Edward Sang. By D. Bruce Peebles.

(Read May 17, 1897.)

In the latter half of the seventeenth century a farmer of the name of Sang lived in Aberdeenshire, of whom little is known. But we know that he had a family, and that his youngest son, Robert, was born in 1700, and in course of time married Margaret Mitchell, who was an exceptionally clever woman. There is no record of what position he occupied or what business he followed, but very likely it was farming or gardening. He died in 1793, aged 93 years. His son, David, was born in 1749. He was a gardener, and married Mary Chalmers, a daughter of Dr Chalmers of Stonehaven.

Edward, one of his sons, left home when quite young, and settled in Kirkcaldy as a market gardener. He gradually extended his business, and ultimately became a nurseryman and seedsman. In his spare hours he managed by himself to acquire a good knowledge of Greek and Latin, and the difficulties he must have met with no doubt impressed strongly on his mind the advantage of having a good teacher. Business prospered with him, and he married Jean Nicol, a relative of Nicol whose prism is so well known. While giving attention to his business, he must have taken a good deal of interest in municipal affairs, as he was elected Provost of Kirkcaldy, a position he held for a number of years. His son, Edward, our late Fellow, was born on the 30th of January 1805. When he got to the age of seven years it was thought that he should be sent to school. But there were no schools in Kirkcaldy of very high standing at that time, and Provost Sang, along with other gentlemen, started a subscription school in 1812, and engaged Edward Irving as headmaster. After doing so it was natural that they should do what lay in their power to provide pupils for the new school, so we find that three of the Sang family, David, Edward, and their sister, were placed with Irving. The promoters were not wrong in their choice of a teacher, for Irving was a thoroughly able and conscientious

master. In a book lately published, the author, Æ. J. G. Mackay, Sheriff of Fife and Kinross, speaking of Irving, says that "his eloquence, earnestness, and high ideal aims had been known in Kirkcaldy, where it was remembered that he took his pupils to the sands to watch the stars." It may be mentioned, in passing, that in 1816 Thomas Carlyle came to Kirkcaldy, so that we find two men who were to become famous, and a boy who, in his own way, was to rise to eminence, living in close proximity. The boy, young as he was, no doubt felt the influence of his teacher, and possibly of Carlyle also; for two such spirits in a small place like Kirkcaldy could not fail to stir up old and young who came in contact with them, especially those who had any leanings toward science and literature. The boy Sang must have been deeply impressed with Irving's method of teaching, for in after years he used, when occasion served, to give at the meetings of the Royal Scottish Society of Arts vivid descriptions of his school life at Kirkcaldy. He told how Irving made the boys throw down their books and go off with him to field or sea shore to work out problems in land-surveying, astronomy, and navigation. To young and ardent minds no method of teaching could be more fascinating. The pupils were fortunate in having such a teacher. Of the progress the boy made during the first two years of his school life we may judge by referring to a prize he then obtained. We find the following inscribed on a copy of Vince's *Hydraulics*:—

"This book is presented to Edward Sang by the Patrons of the Kirkcaldy Subscription School for his progress in the Higher Class of Mathematics.
(Signed) "EDWARD IRVING."

Sang was not much of a letter-writer, so that there is little in the way of correspondence to weave into such a notice as this. There is one letter, however, the first he wrote, which is peculiarly interesting, as it gives a good idea of what he, a lad of eleven years of age, and his brother, a little older, were thinking about. The letter is addressed, "Mr David Sang, Brucefield, Dunfermline," and is as follows:—

"KIRKCALDY, *7th December* 1816.

"DEAR BROTHER,—I have sent with the cart the Algebra, Mechanics, and Astronomy. I cannot send the Geometry, Conic Sections, or French Dictionary. I do not know whether you wish me to send any Latin books

or not. You may send word in your next letter. I understand what you mean by some of your work; it was a cissoid. I intend to come at New Year, if I get play for a week, if Father consents. Excuse my letter, being the first I have written.—I am, dear David, your affectionate brother,

(Signed) "EDWARD SANG."

Here we have a boy eleven years of age—an age at which many boys are scarcely out of the nursery—getting a prize for his progress in the higher class of mathematics, and evidently busy with mechanics, astronomy, French, and Latin. Continuing his school life, and no doubt working hard, he obtained another prize in 1817, on which was inscribed:—

"This book was presented to Edward Sang by the Patrons of the Kirkcaldy Subscription School as a reward for his diligence, and a testimony of his success in the study of Mathematics during the Session that closed this day, the 1st November 1817. (Signed) "EDWARD IRVING."

The book was Legendre's *Eléments de Géométrie*. The boy was then twelve years of age. Before leaving the story of his school life, a short extract from a letter written by the Rev. Dr Martin a number of years after may be read. He says:—"Mr Edward Sang has been known to me from his infancy. He began very early to show an uncommon inclination toward mathematical science, and a peculiar aptness for it. As an amateur of that science, I had my attention drawn to his talent for it while he was, with some of my own family, a pupil in an academy in this town in which mathematics were then taught with remarkable success. I still remember the surprise excited by the acute and comprehensive solutions he gave of the problems, theorems, and questions presented to his class."

The Subscription School was broken up in 1818, and in the first year of his teens the boy joined the University of Edinburgh under Professor Leslie, and had to take the second class for mathematics, there being no third or advanced class for that session. He was small for his age, and on his appearance in the class-room he was greeted with laughter by his fellow-students, big fellows, who no doubt wondered what such a youngster was to do in such a class. But the laughter soon gave place to surprise and admiration. Next session there was still no advanced class, and he had again to take the second mathematical class, under Professor Wallace. During

the next four sessions he studied Natural Philosophy under Professor Leslie, two as a regular student and two as a holder of a perpetual ticket. His career in the University was one of uninterrupted progress, and on its termination he received the following certificate:—

"COLLEGE OF EDINBURGH, 20*th April* 1822.—I hereby certify that Mr Edward Sang has most regularly attended the Natural Philosophy Class during the whole of the Session now closed, that his application was ardent and unremitting, and the talents, ingenuity, and penetration which he displayed place him decidedly above all his fellow-students.

(Signed) "JOHN LESLIE."

After leaving college he commenced and continued for some years the practice of surveyor and civil engineer, and then became a teacher of mathematics in Edinburgh. In 1828 he was elected a Fellow of the Royal Scottish Society of Arts, and during his long connection with that body he brought before it some of his most valuable papers. We have here a list of his writings, 112 in number, on a great variety of subjects connected with Mathematics, Natural Philosophy, Horology, Astronomy, Engineering, etc., besides his great work on Logarithms. A glance over these gives one an idea of the diversified character of his studies, and suggests the query as to whether his work as a whole would not have been more valuable had he confined himself to a few, instead of dealing with so many subjects. It has been said that there is plausibility in asking, "not if a man can do many things well, but if he has done one thing supremely." There are some minds—minds of a high order, too—fitted to attack and stick to special work, and it would be a wonder and a disappointment if their work was not supreme. Sang was not one of those; nevertheless, he did many things supremely, but could not be a specialist. The variety of his papers shows that clearly; and while there is no time even to read over their titles, a few may be noticed in their order as having been received with marked favour and approbation. In 1829 he published a small work containing an account of a new method of solving numerical equations, the first of the many works that came from his hands. About 1830 Professor Wheatstone exhibited a very beautiful series of curves, produced by fixing a polished ball on the end of a wire and causing it to vibrate. This

was shown in Edinburgh, and attracted Sang's attention. He went into a consideration of the subject, and made the discovery that every wire, no matter what may be the form of the hole through which it has been drawn, has one direction in which the flexibility is greatest, and another, at right angles, in which the flexibility is least. He tells that he was startled by finding that theory led him to believe that, if the rapidity of vibrations in the one direction be double of that in the other, the common parabola should be the result. He then drew out curves according to theory, and manufactured wires of the proper proportions as indicated by the theory, and found a perfect coincidence between the actual and computed phenomena. In 1831 he exhibited before the Royal Scottish Society of Arts these wires, with their silver knobs, and the beauty of the curves was much admired. They never ceased to charm him, and he often brought them out to show to friends. In 1889, after a lapse of fifty-eight years, he again brought the subject before the Society of Arts, and exhibited a new set of wires he had recently made. In 1838 he read a paper to the Society of Arts, describing a Dioptric Light erected at Kirkcaldy, and he also gave a description and exhibited drawings of the apparatus he used for cutting the annular lens to the true optical figure. For that paper he was awarded the Keith Medal, value 20 sovereigns. In presenting the medal, Sir John Graham Dalziel stated that "no opportunity had hitherto occurred since the Keith fund came into the possession of the Society for awarding that prize. Now, however, it was highly gratifying to find one of the most scientific, useful, and meritorious of all who had been connected with the Society, Mr Edward Sang, entitled to this eminent distinction. His skill, his labours and unremitting exertions in various scientific departments, were too well known to be embellished by any commentary." In 1840 he was presented with the Society of Arts Silver Medal, value 10 sovereigns, for his papers on the "Construction of Circular Signal Towers," on the "Effects of the Curvature of Railways," and for his valuable essays on Life Assurance. For a period of several years he lectured on Natural Philosophy, and in 1841 he became Professor of Mechanical Science in the Manchester New College. Shortly after, he went to Constantinople to assist in establishing schools of civil engineering, and in laying out railroads

in Turkey. He assisted in the erection of ironworks at Zeïtun Buruni, and was afterwards engaged in several colleges completing the courses of education. In addition to this work, he proceeded to compile treatises on the Method of Co-ordinates, on the Differential and Integral Calculus, on Mechanics, Hydrostatics, and the Elements of Physical Astronomy. This arduous task was accomplished by means of oral lectures in the Turkish language, of which full notes were taken by the pupils, and these notes were extended by the students and compared. By this means accuracy of idiom was secured, and the precision of the technical terms taken from the Arabic was ascertained.

Sang never let an opportunity slip of helping truth or correcting error, and he took advantage of an approaching event which led to this, and also gave him a chance of dispelling fear and removing superstition. The solar eclipse of 1847 was close at hand, and was to be annular, and almost central as seen from Constantinople, so he computed its details with great care, and prepared large drawings, with a descriptive notice in French. These were presented to the Sultan through his Excellency Lord Cowley. Notices were also given in the Turkish and French newspapers, accompanied by lithographed drawings on a smaller scale, while prints were freely distributed with an explanation in Turkish. With a great roll of these prints under his arm for distribution, Sang often went to out-of-the-way places and into the bazaars. The priests were the worst to deal with, and were most unwilling to touch the prints, saying that no man should meddle with such matters, as they belonged to God alone. But many of them listened to reason, and a little sensible talk helped generally to overcome their scruples. The excitement was great, and the preparation of pieces of smoked glass was carried on most vigorously by large numbers of the community as the predicted time of the eclipse approached. In those days the uneducated classes looked upon eclipses with terror and consternation, as in many places they still do. There are few amongst ourselves, even now, who are free from awe and wonder when there is a total eclipse, and the weird and unnatural darkness sends bird and beast into seclusion, and brings about a deathlike silence.

The Sultan had made great preparations for the coming event.

With telescope and chronometer at hand, he waited for the second which Sang foretold that the commencement of the wonderful phenomenon would take place. And, sure enough, the very moment which was predicted saw the beginning of the interference with the sun's light. The firing of guns, the blowing of trumpets, the beating of drums and shouting now began, that being thought the only way to bring back the sun's light and prevent the disasters which it was believed would certainly follow. The Sultan at once sent out his officers to stop all that, and the end of Sang's labours was accomplished by the dispelling of superstition and the quelling of the alarm which had always accompanied the natural but uncommon deviation from the daily gradation of light and darkness. This was a triumph of science which had a lasting effect on the population of Constantinople, for it not only removed in a great measure the superstition of the people, but also excited and stimulated the students to such a degree that it led to the details of the next eclipse of 1851 being computed and prepared by Sang's fifth class, at the Imperial College, Muhendis-hana Berri.

In 1849 he was elected a Fellow of this, the Royal Society of Edinburgh, and there were few, perhaps, who took a keener interest in its proceedings. In 1851 he was invited by a circular of the British Association, sent at the instance of the secretary of the Royal Scottish Society of Arts, Edinburgh, to proceed to Russia for observing the total eclipse of the sun. The shortness of the notice, and the calmness of the weather during the voyage, prevented him from reaching farther than Sevastopol, where he arrived just when the eclipse began, so that he could do nothing; and the only result of the trip was to intensify a longing to return to Scotland. There are few Scotsmen who go abroad but are at some period of their exile seized with nostalgia, and sometimes in such a fashion as not to be conquered. It was so in this case. He resigned his situation, greatly against the wish of Fethi Pasha, who would not give his formal consent. Nevertheless Sang returned to Edinburgh in 1854, and resumed his former occupation as a teacher of mathematics.

During his residence in Constantinople he won the respect and esteem of all the various nationalities with whom he came in contact, notwithstanding the differences of religion and race, and

amongst the Turks his name to this day is held in reverential remembrance in the colleges in which he was professor. In 1889, shortly before his death, a young Turkish gentleman being in this country, could not leave it without finding out and coming to see the *hodja*—the teacher—of whom he had heard so much, and who was always spoken of with admiring respect and affection.

In 1856 he read a paper on the Gyroscope in relation to his suggestion of a new experiment which would demonstrate the rotation of the earth, and he claimed that he had clearly proved this by experiment in 1836,—eighteen years before Foucalt performed his experiment at the meetings of the British Association in Liverpool. In connection with this, Professor Baden Powell, in a letter to Professor Piazzi Smythe, says:—"I have just received, I presume through you, a copy of Mr Sang's paper on 'Rotation.' Pray thank him from me if you have an opportunity. It is extremely interesting to find how completely he anticipated the idea so long ago." Professor Chevallier also wrote to Smythe, saying:—"If poor Arago were still alive he would, as a Frenchman, feel himself 'like a woodcock caught in his own springe,' for he always held that a paper communicated to a recognised public scientific society, and regularly entered in their Proceedings, was to all intents and purposes a publication to the world of an invention. Mr Sang must be no ordinary man to have conceived so clearly the solution of so difficult a question by mechanical means. I hope that his claims will now be made generally known."

In 1862 Provost Sang, one of the promoters of the Kirkcaldy Subscription School, died, aged 91 years.

In 1879 the Institution of Civil Engineers, London, awarded to Sang the Telford Premium for his paper on "A Search for the Optimum System of Wheel Teeth." The paper contains elaborate and intricate calculations, undertaken with a view to discover the best form of wheel teeth to adopt as a standard, in order to avoid the lack of uniformity which had so long existed. Another important paper was read by him to the Society of Arts in 1861 on "The Determination of the Form of a Ship's Hull by means of an Analytic Expression," its object being to improve naval architecture and substitute a scientific method, instead of rule-of-thumb and guesswork, in hull construction The Society referred the paper to

a committee, which reported most favourably upon it, stating that "the author deserved great credit for the analytical skill and ingenuity he had displayed in the investigation, which must have cost him much thought and great labour in computing from their equations, and tracing a large number of curves, in order to obtain the requisite familiarity with the use of the formulæ, and also for the purpose of constructing the model which was shown." This paper was awarded the highest prize the Society could give, viz., the Keith Prize, value 30 sovereigns. On the 8th December 1873 he read a paper giving a description of a new machine for the hand-spinning of rope yarn. The machine was shown in action, and the work performed by its beautiful compound motion was much admired. Any length of rope could be spun without having recourse to the long rope-walks commonly used. In 1886 this Society presented him with the Macdougall-Brisbane Prize for 1882–84 for his paper "On the Need for Decimal Sub-Divisions in Astronomy and Navigation, and on Tables therefor."

Our learned Fellow, Lord M'Laren, who was in the chair, in making the presentation, said that "Dr Sang's paper covers a wide range of inquiry, embracing various branches of pure mathematics, mechanics, and optics, as well as the applications of these sciences to practical astronomy, chronometry, and naval architecture. No considerations, save zeal for the advancement of science and a benevolent desire to lighten the labours of future computors, could have induced Dr Sang to undertake such a gigantic task, or have sustained him through the wearisome mass of mechanical detail which overlaid the more interesting parts of his occupation." Time will not permit of noticing more of his work, a fraction of which has only been touched upon, but enough to show what the man was and what he did. Everything he gave his mind to as a philosopher was undertaken with an honest, conscientious, and single-minded devotion to science; and everything he put his hand to as a craftsman was marked with a beauty of design, a faultless precision and delicacy of finish, of which the most skilled workman might be proud. It may be added that accuracy in workmanship was insisted on by him to an extent that to many in times past seemed useless. In this he surely anticipated what was coming, for the marvellous development of automatic machinery—

especially in America, in which accuracy is a *sine qua non*—shows how true his instinct was, and how correct his views were regarding the designing and construction of tools. Even in small matters his love of accuracy crops up, for he had his drawing-pens fitted with screws having divisions on their milled heads representing lines of different breadths, to which the pen could at once be set. He was a beautiful draughtsman, and was never at a loss, with pen or pencil, in making clear even complicated pieces of mechanism.

The man of science goes to nature and asks pointed questions. To these, up to a certain limit, he gets answers of precision; but philosophers are not inclined to stop there, and they go on asking questions to which they can get no possible solution. Why? is ever on their lips; and when there is no answer, or one which is unsatisfactory, belief in the existence of a Supreme Ruler and Governor is apt to vanish, and Doubt sits down in the empty chair from which Faith has been driven. Sang was not one of that class. He never obtruded his opinions on such matters, but his belief in a Supreme Ruler was strong and unequivocal. If he alluded to the subject it was always in the most reverential spirit. In an address he gave at the Jubilee of the Society of Arts in 1867, he said:—"The exquisitely-carved shell of the minutest diatom reveals arrangements and contrivances infinitely beyond all that man has done or ever will do; and we place our hands upon our mouths, our faces in the dust, in the presence of a wisdom that we cannot begin to comprehend, of a goodness that overwhelms us." In a paper he read in 1884 he concluded by saying:—"Of the untiring goodness, the unfathomable wisdom evident in all that passes around us, even in the mysterious complexity of human life, let us recall what has been said,—'He rewardeth the searcher and the keeper of His laws.'" On a large telescope which he mounted himself for the Wray lens he got from the Institution of Civil Engineers, he inscribed that precious motto in golden characters in Turkish and English; and when, in times of depression, from which few are exempt, he felt that his labours were not rewarded as they should have been, we can fancy him getting comfort and inspiration from such a motto. But he was a busy man, and the busy man is generally a happy man. Reason rather than memory was what he valued and appealed to in connection with his pupils. Parrot-work

he hated, and the palaver of a dilettante he met with ridicule. He received a grant of £100 per annum from Government as a recognition of his valuable scientific work; and the associated Scottish life assurance offices, feeling that some substantial recompense was due to him for his logarithms and actuarial tables, at a meeting in 1878 resolved to recommend to the offices the payment of an annuity of £100 . . . . for the remainder of his life, which was agreed to and subscribed by the offices. His great work on Logarithms, faultless as it is believed to be for accuracy, is a monument not only to his mathematical skill, but to his tenacity of purpose and love of science. There are few who for forty years could have, with what may be termed intermittent continuity, persevered with such a colossal work; and the pity of it is that forty-seven volumes of such valuable matter for astronomers, navigators, and others should be lying uncared for and useless. Such a work surely demands the care of Government or some of the learned societies. In 1881 he was made corresponding member of the Royal Tunis Academy. In 1883 he was honoured by being made an LL.D. of Edinburgh University, and in 1884 he was made an honorary member of the Franklin Institute, Philadelphia. In 1889, feeling his advanced age telling upon him, he resigned the post of secretary to the Society of Arts; and his last paper to this Society, on "The Extension of Brouncker's Method," was written in 1890, and read by Professor Tait on the 15th December. For some months before this he had been failing in bodily health, but his mind was clear and undisturbed. Four days before his death he dictated letters, and also wrote some himself; but the end was near, and on the 23rd of December he died, within a few days of reaching the age of 86 years. A long-lived race were the Sangs—Robert reached 93 years, David 88, Edward of Kirkcaldy 91, and Edward, our late Fellow, 86, giving an average of close on 90 years for the four generations.

The mourners, who were honoured in having been chosen by himself to attend and pay the last tribute of respect at his funeral were all sincere friends, who grieved over the loss they had sustained in the departure of one whom they loved and deeply respected, not only as a man, but also for his great and varied learning.

Few have lived with such an enthusiastic and single-minded

devotion to science, and his rewards in a worldly sense were far from commensurate with his great and valuable labours. In writing this paper a free use has been made of the *Transactions of the Royal Scottish Society of Arts*, a short autobiography, and information kindly supplied by the Misses Sang. It may fitly be closed by a quotation from one of Miss Sang's letters. Speaking of her father, she says:—"To investigate, to endeavour if possible to reach the fundamental principle, and so be able to build a firm superstructure on a sure foundation, was a passion with him in which he found a happiness few could realise, and he was always ready to communicate to others from his own stores." Let us hope that the promise of his favourite motto is being amply fulfilled in another sphere, where he now finds how true it is, with reference to a Supreme Ruler, that "He rewardeth the searcher and the keeper of His laws."

LIST OF WRITINGS.

1. "Solution of Algebraic Equations of all orders, whether involving one or more Unknown Quantities." Edinburgh, 1829.
2. "On a Remarkable Analogy between the Primitives and Derivatives of the Product of two Monome Functions." *Annals of Philosophy*, August 1829.
3. "Observations on the Theory of Capillary Action given in the supplement to the 'Encyclopædia Britannica.'" *Edinburgh Philosophical Journal*, February 1830.
4. "On the Adaptation of the Fly-wheel and Pully of the Turning-Lathe to a given length of Band." April 1831.
5. "Experiments made to Determine the Thermal Expansion of Marble." June 1831.
6. "A New Solution of that case in Spherical Trigonometry in which Two Sides and the Contained Angle are given." 1832.
7. "Analysis of the Vibration of Straight Wires." *Edinburgh Philosophical Journal*, April 1832.
8. "A few Remarks on the Relation which subsists between a Machine and its Model." November 1832.
9. "Remarks on some Prevailing Misconceptions concerning the Actions of Machines." January 1833.
10. "On the Advantages of a Short Arc of Vibration for the Clock Pendulum, with a Table of Corrections of the Daily Rate." July 1833.

11. "Meteorological Observations made at Edinburgh during the Solar Eclipse of 7th July 1833."

12. "A Method of Freeing the Determination of the Latitude of an Observatory from the consideration of Atmospheric Refraction." August 1833.

13. "First Essay preliminary to the series of Reports ordered by the Society of Arts for Scotland." September 1834.

14. "On a certain Relation between the Successive Prime Numbers." Laid before the British Association.

15. "The First Book of the Geometry of Sines of the Third Order." Laid before the British Association.

16. "Second Essay preliminary to the series of Reports ordered by the Society of Arts, Scotland." October 1834.

17. "Report on the Recent Improvements in the Carpet Manufacture." August 1835.

18. "On the Manner in which Friction affects the Motions of Time-keepers." July 1835.

19. "Suggestion of a New Experiment whereby the Rotation of the Earth may be Demonstrated." January 1836. Read before the Society of Arts, 9th March 1836.

20. "Account of an Improvement in the Construction of Wollaston's Goniometer." Society of Arts, 1836.

21. "Annual Report on the State of the Useful Arts." Read before the Society of Arts, 7th December 1836.

22. "On the Construction of a Solid Achromatic Eyepiece." Royal Society, 6th January 1837.

23. "On the Construction of Eyepieces for Transmitting only one of the Pencils of Polarised Light." 30th January 1837.

24. "Second Report on the Progress of Exactitude in the Manufacture of Machines." June 1837.

25. "Notice of Precautions to be taken while using Mr Adie's Anemometer." Society of Arts, January 1838.

26. "Notice of a Method of Determining the Velocity of the Wind." Society of Arts, 10th January 1838.

27. "Notice of a Singular Phenomenon connected with the Rotatory Motion of Fluids." Society of Arts, 14th February 1838.

28. "Description of the Phoroscope, an instrument for Measuring Time and Velocity." Society of Arts, 28th February 1838.

29. "Description of an Improved Nut for Leading Screws." Society of Arts, 28th March 1838.

30. "Notice of a Dioptric Light erected at Kirkcaldy Harbour, with a Description of the Apparatus used in Making the Lens." April 1838.

31. "Theory of the Construction of Oblique Arches." May 1838.

32. "Notice of an Erroneous Method of using the Great Theodolite, practised by the Ordnance Surveyors, with a Strict Analysis." November 1838.

33. "On a Method of obtaining the greatest possible Exactitude from the Data of a Survey." January 1839.

34. Description of a new Waving Bar for Engravers' Ruling Machines." Society of Arts, 30th January 1839.

35. "Essay on the Law of Mortality in England and Wales, deduced from the Return by the Registrar-General." December 1839.

36. "Essay on the Bonus System of Life Assurance Offices." January 1840.

37. "On what Constitutes the Profits of a Mutual Assurance Society, and on the only Equitable Method of Distributing these among its Members." January 1840.

38. "On the Money Values deduced from the various Bills of Mortality." March 1840.

39. "On the Construction of Circular Signal Towers." April 1840.

40. "On the Effects of the Curvature of Railway." May 1840.

41. "On the Nodus Rosi (a Phenomenon Exhibited by some Specimens of Calcareous Spar brought from Iceland by Mr Rose)." Society of Arts, 8th March 1841.

42. "On the Proper Form for the Convertible Pendulum." July 1841.

43. "On an Erroneous Deduction from Captain Kater's Experiments on the Flexure of Thin Bars." April 1841.

44. "On a Method of Registering the Force transmitted by a Driving-Belt."

45. "Life Assurance and Annuity Tables—One Life in Folio." Edinburgh 1841.

46. "Delineation of the Annular Eclipse of the Sun as it will be seen from Constantinople on Saturday, October 9th, 1847." May 1847.

47. "Account of Observations made at Sevastopol on the Solar Eclipse of July 18th, 1851." Read before the Royal Scottish Society of Arts, 24th November 1851.

48. "A New General Theory of the Teeth of Wheels." Edinburgh, 1852.

49. "Description of a Chronofore or Hackwatch for Measuring to Minute Intervals of Time." Society of Arts, 27th November 1854.

50. "Theory of Driving-Belts." Society of Arts, January 1855.

51. "On an Improved Mode of Constructing Standards of Weight." Society of Arts, December 1855.

52. "On an Inaccuracy (having its greatest value about 1″) in the usual Method of Computing the Moon's Parallax." Read before the Royal Society of Edinburgh, 19th February 1855.

53. "On the Accuracy attainable by Multiplied Observations." Read before the Royal Society of Edinburgh, 30th April 1855.

54. "Geometry is a purely Experimental Science." Read before the Royal Society of Edinburgh, 7th January 1856.

55. "On the use of the Altitude and Azimuth Circle for Stereometric Surveying." Society of Arts, January 1856.

56. "On Turkish Weights and Measures." Read before the Royal Society of Edinburgh, 4th February 1856.

57. "Theory of the Free Vibration of a Linear Series of Elastic Bodies." Read before the Royal Society of Edinburgh, 18th February 1856, and published in the *Edinburgh Philosophical Journal.*

58. "Elementary Arithmetic." W. Blackwood & Sons, 1856.

59. "The Gyroscope." 1856.
60. "Higher Arithmetic." W. Blackwood & Sons, 1857.
61. "On the Exhibition of Both Roots of a Quadratic Equation by One Series of Converging Fractions." Read before the Royal Society of Edinburgh, 18th January 1858.
62. "Life Assurance and Annuity Tables, for every Combination of Two Lives." Vol. ii. 1859.
63. "Pendulum." *Encyclopædia Britannica*, vol. xvii.
64. "Perspective." *Encyclopædia Britannica*, vol. xvii.
65. "Saw and Saw Mill." *Encyclopædia Britannica*, vol. xix.
66. "Skew Bridge." *Encyclopædia Britannica*, vol. xx.
67. "Determination of the Form of a Ship's Hull." 1861.
68. "Sefinet Equation for Determining the Form of a Ship's Hull." 1863.
69. "Catadioptric Altitude and Azimuth Circle." 1862.
70. "Deflection of Plummet." 1862.
71. "Roots of Equations." 1863.
72. "Roots of Equations." 1864.
73. "Crystal Pointer Clock Adjustment." 1864.
74. "Commensurables." 1864.
75. "Motion in a Circle." 1865.
76. "Epicycloid." 1865.
77. "Recurring Functions." 1866.
78. "Brouncker's Method." 1870.
79. "Motion in a Circle." 1870.
80. "Account of New Table of Logarithms to 200,000." 1872.
81. "Crystal Cavities." 1873.
82. "Rope Yarn Machine." 1873.
83. "Canon of Sines." 1877.
84. "Revolving Wire." 1877.
85. "Unround Discs." 1877.
86. "Wheel Teeth." Part ii. 1877.
87. "Ballistic Curves." 1878.
88. "Earth's Density." 1878.
89. "Approximating Fractions." 1878.
90. "Optimum Wheel Teeth." 1879.
91. "Noveau Calcul." 1879.
92. "Addition on Noveau Calcul." 1880.
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