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Correlation-induced self-focusing and self-shaping effect of a partially coherent beam

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Abstract

A new specially correlated partially coherent beam named nonuniform multi-Gaussian correlated (NMGC) partially coherent beam is introduced. The correlation functions of such beam in x and y directions are different from each other, i.e., nonuniform correlation function in one direction and multi-Gaussian correlated Schell-model function in the other direction. The propagation properties of an NMGC partially coherent beam in free pace are demonstrated, and we find that the intensity distribution of such beam exhibits self-focusing and self-shifting effect in one direction and self-shaping effect in the other direction on propagation. The correlation-induced self-focusing and self-shaping effect will be useful in some applications, where the high power and shaped laser is required, such as material thermal processing and laser carving.

Keywords: correlation function; partially coherent beam; self-focusing effect; self-shaping effect

1. Introduction

Recently, modulation of the correlation functions of partially coherent beams has been studied widely[1–9]. Correlation function provides a new freedom for manipulating optical field, e.g., creation of light beam with prescribed far-field intensity distribution. Various partially coherent beams with prescribed correlation functions have been investigated in both theory and experiment since Gori and collaborators derived the sufficient condition for constructing the genuine correlation functions[2, 3]. Recent studies have shown that specially correlated partially coherent beams display many unique but interesting properties, e.g., self-splitting effect appears in a Hermite–Gaussian correlated Schell-model beam on propagation[9–11], partially coherent beams with multi-Gaussian, Bessel–Gaussian and cosine-Gaussian correlated Schell-model functions exhibit prescribed far-field intensity distributions[12–15], Laguerre–Gaussian correlated Schell-model beam produces an optical cage near the focal plane[16, 17]. Due to those extraordinary properties, specially correlated partially coherent beams are useful in some applications, such as optical imaging, particle trapping, atom guiding and free-space optical communications. We presented a review on specially correlated partially coherent beams in Ref. [4].

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The nonuniform correlated partially coherent (NCPC) beam was first introduced in Ref. [18], and such beam exhibits self-focusing and self-shifting effect in free-space propagation. Propagation properties of an NCPC beam in the uniformly correlated media were studied in Ref. [19]. The evolution properties of the scintillation of an NCPC beam in atmospheric turbulence was illustrated in Ref. [20], and it was found that an NCPC beam can not only have lower scintillation but also higher intensity than a conventional Gaussian–Schell-model (GSM) beam, and it will be useful for long-distance free-space optical communications. Recently, the scalar NCPC beam was extended to the electromagnetic case[21] and the statistical properties of such beam in turbulent atmosphere were also investigated[22]. Multi-Gaussian correlated Schell-model (MGCSM) beam introduced in Ref. [12] displays far-field flat-topped beam profile although it has a Gaussian beam profile in the source plane[12, 23]. Both theoretical and experimental results have demonstrated that an MGCSM beam exhibits lower scintillation than a GSM beam in turbulence[24–26].

All above-mentioned specially correlated partially coherent beams have the same correlation functions in x and y directions. However, in some specific applications, such as trapping and guiding anisotropic particles, beams with different intensity distributions in x and y directions are required. In this paper, we introduce a new specially correlated partially coherent beam whose correlation functions in x and y directions are different, which is named nonuniform multi-Gaussian correlated (NMGC) partially coherent beam. The evolution properties of the intensity distribution of this new beam in free space are illustrated numerically, and we find that the NMGC partially coherent beam exhibits self-focusing and self-shifting in one direction and self-shaping in the other direction. The interesting properties of this new beam may find some applications, e.g., material thermal processing and laser carving, where a laser with high power and prescribed beam shape is required.

2. Basic theory for constructing specially correlated partially coherent beam

In the space-time domain, partially coherent beam is characterized by the mutual intensity, which is restricted by the nonnegative definiteness requirement and can be expressed in the following form[1–3],

$$J_0(\mathbf{r_1}, \mathbf{r_2}) = \int I(\mathbf{v}) H^*(\mathbf{r_1}, \mathbf{v}) H^*(\mathbf{r_2}, \mathbf{v})\, d^2\mathbf{v}, \tag{1}$$

where I is a function that satisfies the nonnegative condition and H is an arbitrary function. Various specially correlated partially coherent beams can be defined through choosing suitable functions of H and I.

In this paper, we consider a more general case. The correlation functions in x and y directions are different from each other, and the mutual intensity of this new beam is given as $J_0(\mathbf{r_1}, \mathbf{r_2}) = J_{0x}(x_1, x_2) J_{0y}(y_1, y_2)$. Then, to satisfy the nonnegative definiteness requirement, $J_0(\mathbf{r_1}, \mathbf{r_2})$ can be expressed in the following form,

$$\begin{aligned} J_0(\mathbf{r_1}, \mathbf{r_2}) = &\int I_x(u) H_x^*(x_1, u) H_x^*(x_2, u)\, du \\ &\times \int I_y(v) H_y^*(y_1, v) H_y^*(y_2, v)\, dv. \end{aligned} \tag{2}$$

Under the condition of $I_x = I_y$ and $H_x = H_y$, Equation (2) reduces to Equation (1).

After some operation, Equation (2) can be rewritten as

$$\begin{aligned} J_0(\mathbf{r_1}, \mathbf{r_2}) = &\int \Gamma_x(u_1, u_2) H_x^*(x_1, u_1) H_x^*(x_2, u_2)\, du_1\, du_2 \\ &\times \int \Gamma_y(v_1, v_2) H_y^*(y_1, v_1) H_y^*(y_2, v_2)\, dv_1\, dv_2, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \Gamma_x(u_1, u_2) &= \sqrt{I_x(u_1) I_x(u_2)}\,\delta(u_1 - u_2), \\ \Gamma_y(v_1, v_2) &= \sqrt{I_y(v_1) I_y(v_2)}\,\delta(v_1 - v_2). \end{aligned} \tag{4}$$

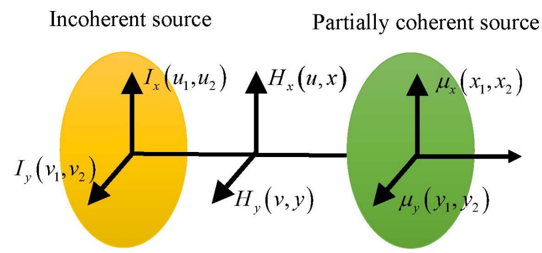

Figure 1. Schematic for forming a partially coherent beam whose correlation functions in x and y directions are different through propagation.

One sees from Equations (3) and (4) that $J_0(\mathbf{r_1}, \mathbf{r_2})$ and $\Gamma_x(u_1, u_2)\Gamma_y(v_1, v_2)$ represents the output and input mutual intensities, respectively. H_x and H_y denote the response functions of the optical paths in x and y directions between two planes, respectively. $\Gamma_x(u_1, u_2)$ and $\Gamma_y(v_1, v_2)$ denote the mutual intensities of the incoherent source in x and y directions, respectively. Thus, one can generate a specially correlated partially coherent beam whose correlation functions in x and y directions are different by varying H_x, H_y, I_x and I_y through propagation (see Figure 1).

3. Nonuniform multi-Gaussian correlated partially coherent beam

In this section, we introduce a typical kind of partially coherent beam whose correlation functions in x and y directions are different named NMGC partially coherent beam, and explore its free-space propagation properties.

The mutual intensity of a partially coherent beam whose correlation functions in x and y directions are different can be expressed as

$$J_0(\mathbf{r_1}, \mathbf{r_2}) = \sqrt{I(\mathbf{r}_1) I(\mathbf{r_2})}\,\mu_x(x_1 - x_2)\mu_y(y_1 - y_2), \tag{5}$$

where I denotes the intensity distribution, μ_x and μ_y are the correlation functions in x and y directions, respectively. Now we introduce a new specially correlated partially coherent beam which has nonuniform correlation function in one direction and MGCSM function in the other direction, and we call this new beam as NMGC partially coherent beam. The intensity and correlation functions of an NMGC partially coherent beam at $z = 0$ are written as

$$I(\mathbf{r}) = \exp\left(-\frac{\mathbf{r}^2}{2\sigma_0^2}\right), \tag{6}$$

$$\mu_x(x_1, x_2) = \exp\left\{-\frac{[(x_2 - x_0)^2 - (x_1 - x_0)^2]^2}{w_x^4}\right\}, \tag{7}$$

$$\mu_y(y_1, y_2) = \frac{1}{C_0}\sum_{m=1}^{M}\frac{(-1)^{m-1}}{\sqrt{m}}\binom{M}{m}\exp\left[-\frac{(y_1 - y_2)^2}{m w_y^2}\right], \tag{8}$$

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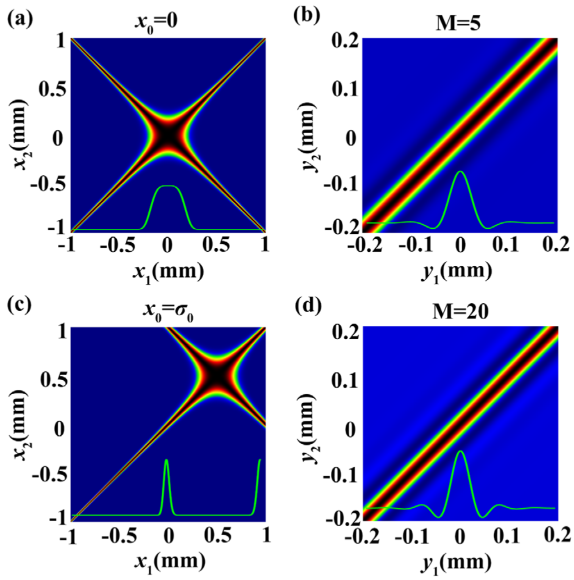

Figure 2. Density plot of the correlation functions of an NMGC partially coherent beam for various x_0 and M in x and y directions.

$$C_0 = \sum_{m=1}^{M} \frac{(-1)^{m-1}}{\sqrt{m}} \binom{M}{m}, \tag{9}$$

where σ_0 represents the beam width, w_x and w_y denote the coherence widths along x and y directions, respectively, x_0 is a real constant and $\binom{M}{m}$ stand for binomial coefficients.

Figure 2 shows the density plot of the correlation functions of an NMGC partially coherent beam for various x_0 and M in x and y directions. The green lines in Figure 2 denote the cross-lines with $x_2 = 0$ and $y_2 = 0$, respectively. During calculation, we set $w_x = w_y = 0.2$ mm, and the values of other parameters are attached in Figure 2. We see from Figure 2 that the correlation function of the NMGC partially coherent beam in x direction is totally different from that in y direction, i.e., the correlation function in x direction is locally varying, but the correlation function in y direction is independent of the lateral coordinate. Comparing Figures 2(a) and 2(c), we find that the highest value of the correlation function shifts to the position x_0. Comparing Figures 2(b) and 2(d), we find that the number of the lobes of correlation function in y direction increases as the beam order M increases. Due to the special distribution of the correlation functions, the NMGC partially coherent beam exhibits self-focusing and self-shaping effect during propagation as shown later.

Paraxial propagation of the mutual intensity of an NMGC partially coherent beam through a stigmatic ABCD optical system can be studied by use of the generalized Collins formula[27],

$$\begin{aligned} J(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= \frac{1}{(\lambda B)^2} \exp\left[-\frac{ikD}{2B}(\boldsymbol{\rho}_1^2 - \boldsymbol{\rho}_2^2)\right] \\ &\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J_0(\mathbf{r_1}, \mathbf{r_2}) \\ &\times \exp\left[-\frac{ikA}{2B}(\mathbf{r}_1^2 - \mathbf{r}_2^2)\right] \\ &\times \left[\frac{ik}{B}(\mathbf{r}_1 \cdot \boldsymbol{\rho}_1 - \mathbf{r}_2 \cdot \boldsymbol{\rho}_2)\right] d^2\mathbf{r}_1\, d^2\mathbf{r}_2, \end{aligned} \tag{10}$$

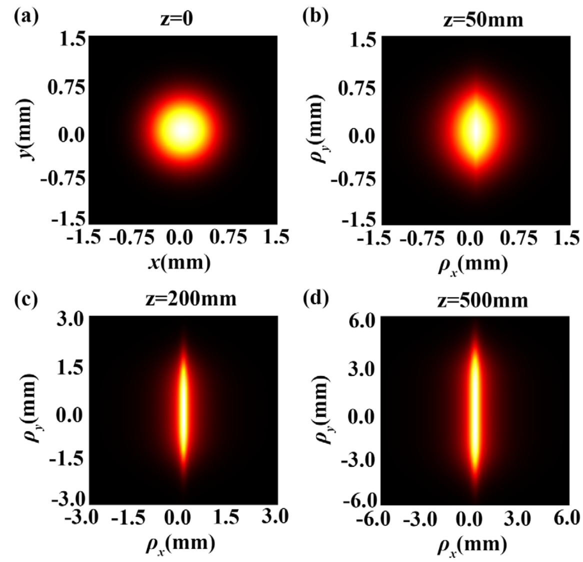

Figure 3. Normalized intensity distribution of an NMGC partially coherent beam on propagation in free space with $x_0 = 0$ and $M = 5$.

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are two arbitrary position vectors in the output plane, $k = 2\pi/\lambda$ and λ is the wavelength, A, B, C, and D are the elements of the optical system transfer matrix. The intensity distribution in the output plane is obtained as

$$I(\boldsymbol{\rho}, z) = J(\boldsymbol{\rho}, \boldsymbol{\rho}). \tag{11}$$

Let us consider the propagation of an NMGC partially coherent beam in free space. The elements of the transfer matrix between the source plane (at $z = 0$) and output plane (at z) in free space are given as

$$A = 1, \quad B = z, \quad C = 0, \quad D = 1. \tag{12}$$

By substituting Equations (5)–(9), (11) and (12) into Equation (10), we can easily study the evolution properties of the intensity distribution of an NMGC partially coherent beam on propagation in free space numerically. Figures 3 and 4 show the normalized intensity distribution of an NMGC partially coherent beam on propagation in free space for various x_0 and M. During calculation, σ_0 is set as 0.5 mm, w_x and w_y are equal to 0.2 mm. One finds from

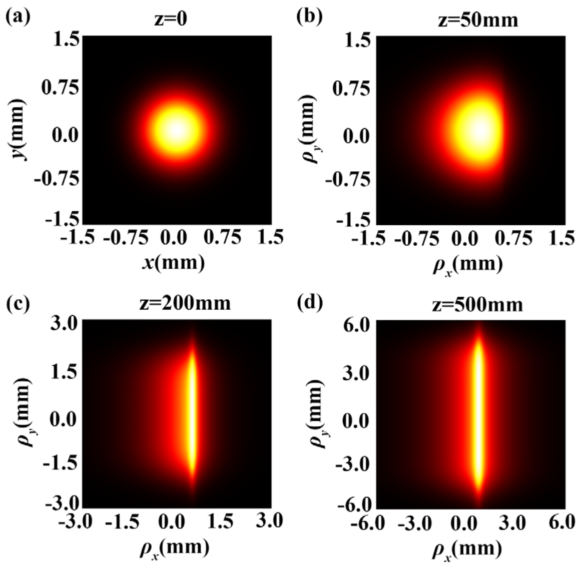

Figure 4. Normalized intensity distribution of an NMGC partially coherent beam on propagation in free space with $x_0 = \sigma_0$ and $M = 20$.

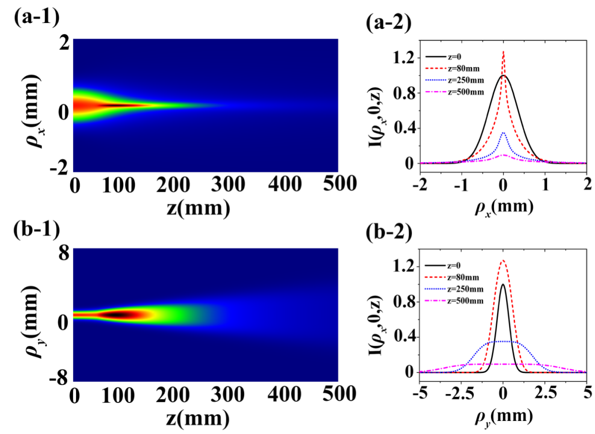

Figure 5. Normalized intensity distribution of an NMGC partially coherent beam on propagation in free space (a-1) in ρ_x–z plane, and (b-1) in ρ_y–z plane with $x_0 = 0$ and $M = 5$ and the corresponding cross-line.

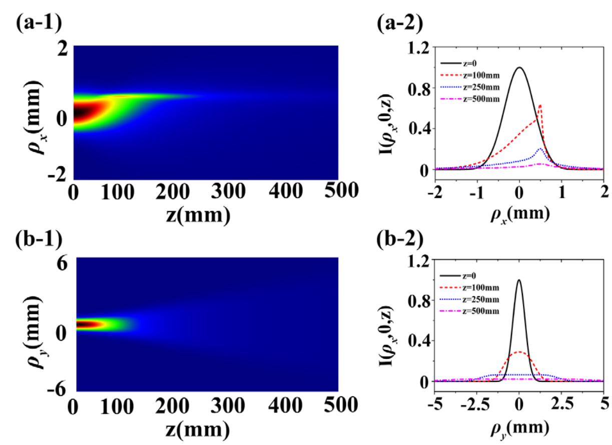

Figure 6. Normalized intensity distribution of an NMGC partially coherent beam on propagation in free space (a-1) in $\rho_x = z$ plane, and (b-1) in $\rho_y = z$ plane with $x_0 = \sigma_0$ and $M = 20$ and the corresponding cross-line.

Figures 3 and 4 that the NMGC partially coherent beam has a Gaussian beam profile in the source plane, and the Gaussian beam profile generally evolves into needle-like beam profile on propagation, and the length of the needle increases as the beam order M increases. When $x_0 = \sigma_0$, the position of maximum intensity shifts during propagation. Above propagation properties of an NMGC partially coherent beam in free space are also called self-shaping effect. Thus, modulating the correlation functions of x and y directions in different ways provides an effective method for manipulating the optical field, and it will be useful in various beam shaping applications.

In order to observe the correlation-induced self-focusing effect, we calculate in Figures 5 and 6 the normalized intensity distribution of an NMGC partially coherent beam on propagation in free space in ρ_x–z plane and ρ_y–z plane for various x_0 and M and the corresponding cross-line. We find from Figure 5 that the NMGC partially coherent beam exhibits self-focusing effect in ρ_x–z plane, i.e., the intensity at $z = 80$ mm is narrower and more powerful than that in the source plane, and exhibits self-shaping effect in ρ_y–z plane, i.e., the intensity distribution evolves from a Gaussian beam profile to a flat-topped beam profile during propagation. From Figure 6, we see that the position of the maximum intensity shifts in ρ_x–z plane under the condition of $x_0 = \sigma_0$, and this phenomenon is called self-shifting[18].

4. Conclusions

As a summary, we have introduced a new specially correlated partially coherent beam named NMGC partially coherent beam whose correlation functions in x and y directions are totally different, and have outlined briefly the nonnegative definiteness for constructing the mutual intensity of this new beam. The propagation properties of this new beam in free space have been demonstrated numerically. Our results have shown that an NMGC partially coherent displays self-focusing and self-shifting effect in one direction and self-shaping effect in the other direction, and a needle-like beam profile can be formed on propagation. Our results suggest a new approach for optical field manipulation and will be useful in various beam shaping applications. The correlation-induced self-focusing and self-shaping effect will be useful in some applications where the high power and shaped laser is required.

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