An invitation to combinatorics by Shariar Shahriari, pp 612, £36.99 (hard), ISBN 978-1-10847-654-6, Cambridge University Press (2021)

Combinatorial analysis is 'the branch of mathematics concerned with the theory of enumeration, or combinations and permutations, in order to solve problems about the possibility of constructing arrangements of objects which satisfy specified conditions' (Collins dictionary, accessed online 25/02/2023). As a mathematical discipline, it is relatively young—its origins are usually attributed to G.W. Leibniz's *Dissertatio de arte combinatoria* from 1666/1690—but with the ascent of discrete mathematics, there has been substantial progress since the 1950s. Teaching combinatorics can be difficult, and the title of Leibniz's tract highlights the problem: Combinatorics is more of an art (and a mindset) than a science; it is best taught at school and as early as possible. But, of course, combinatorics is part of every undergraduate curriculum and Shahriari's new textbook addresses this need. After a brief introduction on tools from discrete mathematics, a glimpse into Ramsey theory and the pigeon-hole principle (Chapters 1 & 2, 75 pages), the main part (Chapters 3–9, 280 pages) of the book is devoted to explaining the 'twelve-fold way' (the name is due to Joel Spencer, the classification system to Gian-Carlo Rota), that covers all enumeration problems between finite sets, placing (un-)distinguishable balls into (un-)distinguishable boxes with (or without) restrictions on the number of balls in any box; in fact, Shahriari expands the twelvefold way into a table with 16 fields. The last two chapters (160 pages) cover some applications in graph theory, posets, matchings and Boolean lattices. There are over 1200 exercises, of which about 240 have hints, 200 come with short answers, and 100 are fully solved. There are also more demanding exercises ('collaborative mini-projects' and 'scaffolded problems') for deeper engagement with the subject and, in the later chapters, collections of (open) problems to initiate undergraduate research. The book is deliberately written in classroom style, in the hope that this makes it more interesting and accessible to students. This, however, comes at a price: the pace is slow, sometimes circumstantial and not very systematic. It lacks the conciseness of Stanley (*Enumerative Combinatorics*, Wadsworth 1986, Cambridge 1997) or Riordan (*An Introduction to Combinatorial Analysis*, Princeton 1978), and the elegance and spirit of Bóna (A walk through combinatorics, World Scientific 2005). The *art* in combinatorics has had to make way for *didactical finesse*.

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Modern mathematical logic by Joseph Mileti, pp. 550, £49.99 (hard), ISBN 978-1-10883-314-1, Cambridge University Press (2022)

This is a new, quite sophisticated, introduction to mathematical logic at the upper undergraduate/beginning graduate level. It is nicely written, adaptable to both semester and year-long courses, and organizationally versatile, and it contains more applications to other branches of mathematics than is typical in books at this level. However, although the author makes a considerable effort to motivate the discussion, it is a demanding book, one that may be too demanding for undergraduates at an 'average' university.

I think of this book as comprising five parts. The first part, consisting of the first two chapters, is prefatory. Chapters 3 to 6 constitute a 'core' course in mathematical logic: propositional and first-order logic and their basic theorems (completeness, compactness). From here the book branches off into three largely independent parts covering model theory (chapters 7 and 10), axiomatic set theory (chapters 8 and 9), and computability and undecidability (chapters 11 and 12). I say 'largely independent' because chapter 10 uses set theory to discuss aspects of model theory such as the Lowenheim-Skolem theorem (previously proved for countable structures, now extended to uncountable ones) and ultraproducts.

The sophistication of the book derives largely from the wealth of material included and, especially, the extensive account of applications to other branches of mathematics. For example, the reader will see here discussion of topics from abstract and linear algebra (e.g. vector spaces and (possibly infinite) bases, ordered abelian groups, Cauchy's theorem on elements of order p in a group, algebraically closed fields, the Jacobson radical, transcendence bases), analysis (open and closed sets, perfect sets) and discrete mathematics (partially ordered sets and random graphs). To appreciate this material fully, of course, the reader will have to have some considerable background in these subjects, particularly abstract algebra. An Appendix ('Mathematical Background') attempts to cover the necessary background, but is fairly short (12 pages) and so, as is typically the case, is no substitute for some real experience in these areas. An instructor who has concerns about the extent of the background knowledge of his or her students may wish to use a somewhat gentler text, such as Enderton's *A Mathematical Introduction to Logic* or Leary's *A Friendly Introduction to Mathematical Logic*.

This text begins with a nice introductory chapter that discusses the nature of the subject, mathematical language, semantics and syntax, etc. So far, so good. However, any momentum that may have developed in this opening chapter then comes to a screeching halt in chapter 2, which provides a general framework for induction and recursion. This chapter operates at a fairly high level of generality, and because it does so quite early in the book, runs the risk of exposing students to a level of sophistication for which they may not be quite ready. The author himself recognizes the potential problem here. He writes: "I understand that many [readers] can find that level of detail and generality a bit dry" and, for that reason, when he teaches the course, "I cover that chapter quickly, and leave most of the details for the curious and engaged reader." In view of the fact that the material covered in this chapter has strong intuitive appeal, a natural question to ask at this point is why the author didn't relegate this material to an appendix, while discussing this material at a more intuitive level, as he does in class.

In chapters 3 to 6 the general framework for induction and recursion that was developed in chapter 2 is used repeatedly (for example, to give a fairly technical definition of free and bound variables); personally, if I were teaching a course in logic, I'd probably rely more on the students' intuition for these concepts. Another point that deserves mention is Mileti's approach to the rules of inference for the propositional calculus. Mileti gives a number of these rules and then goes on to prove the completeness theorem, namely that any tautology can be derived from these rules. This is not the way that, for example, Enderton or Leary do it; they just assume all tautologies as axioms and use modus ponens as the sole rule of inference. Mileti's approach has the advantage of helping prepare the students for the completeness theorem for first order logic; the approach taken by Enderton and Leary, on the other hand, seems simpler and saves time for more interesting things down the road.

After finishing chapter 6, an instructor can, time permitting, branch off into one or more special topics. Model theory is the subject of chapter 7, and it is here that we really see a difference between this book and its competitors. Mileti goes much more

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deeply than do Enderton or Leary into such things as quantifier elimination and nonstandard models of arithmetic and analysis, and he also provides some fairly sophisticated applications, including the Ax-Grothendieck theorem on polynomial maps.

Axiomatic set theory comprises two chapters and about 100 pages of text, or about one-fifth of the book. Most introductory logic textbooks don't cover this topic at all, so the presence of this material definitely enhances the versatility of this book. By and large the discussion here is clear and well written, but some choices made by the author puzzle me. Although, for example, he refers to the axiom of choice as 'infamous', he doesn't really give much reason why. Likewise, the Continuum Hypothesis receives rather short shrift, just a brief mention with one sentence pointing out that it can neither be proved nor refuted from the axioms of set theory. By contrast, perfect sets receive several pages of discussion, presumably so that the author can show how set theory can say things about the real numbers. This seems a rather large detour to me, and I can't help but feel that some of these pages would have been better spent, for example, discussing some of the interesting history of the Axiom of Choice.

The final part of the book covers completeness and decidability. Computable functions are defined in two ways, that are later shown to be equivalent: inductive generation and by means of unlimited register machines (similar to, but not the same as, Turing machines). Godel's two incompleteness theorems (for language of semirings) are proved.

In summary: this is an interesting and well-written text, but may be a bit too demanding for many undergraduates.

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50 mathematical ideas you really need to know by Tony Crilly, pp 209, £9.99 (paper), ISBN 978-1-52942-804-9, Quercus Publishing plc (2023)

This lovely little book provides a concise overview of some of the most important mathematical ideas developed over the centuries. Of course, it is a personal, subjective selection by the author who undertook a mammoth task in attempting to condense into 50 short vignettes some of these developments, but he has succeeded fairly well in his mission.

It deals with 50 short, digestible snippets about ideas involving counting, number theory, geometry, algebra, relativity, game theory, chaos, fractals, graph theory, probability, coding theory, non-Euclidean geometry, linear programming, Fermat's Last Theorem, and so on. Right at the end there is a short discussion of the Riemann Hypothesis. At the end of each section there is a short historical overview, and, where possible, the author also mentions some real world applications.

The book is not aimed at the specialist but rather at the general public, teachers, parents, and school students. The over-arching intention is to create interest in the broad scope and applicability of mathematics to many aspects of modern day life. It is hoped that some high school students will become intrigued by some of the topics, and become motivated to study mathematics further.

No advanced knowledge of mathematics is required, and the reading is easy and non-technical. It is therefore highly recommended as a school library asset or for use by a mathematics teacher or parent who wants to show their learners some of the