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Computations of static white dwarf models: A must for asteroseismological studies

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Abstract

We present briefly a new generation of white dwarf models incorporating the latest developments of the constitutive physics. These are static models especially designed for accurate seismological studies.

32.1 Introduction

The main goal of asteroseismology is the determination of the internal structure of a pulsating star through the analysis of its observed pulsation properties. One way to fulfill this goal is by producing a stellar model that reproduces to high accuracy the observed periods of oscillation. This is generally not possible through full evolutionary calculations as the parameters of a model must be tuned rather finely to satisfy the requirement of accuracy. However, computations of static models can be used with profit here. We have therefore developed the capacity to rapidly build complete static models of stratified H-rich (DA) or He-rich (DB) white dwarfs, especially suited for asteroseismological studies, by specifying the stellar mass, the H-layer thickness, the He-layer thickness, the convective efficiency and the effective temperature.

32.2 Method

To build our models, we integrate with the help of a Runge-Kutta technique the equations of stellar structure and stellar grey atmosphere (see, *e.g.*, Cox

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& Guili 1968 and Mihalas 1978) from the high atmosphere ($\rho \lesssim 10^{-13}$) down to the center of the star. We iterate this procedure until we find a model with $M_r = 0$ at $r = 0$. To have a good spatial resolution both in the interior and the external regions, we use the integration variable $x[\equiv \ln(r/P)]$. This is the same integration variable that we use when we solve the nonradial oscillation equations to find the oscillation periods of a stellar model (Brassard *et al.* 1992).

Because we do not know the history of the star, we assume that $L(r)$ is proportional to M_r. This approximation is very good for white dwarf stars (see, *e.g.*, Lamb & Van Horn 1975). We also assume that the composition transition zones are given by diffusive equilibrium profiles (Vennes *et al.* 1988)†.

In order to integrate the equations throughout the entire model we use a version of the mixing-length theory that can be also used in optically thin regions (Bergeron *et al.*, 1992). We also use the following expression for $\nabla_{\mathbf{rad}}$:

$$\nabla_{\mathbf{rad}} = \frac{3\kappa P}{16 g_0}\left(\frac{T_{\mathrm{eff}}}{\mathrm{T}}\right)^4 [1 + q'(\tau)] \quad , \tag{1}$$

where $q'(\tau)$ is the derivative of the Hopf function $[q(\tau)]$ for grey atmospheres (Mihalas 1978). In optically thick regions ($\tau \gg 1$), we have $q'(\tau) = 0$ and we find the usual expression for $\nabla_{\mathbf{rad}}$ in the diffusion approximation.

32.3 Equation of state

For the low-density regions we use an analytic treatment of the equation of state by solving the Saha equations for a mixture of radiation and an ideal, nondegenerate, partially ionized gas for a mixture of H and He. In the region of partial ionization where nonideal and partial degeneracy effects are important we use, at our choice, an improved version of the tables of Fontaine *et al.* (1977, hereafter named FGV) or the new equation of state of Saumon & Chabrier (1993). In the deep interior, we use an improved version of Lamb's (1974) equation of state for a completely ionized pure plasma. Also, we have made sure that the transition between the envelope and the deep interior is made smoothly.

For pulsation calculations, it is very important to have a model without

† Equation (14) of that paper contains a misprint; this equation should read
$$\frac{\partial \gamma}{\partial \ln p} = \left[E(\gamma)(A_2 Z_1 - A_1 Z_2) + D(\gamma)(Z_2 - Z_1)\right] \Big/ \left[\frac{C(\gamma)D(\gamma)}{\gamma B(\gamma)} + \frac{D(\gamma)(Z_2 - Z_1)^2}{B(\gamma)E(\gamma)}\right] .$$

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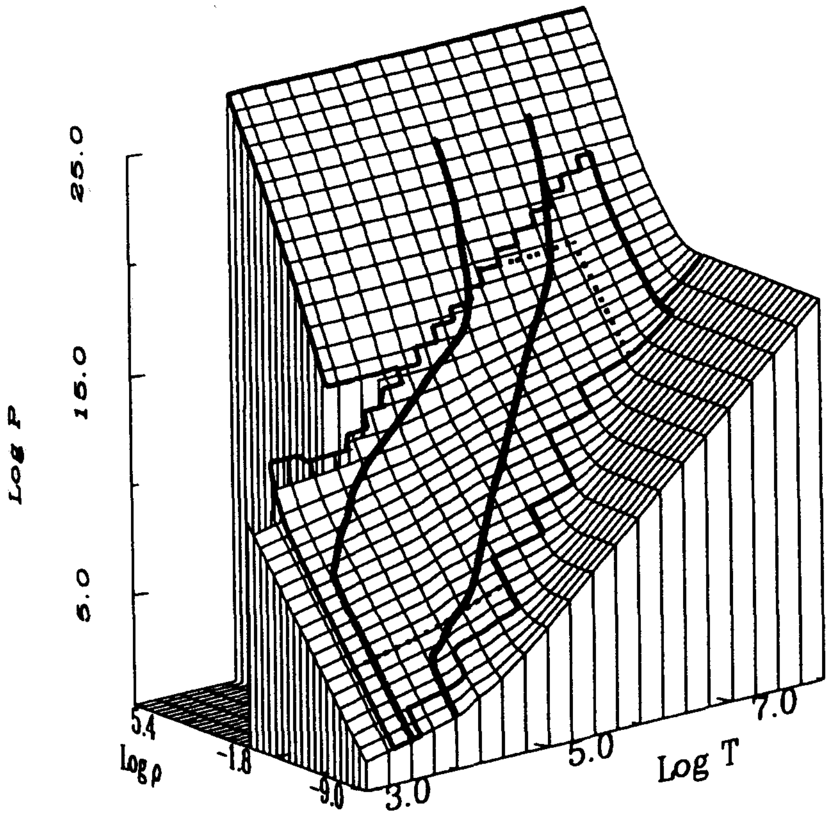

Fig. 32.1 Temperature-density diagram for the equation of state of He. The first two regions indicate from which table the data are extracted: Lamb (high density region), FGV (medium density region). Data from the Saumon-Chabrier equation of state is enclosed in the FGV region and is delimited by dots. Formalism for perfect gases is used in the low density and high temperature region of the diagram. The two thick lines refer to a typical (ρ, T) excursion of a pulsating DA white dwarf ($T_{\rm eff} = 12,500$ K, right line) and of a cool DA white dwarf ($T_{\rm eff} = 4000$ K, left line).

any artificial discontinuity. Composition interpolations are made with the "additive-volume" method of FGV.

32.4 Opacities

At our choice, three set of radiative opacities can be used. For comparison with previous available models, we have included the older Cox and Stewart opacities (1970) and the Huebner (1980) opacities. The third set is a combination of the new OPAL opacities of Rogers & Iglesias (1992) for T≥6000 K

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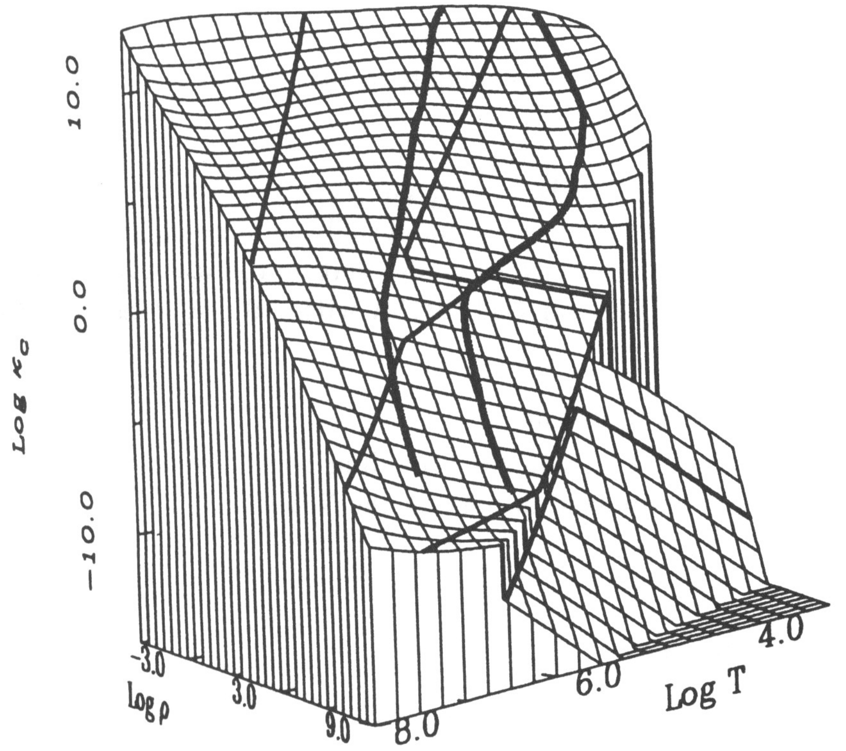

Fig. 32.2 Temperature-density diagram for the conductive opacities of He. The three regions indicate from which table the data are extracted: Hubbard & Lampe (low density), liquid metal phase of Itoh *et al.* (medium density) and crystalline phase of Itoh *et al.* (high density). Values outside these three regions are extrapolated conductive opacities. The two thick lines refer to a typical (ρ, T) excursion of a pulsating DA white dwarf ($T_{\rm eff} = 12,500$ K, left line) and of a cool DA white dwarf ($T_{\rm eff} = 4000$ K, right line). We can point out here that the center of the cool DA white dwarf is crystallized. This is not apparent in this diagram because the crystallisation arises in C composition at higher density than He. We can also note that the low temperature model relies heavily on extrapolated data.

and of the molecular opacities of Lenzuni, Chernoff & Salpeter (1991) for T$\leq$6000 K. At high densities, where no new data were available, opacities were obtained, in order, from the Huebner data, the Cox & Stewart data, or from linear extrapolation.

For the conductive opacities, we use the tables of Itoh *et al.* (1983) in the liquid metal phase with the low-temperature quantum corrections of Mitake *et al.* (1984). In the crystalline lattice phase, we use the data from Itoh *et al.* (1984). At low density, conductive opacities are supplemented with the older tables of Hubbard & Lampe (1969). Finally, in the remaining uncovered regions, conductive opacities are extrapolated from a fourth order polynomial fitted with data available in the valid regions.

32.5 Conclusion

Applications for our models are potentially numerous. As an example, we have already built a white dwarf model that reproduces the observed periods of pulsation of the DA star G117-B15A within 0.1 s (Brassard *et al.* 1993).

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