ON THE SOLUTION OF THE EXTERIOR BOUNDARY VALUE PROBLEM WITH THE AID OF SERIES

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ABSTRACT

The exterior gravitational field depending on the Earth's non-sphericity is usually determined from the analysis of satellite data or by the solution of the exterior boundary value problem. In the latter case some integral equations are solved which correlate the exterior potential with the known vector of gravity and the shape of the Earth's surface (molodensky problem). In order to carry out the integration the small parameter method is applied. As a result, all the quantities which involve the equations should be expanded in powers of a certain small parameter, among these being the heights of the Earth's surface points as well as the inclination α of the Earth's physical surface. Since the angle α can be significant, especially in mountains, and in fact does not depend on any small parameter then the solution of integral equations is possible only for the Earth's surface which is smoothed enough.

Different authors expressed the desire to represent the Earth's potential V by a unified mathematical model valid both on the Earth's surface and outside the Earth. The widely accepted form of such a kind is the expansion in spherical harmonics

$$V = fMr^{-1} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n P_n^m(\sin\psi) \, [C_{nm} \cos m\lambda + S_{nm} \sin m\lambda] \qquad (1)$$

where f, M, R are the gravitational constants, the Earth's mass and its mean radius and r, ψ, λ represent the spherical coordinates of the point considered. The series (1) is known to converge outside the sphere enclosing the Earth. But for the possibility of applying the terrestrial gravity data in the evaluation of the coefficients C_{nm} and S_{nm} this expansion is extended up to the Earth's surface. The problem of the convergence of the series (1) on the Earth's surface has not yet been solved but even if it does converge another problem emerges whether or not the sum of the series tends to the potential V.

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In the present paper, spherical harmonic expansion is developed which generalizes (1). It converges and represents the potential both on the Earth's surface and in the outer space. While the series (I) is based on the expansion

$$\Delta^{-1} = r^{-1} \sum_{n=0}^{\infty} \left(\frac{r_1}{r}\right)^n P_n(\cos i) \tag{2}$$

the new one is derived as a result of the analytical continuation of the series (2) which in the real domain corresponds to

$$\Delta^{-1} = 2 \sum_{n=0}^{\infty} \frac{(4rr_1)^n}{[r + r_1 + |r-r_1|]^{2n+1}} P_n(\cos i)$$

The exact formula in comparison with (1) has some additional terms with the coefficients of different structure than those corresponding to (1), that is

$$V = fMr^{-1} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n P_n^m(\sin\psi) \, [C^*_{nm}\cos m\lambda + S^*_{nm}\sin m\lambda] \tag{3}$$

where

$$C^*_{nm} = C_{nm} + \Delta C_{nm}(r), \quad S^*_{nm} = S_{nm} + \Delta S_{nm}(r) ,$$

$$\Delta C_{nm}(r) = \sum_{k=0}^{\infty} C^{(k)}_{nm} T^*_k \left(\frac{R_i}{r}\right), \quad \Delta S_{nm} = \sum_{k=0}^{\infty} S^{(k)}_{nm} T^*_k \left(\frac{R_i}{r}\right) .$$

Here $T^*_k(\kappa)$ are the shifted Chebyshev polynomials and R_i means the shortest distance of the Earth's surface from the origin of coordinates.

The additional terms $\Delta C_{nm}(r)$ and $\Delta S_{nm}(r)$ represent the errors of the model (I) on the Earth's surface. They have the order of the square of the Earth's flattening and vanish outside the enveloping sphere. Thus no satellite observations can in principal provide these quantities. The values of the complements $\Delta C_{nm}(r)$ and $\Delta S_{nm}(r)$ are the larger, the lower is the point on the Earth's surface where the gravity is measured.

The series (3) may be applied for the well-grounded evaluation of the potential through a combination of terrestrial and satellite data.

All the consideration remains obviously true for the potential of any other planet in the solar system.