Properties of Matter, by F.C. Champion and N Davy. Third edition, Philosophical Library, New York, 1959. xvi + 334 pages. $10.00.

The distinguishing feature of this book lies in the thorough treatment of an assortment of topics which are only mentioned or even omitted from the usual general physics texts. The danger always inherent in such a selection is that in its random specialization it does not cater for any one class of scientist. However in this case, approximately one-third of the chapters are devoted to topics which can be construed as being of direct interest to geophysicists.

Among these might be mentioned the sections on gravity, elasticity, compresibility of solids and liquids, seismic waves, and statistics of errors in measurement. This latter chapter, and those on units and dimensions, capillarity, kinetic theory, osmotic pressure, diffusion and viscosity, as a group have less of a specialist interest, and contain many informative paragraphs not suggested by the book's title. A case in point is the comprehensive section on the production and measurement of high vacua. The chapter on units and dimensions is brief, but provides several interesting examples on the method of dimensional analysis.

The most widespread use for the work might well be as a useful supplement to reading provided by general texts for advanced students of physics.

Peter Schwerdtfeger, McGill University

Eléments d'histoire des mathématiques, by N. Bourbaki. Collection "Histoire de la pensée - IV" Hermann, Paris 1960. 276 pages. 18 NF.

Mathematicians will be pleased to have the historical notes of the "Eléments de Mathématique" collected in a small volume. In an accompanying circular the publisher introduces the work as follows. "Les données les plus importantes de l'histoire des mathématiques révélées par le plus grand mathématicien contemporain. Ces réflexions, qui n'ont pas la prétention de constituer une histoire complete des mathématiques, offrent aux mathématiciens et aux historiens des sciences un tableau remarquablement clair de l'histoire et du développement d'une partie des mathématiques. Le public cultivé y verra comment, depuis les Babyloniens jusqu'à nos jours, se sont affermies et amplifiées les notions de nombre, d'espace, de relations et de structure.

The book contains a really penetrating study of the historical development of a number of mathematical disciplines; not a "History of Mathematics", rather a collection of essays, in each of which "on retrace le développement" of a special branch: Foundations, Logic, Sets; The evolution of Algebra; Linear Algebra; Non-commutative Algebra; Topological Spaces; Exponentials and Logarithms; Infinitesimal Calculus; Topological Vector Spaces; Integration; and some others. Each essay can be read without regard to the earlier ones and reading will be a pure joy for most mathematicians and for anybody with a mathematical education of some depth. Only those parts of mathematics are covered which have been dealt with in the "Eléments de Mathématique"; thus no systematic treatment is given of the history of Differential Geometry, Algebraic Geometry, Calculus of Variations, Theory of Numbers, and others; to quote from the preface: "... ces lacunes deviennent-elles plus nombreuses et plus importantes quand on arrive à l' époque moderne Il va sans dire qu' il ne s' agit pas là d' omissions intentionelles; elles sont simplement dues au fait que les chapitres correspondants des "Eléments" n' ont pas encore été publiés. Enfin, le lecteur ne trouvera pratiquement dans ces Notes aucun renseignement biographique ou anecdotique sur les mathématiciens dont il est question; on a cherché surtout, pour chaque théorie, à faire apparaître aussi clairement que possible quelles en ont été les idées directrices et comment ces idées se sont développées et ont réagi les unes sur les autres".

Naturally it is not possible to give a detailed account of each of the 21 essays; but let us consider the one on "Quadratic Forms; Elementary Geometry", pp. 129-145. A systematic theory of quadratic forms begins only after the middle of the 18th century to satisfy the requirements of number theory, analysis, and mechanics. Quadratic forms have appeared, however, much earlier in euclidean geometry "dont elles forment l' armature"; thus, in antiquity: "... c' est le principal but de cette Note que de faire voir comment, très graduellement, les mathématiciens ont pris conscience des parentés entre questions d' aspect souvent très différent". Leaving aside the discovery by the Babylonians of the solution of a quadratic equation, the beginning is found in Pythagoras' theorem and its generalizations (Euclid, Ptolemy) and in the theory of conics (from Archimedes and Appollonius to Fermat and Descartes). Only with the development of three-dimensional analytic geometry the main problem appears: the reduction to principal axes. Still Euler was unable to prove the reality of the eigenvalues in the case $n = 3$, a fact proved for any n by Cauchy. About this time enters the idea of a group of motions, thus introducing the "golden age of geometry", i. e., the time between the publication of Monge' s "Géométrie descriptive" and Klein' s "Erlanger Programm". A number of geometrical ideas, mostly of earlier origin, are now made use of effectively: the infinite elements (Poncelet); the imaginary elements (Poncelet, Plücker, Chasles); point transformations and their composition; duality, angle and distance in projective geometry (Cayley, Laguerre); non-euclidean geometry; all in connection with quadratic forms. Soon follows the decline of synthetic geometry and the rise to power of analytic geometry (Bobillier, Plücker, Möbius) supported by the fundamental thesis of

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Klein's "Programm". Simultaneously the "age of algebra" begins with the papers of Cayley, Hamilton, Frobenius, and continues with ever increasing importance given to the abstract, up to the "sesquilinear forms" treated in one of the last issues of Bourbaki's "Eléments" from which this essay is the historical Note (cf. the review, this Bulletin 4, p.).

There is an abundance of interesting sidelights in these essays. Some of these may well be commonplace to the specialist or to the historian of mathematics; but some seem to be quite original, as for instance the remark that Grassmann, in essence, knew that every complete matrix is similar to a triangular matrix; (cf. p. 116); or, that the Greeks used a kind of coordinate system, ineffectively indeed, because of their lack of algebra (cf. p. 132). In the history of the real numbers (p. 157) attention may be drawn to the quotation of Plato's "Republic" (Book VII, para. 525, p. 293 in the Penguin Classics Translation by H. D. P. Lee) where "Platon se moque des calculateurs 'qui change l'unité pour de la menue monnaie' et nous dit que, là où ceux-ci divisant, les savants multiplient: ce qui veut dire que, par exemple, pour le mathématicien, l'égalité de deux rapports a/b et c/d se constate, non en divisant a par b et c par d, ce qui conduit en general à un calcul de fractions (c'est ainsi qu'auraient opéré aussi les Egyptiens ou les Babyloniens) mais en vérifiant que $a \cdot d = b \cdot c$; et autres faits sembables". It would not be difficult to enumerate many other equally interesting remarks, as well as references of interest for the more general reader, as for instance that concerning the number system of the Mayas in the short article "Numération; Analyse Combinatoire" (p. 65-67).

Extremely sound judgement seems to be displayed in places where up to recent times unity of opinion has not been reached, as for instance in the question of priority of the Calculus: "... les recherches les plus récentes, fondées sur l'analyse des manuscrits, ont-elles mis en évidence, d'une maniere qui semble irréfutable, un point que des querelles partisanes avaient quelque peu obscurci: c'est que, chaque fois que l'un des grand mathématiciens de cette époque a porté témoignage sur ses propres travaux, sur l'évolution de sa pensée, sur les influences qu'il a subies et celles qu'il n'a pas subies, il l'a fait d'une manière honnête et sincère, et en toute bonne foi; (footnote: Ceci s'applique par exemple à Torricelli et à Leibniz.); ces témoignages précieux, dont nous possédons un assez grand nombre peuvent donc être utilisés en toute confiance, et l'historien n'a pas à se transformer à leur egard en juge d'instruction. Au reste, la plupart des questions de priorité qu'on a soulévées sont tout à fait dépourvues de sens" (cf. p. 184). The basic ideas of the Calculus are traced carefully from Archimedes, whose Works, edited for the first time in 1544, had influence on Kepler, and "tous, de Fermat à Barrow, le citent à l'envi", up to Leibniz and Newton, the whole story based upon well-documented facts, with no such lapidary statements concerning the exclusive merit of one or the other, as are to be found even in quite recent books on History of Mathematics (cf. J.E. Hofmann, Geschichte der Mathematik II, pp. 62, 70; Berlin 1957, Sammlung Göschen 875; reviewer's quotation). The more recent developments in Analysis are covered in the subsequent essays, mainly "Espaces vectoriels topologiques" and "Intégration".

For every essential statement a reference is given to the literature, collected in the bibliography at the end of the book; 253 authors, many being represented by several items. Most historical references are of course to the standard works by Neugebauer, Tropfke, etc., and to the collected works of the great mathematicians of the past. But also outstanding papers of a great number of more recent and contemporary authors are referred to and in some cases discussed.

In conclusion: this is a book on the history of mathematics as the active as well as the teaching mathematician wants it; to the best knowledge of the reviewer, nothing similar has been available so far. If anything remains to be desired it is, that "the greatest mathematician of our time" should soon find it possible to complete his work by a second volume dealing with those topics that had to be omitted in the present volume.

H. Schwerdtfeger, McGill University

Calculus, by R.L. Jeffery. Third edition, University of Toronto Press, 1960. xii + 298 pp. $4.95.

There are many students who must learn calculus, who are indeed capable of using it effectively in their several sciences, but for whom a full treatment of ε's and δ's at the very beginning is too hard. In the middle of the twentieth century these students need something considerably less unrigorous than books like the famous "Calculus made easy": the present book bridges the gap. Although the initial stages are largely intuitive, rigour being postponed until just before the treatment of series (which is pointless if unrigorous), theorems are correctly stated: e.g. we read that a continuous function attains its bounds on a closed interval, the words "continuous" and "closed" not being omitted. Similarly the existence of the definite integral is stated (and later proved) for continuous functions: the student is carefully warned that he cannot rely on every function being integrable. After a preliminary review of the number-system and the concept of function, differentiation is motivated by a discussion of speed, leading to a very brief discussion of limits. Next comes some practical differentiation and then differentials and anti-differentials. Here, conveniently earlier than usual, we meet separable differential equations. The anti-differential is the primitive in differential (rather than derivative) notation, and the concept is very properly divorced from that of integral. (Would that the author had