2)_{y^k}. \tag{4.3}$$

$$2(A_{x^k} - 4\eta_{y^k} A - \eta A_{y^k}) = 2B\{B_{x^m y^k} y^k - B_{x^k} - 2\eta B_{y^k}\} - (B_0 - 4\eta B)B_{y^k}. \tag{4.4}$$

Proof Assume that F is projectively flat. Then A and B satisfy (4.1) and (4.2), respectively. It follows from (4.1) that

$$M_k A = (2ABB_{y^k} + 2B^2 A_{y^k} - 3AA_{y^k})A_0, \tag{4.5}$$

where $M_k := 4(B^2 - A)(A_{x^m y^k} y^m - A_{x^k}) - 2(BA_{y^k} - 2AB_{y^k})B_0$.

First we claim that A_0 is divisible by A. If this is not true, then from (4.5), one can see that $2ABB_{y^k} + 2B^2 A_{y^k} - 3AA_{y^k}$ is divisible by A since A is irreducible. That is, there is a 3-form ω_k such that $2ABB_{y^k} + 2B^2 A_{y^k} - 3AA_{y^k} = \omega_k A$. Rewriting the above equation as follows, $2B^2 A_{y^k} = (\omega_k + 3A_{y^k} - 2BB_{y^k})A$. The right-hand side is divisible

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press

by B^2. This is impossible since A is irreducible and $\deg(\omega_k + 3A_{y^k} - 2BB_{y^k}) = 3$. This proves our claim.

Therefore A_0 is divisible by A. There is a 1-form η such that $A_0 = 8\eta A$. Rewrite (4.2) as follows:

$$B_{x^m y^k} y^k - B_{x^k} = \frac{A_0}{4A} B_{y^k} - \frac{BA_0 - 2AB_0}{2A} Y_k. \tag{4.6}$$

Plugging $A_0 = 8\eta A$ into (4.6) yields (4.3).

Rewrite (4.1) as follows:

$$A_{x^m y^k} y^m - A_{x^k} = \frac{A_0}{2A} A_{y^k} + B_0 B_{y^k} + (A_0 - 2BB_0) Y_k. \tag{4.7}$$

Plugging $A_0 = 8\eta A$ into (4.7) yields

$$2(A-B^2)\{2(A_{x^k} - 4\eta_{y^k} A - \eta A_{y^k}) + (B_0 - 4\eta B) B_{y^k}\} = B(B_0 - 4\eta B)(A - B^2)_{y^k}. \tag{4.8}$$

Then (4.4) follows from (4.3) and (4.8).

It is easy to prove that the converse is true, too. ■

Proof of Theorem 1.2 Assume that $F = (A^{1/2} + B)^{1/2}$ is projectively flat. Then by Proposition 4.2, (4.3) and (4.4) hold.

The right-hand side of 4.3 is divisible by $A - B^2$. Since $A - B^2$ is irreducible and $\deg[(A - B^2)_{y^k}] = 3$, we conclude $B_0 = 4\eta B$.Contracting (4.4) with y^k yields

$$A_0 = 8\eta A.$$

By a direct computation, we get the spray coefficients $G^i = Py^i$ with

$$P = \frac{A_0 + 2A^{1/2} B_0}{8A^{1/2}(A^{1/2} + B)} = \frac{8\eta A + 8\eta A^{1/2} B}{8A^{1/2}(A^{1/2} + B)} = \eta.$$

Thus F is a Berwald metric.

In dimension $n \geq 3$, by Theorem 2.1, every Berwald metric of non-zero scalar flag curvature must be Riemannian. Then $B = 0$ and A is a perfect square of a Riemannian metric. This contradicts our assumption. Thus $\mathbf{K} = 0$ and F is locally Minkowskian. ■

References

[1] N. Brinzei, *Projective relations for m-th root metric spaces.* J. Calcutta Math. Soc. **5**(2009), no. 1-2, 21–35.

[2] L. Berwald, *Parallelúbertragung in allgemeinen Räumen*, Atti Congr. Intern. Mat. Bologna **4**(1928), 263–270.

[3] ———, *Über die n-dimensionalen Geometrien konstanter Krümmung, in denen die Geraden die kürzesten sind.* Math. Z. **30**(1929), 449–469. doi:10.1007/BF01187782

[4] ———, *On Finsler and Cartan geometries. III.Two-dimensional Finsler spaces with vectilinear extremals.* Ann. of Math. **42**(1941), 84–112. doi:10.2307/1968989

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press

[5] B. Kim and H. Park, *The m-th root Finsler metrics admitting (α, β)-types.* Bull. Korean Math. Soc. **41**(2004), no. 1, 45–52. doi:10.4134/BKMS.2004.41.1.045
[6] B. Li and Z. Shen, *On a class of projectively flat Finsler metrics with constant flag curvature.* Internat. J. Math. **18**(2007), no. 7, 749–760. doi:10.1142/S0129167X07004291
[7] M. Matsumoto, *Theory of Finsler spaces with m-th root metric. II.* Publ. Math. Debrecen **49**(1996) no. 1-2, 135–155.
[8] M. Matsumoto and K. Okubo, *Theory of Finsler spaces with mth root metric: Connecions and main scalars.* Tensor (N.S.) **56**(1995), no. 1, 93–104.
[9] S. Numata, *On Landsberg spaces of scalar curvature.* J. Korea Math. Soc. **12**(1975), no. 2, 97–100.
[10] D. G. Pavlov (ed.), *Space-Time Structure. Collected papers.* TETRU, Moscow, 2006.
[11] Z. Shen, *Projectively flat Finsler metrics of constant flag curvature.* Trans. Amer. Math. Soc. **355**(2003), no. 4, 1713–1728. doi:10.1090/S0002-9947-02-03216-6
[12] ———, *On projectively flat (α,β)-metrics.* Canad. Math. Bull. **52**(2009), no. 1, 132–144. doi:10.4153/CMB-2009-016-2
[13] H. Shimada, *On Finsler spaces with the metric $L = \sqrt[m]{a_{i_1 i_2 \cdots i_m} y^{i_1} y^{i_2} \cdots y^{i_m}}$*, Tensor (N.S.) **33**(1979), 365-372.

Department of Mathematics, Ningbo University, Ningbo, Zhejiang Province 315211, P.R. China
e-mail: libenling@nbu.edu.cn

Center of Mathematical Sciences, Zhejiang University, Hangzhou, Zhejiang Province 310027, P.R. China

and

Department of Mathematical Sciences, Indiana University Purdue University Indianapolis (IUPUI), 402 N. Blackford Street, Indianapolis, IN 46202-3216, USA
e-mail: zshen@math.iupui.edu

https://doi.org/10.4153/CMB-2011-056-5 Published online by Cambridge University Press