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CLASSICAL AND QUANTUM QUADRATIC HAMILTONIANS

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This thesis is concerned with the properties of dynamical systems with Hamiltonians which are general quadratic combinations of classical or quantum canonical variables or of Boson or Fermion annihilation and creation operators.

Chapter I, consisting of six sections, deals mainly with sets of canonical forms for quadratic Hamiltonians under the action of the group of linear canonical transformations. By viewing the group of C.C.R. Bogoliubov transformations as $\mathrm{Sp}(2N, R)$, a full set of canonical Boson Hamiltonians, labelled by the invariants for symplectic conjugacy classes, is obtained. Since the conjugacy class of a real orthogonal matrix is determined by its eigenvalues, there is only one type of canonical form for quadratic Fermion Hamiltonians. Unlike the Fermion case, indefinite Boson Hamiltonians can not in general be reduced to a sum of quasi-particle number operators. However, it is shown how spectral theory in Krein space rigorizes the full reduction of a strictly positive quadratic Boson Hamiltonian for which the single particle space is infinite dimensional. In the case of infinite degrees of freedom, there are various existence criteria for a unitary operator which implements a given Bogoliubov transformation and the equivalence of these is discussed in Section 6.

Cahpter II, consisting of seven sections, applies the techniques of Chapter I to algebraic quantization. The Boson commutation relations, among formal mode operators obtained in heuristic quantization, are shown to be incompatible with the canonical commutation relations among conjugate variables unless the motion is stable. Segal quantization of any real

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orthogonal dynamics, according to Fermi-Dirac statistics, is shown to be possible. The orthogonal component, in the polar decomposition of the dynamical generator $-\hat{A}$, is a complex structure which enables the classical dynamics to be viewed as being unitary. When a symplectic dynamical group has a strictly positive generator $\hat{A}$, the complex structure J is uniquely determined as the pseudo-orthogonal component in the polar decomposition of $\hat{A}$ viewed as an operator on Krein space with indefinite inner product $iB(\cdot, \cdot)$, B being the symplectic form. Equivalently, $J = -i\left(1-2E(o)\right)$, $E(o)$ being the projection onto the maximal dynamically invariant subspace on which $i\hat{A}$ is negative definite. Unitarization of unstable symplectic dynamics is impossible but for a large class of Hamiltonians, determined in Section 13, the unstable motion can be viewed as being pseudo-unitary.

Chapter III is devoted to applications of the general theory developed earlier. In Section 14, fields with external potential, non-local fields, the Schwinger model and the Thirring-Narnhofer model are considered. Section 15 treats the Ising model and assimilates the theory of quasi-free states of the C.A.R. C^* algebra. A similar analysis of statistical systems of Bosons is discussed in Section 16.

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