# THE ORIGIN AND EVOLUTION OF NOVAE

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Abstract. The cataclysmic binaries are products of the nonconservative evolution of close binaries with large initial mass ratios of components. The accretors in cataclysmic binaries can be helium or carbon-oxygen or oxygen-neon-magnesium white dwarfs. Their annual birthrates are $\sim 0.005$, $\sim 0.005$, and $\sim 0.00005$ respectively. In one-zone approximation of a thin accreting shell we estimate the critical masses of hydrogen and helium shells and recurrence time scales of thermonuclear runaways.

## INTRODUCTION

The modern approach to the nature of Novae and thermonuclear Supernovae was first formulated by Gurevich and Lebedinsky (1946, 1947). Analysing the stability of the nuclear burning they stated: "...The Supernova must be caused by the explosion in the central part of the star, where main nuclear reactions do occur... We base our investigation upon assumption that the explosions of Supernovae and Novae and possibly of Nova-like U Gem stars are caused by thermal runaways initiated by nuclear reactions... The energy generated by explosion is brought to the surface by a shock wave..." Essentially upon the same ideas is based the modern theory of astrophysical thermonuclear explosions.

The components of a typical cataclysmic binary are a Roche-lobe filling low-mass red dwarf and an accreting white dwarf, either helium or carbon-oxygen or neon-magnesium-oxygen one. The binary nature of cataclysmic variables (CV) was first proved by Walker (1954) and Kraft (1959). Kraft (1963) has suggested that activity of CV is caused by accretion by blue star of matter spilt by the red one. The possibility of explosions in accreted hydrogen layer was discussed by Mestel (1952). First numerical models of shell thermonuclear explosions were computed by Giannone and Weigert (1967), Redkoborody (1972), Starrfield et al. (1972). Tutukov and Yungelson (1972), Fujimoto (1982 a,b), Paczyński (1983) and many others assuming accumulative nature of instability have investigated analytically the main properties of thermonuclear runaways in hydrogen burning shells. A comprehensive review of recent developments in numerical models of Novae was published by Shara (1988).

The evolution of CV is determined mainly by the masses of components. The position of all CV with known $M_1$ and $M_2$ in the $q=M_2/M_1$-$M_2$ diagram is shown in Fig. 1. The basic properties of this diagram were already discussed in detail by Tutukov et al. (1982). The borders of regions in this diagram were found comparing the derivatives dlnR/dlnM and $\mathrm{dln}R_{cr}/\mathrm{dln}M$ for different rates of mass loss. $R_{cr}$ here refers to the radius of the Roche-lobe. In the regions HD and TH there would be placed systems with mass exchange occuring on dynamical and thermal time scales, respectively. Both time scales are short and no one of such systems is yet observed. The region A is occupied by systems captured by the orbital angular momentum loss (AML).

The angular momentum may be lost from the system, for example, by means of systemic mass loss or of magnetic stellar wind (MSW) or gravitational waves radiation (GWR). In the region AHD two modes of mass loss are possible: on the AML time scale or on the dynamical one. The latter may be realized if due to some reason mass loss rate rapidly increases so that an irreversible quite adiabatical expansion of the donor starts. As possible reasons for a sudden mass loss rate increase one may envisage mass-transfer pulse due to dynamical instability of convective Roche-lobe-filling secondary suggested by Bath (1975; see also Edwards (1985) and references therein). In densely populated regions hardening of binaries caused by encounters with field stars may increase the mass loss rate (Krolik et al. 1984). A common envelope develops then inevitably. The fate of such systems is unclear.

The masses of most of all secondaries in CV do not exceed $M_\odot$. Therefore the effects of nuclear burning are not very significant for them. The donors of Novae are well "mixed" with donors of systems of other kinds in the $q$-$M_2$ diagram. The occurence of three nova-like systems (UX U Ma, V442 Oph, RW Tri) in the AHD and HD-regions of $q$-$M_2$ diagram is most probably incidental, because of errors in the estimates of their parameters (see Fig. 1). However, one can not exclude the real occurence of some systems in the AHD-region.

The masses of accretors in CV are confined to 0.2-1.25 $M_\odot$. The lower limit agrees well with the lowest masses of degenerate helium dwarfs produced in close binaries. The upper limit is lower than the Chandrasekhar mass. This may be well a simple consequence of uncertainty of mass estimates, but more probably this reflects the erosion of accretor in the course of recurrent shell flashes.

Simple purely empirical estimates clearly show that only a low proportion of all binaries of certain type become Novae and that their outbursts have to be recurrent. The frequency of Novae in the Galaxy is estimated as 76 ± 38 per year (Liller, 1987). The birthrate of stars in the Galaxy is about one per year. This shows the necessity of recurrence of outbursts. On the other hand, taking into account the empirical estimate of mass of Novae ejecta $10^{-5}-10^{-4}$ $M_\odot$ (Gallagher and Starrfield, 1978) we immediately infer, that each Nova experiences in its lifetime $10^4$ to $10^5$ outbursts. Now comparing this number with the annual rate of Novae we obtain that the birthrate of Novae progenitors is $1.5 \cdot 10^{-3}$ - $1.5 \cdot 10^{-2}$ per year. This simple estimate, as we show below, agrees well with much more refined theoretical estimate of this birthrate.

## THE FORMATION AND EVOLUTION OF CATACLYSMIC BINARIES

The systems with Novae most probably do not differ from other cataclysmic binaries, therefore we may discuss all them together. The formation of CV from initially respectively wide binaries with large mass ratios of components through nonconservative evolution was first suggested by Webbink (1975), Ritter (1976), Paczyński (1976). A more detailed scenario of formation of cataclysmic binaries is shown in Fig. 2 (all numbers in this Figure refer to CV with CO- and ONeMg-white dwarfs which produce Novae). This scenario was thoroughly discussed in the paper by Tutukov and Yungelson (1987) and we shall restrict ourselves to a brief review. For all numerical estimates we employ the lifetimes of stars in different stages of evolution according to our computations reported in the paper just mentioned. The mass exchange in a close binary is dynamically stable if $M_1 \gtrsim M_2$ (see Fig.1) and the action of AML through MSW is efficient if $a \lesssim (10-12) R_\odot$. These requirements result in $q = M_2/M_{1init} \approx 0.12 - 0.25$ and $a_{init} \lesssim$ $\lesssim 10^2 - 10^3$ $R_\odot$ depending on $M_{1init}$ (see below). As for formation of CO or ONeMg white dwarf the $M_{1init}$ must exceed $\sim 2.5$ $M_\odot$ the total number of Novae progenitors in the Galaxy is respectively low. After the Roche-lobe overflow (RLOF) by the primary a short phase of common envelope follows. The numerical modelling still does not provide certain estimates of the duration of this stage, however, one may be sure that no more than $10^3$ objects of this kind exist simultaneously. One may expect that their observational appearance is similar to protoplanetary nebulae. The progenitors of Novae emerge from the common envelope as detached red dwarf + white dwarf binaries with $a \lesssim (10-12)R_\odot$. They are similar to the well-known system V471 Tau, but are more close. Several examples of possible pre-Novae have been found by Bond (1988). An additional channel for formation of CV is provided by pair encounters of white dwarfs with main-sequence stars in dense cores of globular clusters and, possibly, in the core of the Galaxy.

Due to the action of AML by magnetic braking and GWR the components go closer and finally the secondary overfills the Roche-lobe. Several tens of evolutionary tracks of secondaries evolving under the action of AML were computed in recent years. Our main numerical results were reviewed in Tutukov and Yungelson (1987); similar computations were recently published by Pylyser and Savonije (1989). Typical tracks of secondaries for three main variations of evolution are shown in $\lg P - \lg M_2$ diagram in the Fig. 3. As a matter of caution, we have to note that all these computations are based on extrapolation to rapidly rotating components of binaries of Skimanich (1972) empirical braking law, which was initially derived for slowly rotating single stars. Until initially unevolved, or slightly evolved ($X_c \gtrsim 0.3$) secondary mass is greater than $0.3M_\odot$ (period exceeds $\sim 3^h$) the main driving force of evolution is AML via magnetic braking. The disappearance of radiative core at $\sim 0.3M_\odot$ breaks the action of magnetic braking. The secondary therefore detaches from the Roche-lobe. This results in formation of the well-known $3^h-2^h$ "period gap" for CV. The system remains detached until GWR brings secondary to contact again. However, one must notice that computations based on Skumanich law were unsuccessful in reproducing the observed width of the gap. Below $P=2^h$ evolution is driven by AML via GWR. The mass exchange rate is low: $\sim 10^{-10}M_\odot$/yr, the total number of such systems is $\sim 10^7$.

The trend of evolutionary tracks from large to small $\dot{M}$ is confirmed by observed decrease of average effective temperatures of accretors with decrease of orbital periods (Sion, 1987).

If at the instant of RLOF the secondary has a helium degenerate core the orbital period can increase to several days.

The third variation of evolutionary path represent secondaries with quite exhausted hydrogen in the core at the instant of RLOF ($X_c \lesssim 0.01$). Rare systems with such secondaries can evolve to ultrashort orbital periods of several minutes (Tutukov et al., 1985; 1987).

Evolutionary scenarios for intermediate mass close binaries predict the existence of systems with nondegenerate helium donors. The estimate of their birthrate in the Galaxy is $\sim 0.007$ (Tutukov and Fedorova, 1989). Their periods are from $12^m$ to $50^m$. The characteristic mass exchange rates are $\sim 3 \cdot 10^{-8} M_\odot$/yr. For such $\dot{M}$ the thermonuclear runaway most probably results in detonation (Nomoto, 1982; Woosley et al., 1986). The energetics and observational appearance of such outbursts would most probably resemble not Novae but some kind of abnormal low energy Supernovae.

The evolutionary computations of secondaries show that their masses can decrease to $\sim 0.01 M_\odot$. Close binaries with such low mass secondaries may be unstable to runaway mass exchange leading to disruption of the secondary on dynamical time scale (Ruderman and Shaham, 1983). The details of such a process are unclear, but if it really occurs, it may result in a single degenerate dwarf surrounded by a massive disk. Accretion from this disk may greatly accelerate the spin of the dwarf. On the other hand, one may as well expect that if an extended envelope forms instead of the disk the spin of the dwarf would be decelerated.

Let us now find the birthrate of CV. The transformation of semimajor axes of orbits inside common envelopes is described by energy conservation law, as suggested by Tutukov and Yungelson (1979)

$$\frac{M_{10}^2}{a_0} = \beta \frac{M_2 M_1}{a_f} , \qquad (1)$$

where $\beta$ is the efficiency of energy expenditure on envelope ejection, $a_0$ and $a_f$ are initial and final semimajor axes of the orbit. We assume $\beta = 1$. We also note that for numerical estimates we shall employ the analytical fits to results of evolutionary computations obtained by Iben and Tutukov (1987). The white dwarfs may be, as it has already been mentioned, composed either of helium (for $0.8 \lesssim M_{init}/M_\odot \lesssim 2.3$), or carbon and oxygen ($2.3 \lesssim M_{1init}/M_\odot \lesssim 9$) or oxygen, neon and magnesium ($9 \lesssim M_{1init}/M_\odot \lesssim 12$). The requirements of (i) avoiding of merger of components, (ii) formation of semidetached system due to AML after the common envelope stage and (iii) of stable mass exchange allow to determine a strip in the $M_1$-a diagram occupied by progenitors of cataclysmic binaries (Fig. 3; Tutukov and Yungelson, 1987). The range of the upper limits of the allowed mass ratios in initial systems is shown in the upper scale of Fig. 3. The birthrate of CV as for other kinds of close binaries we estimate by equation, which takes into account the distributions of new-born binaries over $M_1$ , $q=M_2/M_1$ and a:

$$d^3N = 0.1\, d\lg a\, M_1^{-2.5} dM_1\, dq . \qquad (2)$$

Eq.(2) gives the following birthrates: $\sim 0.005$/yr for systems with He-dwarfs, $\sim$ 0.005/yr for systems with CO-dwarfs,$\sim 5\cdot 10^{-5}$ for systems with ONeMg-dwarfs. If only two latter types of binaries produce Novae the estimate of their birthrate agrees well with observational estimate of this number.

The respective numbers of Novae with dwarfs of the latter two kinds are $\sim 10^6$ and $\sim 10^4$. The presence of ONeMg white dwarfs in some Novae is clearly indicated by the chemical composition of their ejecta (Starrfield et al., 1986; Truran and Livio, 1986).

Neglecting the dependence of mass of helium dwarfs on initial separation of components one may crudely approximate the final-initial mass relation as $M_1/M_\odot \approx 0.22\ (M_{10}/M_\odot)^{0.78}$. The width of CV progenitors strip in the $\lg M_{10}$-lg a diagram (Fig.4) may be approximated as $\Delta \lg a \approx 0.3\ (M_1/M_\odot)^{0.4}$. Then with $\Delta q \approx 0.1$ one obtains from Eq(2):

$$dN \propto M_{10}^{-2.4}\, dM_{10} . \qquad (3)$$

A similar dependence was found by Politano (1988) by numerical modelling of CV formation scenario. Correcting the mass-function of Eq(3) for probability of discovery according to Ritter and Burkert (1985) one gets good agreement with the observed distribution of accretors over masses, constructed after Ritter (1987) data.

## THE RECURRENCE TIMES OF THERMONUCLEAR RUNAWAYS IN THE ENVELOPES OF ACCRETING DWARFS

The accumulative nature of activity of CV confirms the existence of linear empirical relation between the energy radiated in optical range during the outburst by dwarf and recurrent Novae and average interflash period (Antipova, 1987). This relation can be approximated as

$$S \approx 7^{\overset{-3}{+6}}\, \tau_a , \qquad (4)$$

where $\tau_a$ is interflash period, $s=(L_{max}/L_{min})\tau_{max}$, $L_{max}$ and $L_{min}$ are average luminosities of Nova in maximum of brightness and between the outbursts; $\tau_{max}$ is the duration of the outburst. One can easily explain the Eq.(4) by conservation laws. In the minimum of brightness the luminosity is determined by the hot spot at the edge of the accretion disk: $L_{min} = E_1\dot{M}$, where $E_1$ is the energy, radiated away per gram of accreted matter. From the mass conservation law it follows that $L_{max}\,\tau_{max} = \dot{M}\,\tau_a\,E_2$ , where $E_2$ is the average energy liberated during the outburst per gram of the accreted matter. Combining this with former relation for $L_{min}$ one obtains $S = \tau_\alpha (E_2/E_1)$ . From Eq.(4) it follows that $E_2\,/E_1 \approx 7$. This is in agreement with the estimate, that only about one per cent of accreted hydrogen burns out in Nova flash (Tutukov and Yungelson, 1972).

The Eq.(4) holds also for dwarf Novae. It is tempting therefore to conjecture that the outbursts of Novae and dwarf Novae have the same nature. But how one would explain then the $1000^d$ to $10000^d$ gap between interflash periods of these systems ? More plausible explanation is that the mechanisms of outbursts are different and that the coincidence of S-values is casual. For dwarf Novae the disk instability model is more preferable. Then S-value is determined by $r_{disk}/r_{dwarf}$ and is strongly influenced by the bolometric correction.

The instability of the layer of the accreted hydrogen was first considered by Mestel (1952), assuming accretion of interstellar matter; later on analytical or simplified numerical approach to the problem of thermal runaways on the surface of the accreting white dwarf in a close binary was applied by Secco (1968), Saslaw (1968), Starrfield (1971), Tutukov and Yungelson (1972), Tutukov and Ergma (1979), Fujimoto (1982a, b), Paczyński (1983) and many others. In one-zone approximation (Tutukov and Ergma, 1979) one considers the thermal balance of a thin hydrogen layer accreted by a cold white dwarf between the outbursts. The layer is heated by compression of the growing envelope and cooled down by radiative heat conduction.

Evolutionary tracks in the $\lg\rho - \lg T$ diagram of hydrogen and helium shells accreted by $1.3M_\odot$ carbon-oxygen dwarf are shown in Fig.5. As mass is accumulated in the shell, its density and temperature increase. For degenerate matter there exists a single-valued relation between the density of the shell and its mass (the upper scale of Fig. 5). The stationary evolution of the shell continues until it reaches a point where the rate of energy generation by $C + p \longrightarrow N$ reaction begins to exceed the rate of cooling due to radiative diffusion. Further on the shell heats up in the time scale of hydrogen burning. Up to the moment when degeneracy is lifted, the temperature increases under almost constant pressure because the mass of the envelope practically does not change. The pressure at the bottom of a thin spherical envelope does not depend on its thickness. Therefore, the gas which becomes ideal after degeneracy is lifted obeys the law $\rho T$=const (see Fig. 5). The heating of the shell continues until its radius becomes comparable to the radius of the core. Only afterwards the temperature starts to decline.

As the shell expands the rate of hydrogen burning grows until at $T \sim 10^8$K it becomes almost constant because it is limited by the rate of $\beta$-captures. The outcome of the further evolution depends on several factors. The necessary characteristic of Nova is dynamical ejection of $10^{-5}$-$10^{-4}$ $M_\odot$ .This may be achieved in several ways. (i)If the abundance of CNO-elements is about $X_{CNO} \gtrsim 10(X_{CNO})_\odot$; (II) If mass of the accretor is close to the Chandrasekhar one; (iii) If $\dot{M} \lesssim 10^{-10} M_\odot$/yr and (iiii) a close companion is present. But there do exist Novae which do not show any overabundance of CNO elements. The very massive dwarfs have to be rare, and even their existence is questionable. In the orbital period range of the observed Novae theoretically predicted mass accretion rates greatly exceed $10^{-10} M_\odot/y$. Therefore more promising is the assumption of ejection of envelope due to dynamical friction of a binary inside common envelope which is formed after the expanding shell engulfs companion (McDonald, 1980). If one neglects the transport of the angular momentum inside the common envelope and assumes that all drag luminosity is spent on mass loss, then the time scale for the loss of the envelope is (Iben and Tutukov, 1984):

$$\tau \approx 0.3 \left(\frac{R}{R_\odot}\right)^2 \left(\frac{R_\odot}{a}\right)^{1/2} \left(\frac{M_\odot}{M_t}\right)^{1/2} \text{days}, \qquad (5)$$

where R is the radius of the envelope, a - semimajor axis of the orbit, $M_t$ is the mass of the binary. This time scale is much shorter than the time scales of the nuclear burning in the shell

$$\tau_{nuc} \approx 100 \left(\frac{M_{env}}{10^{-5} M_\odot}\right) \left(\frac{10^4 L_\odot}{L}\right) X_H \ \text{yrs} \qquad (6)$$

or radiative wind ejection

$$\tau_W \approx 5 \left(\frac{M_{env}}{10^{-5} M_\odot}\right) \left(\frac{v}{100\ \text{km/s}}\right) \left(\frac{10^4 L_\odot}{L}\right) \text{yrs}. \qquad (7)$$

As an argument in favour of this picture one may consider PUVul-an object in a wide binary system which suffered a Nova-like eruption in 1980 and since then almost retains its luminosity. One can suggest that the absence of close companion is responsible for conservation of the expanded envelope.

However, the enhancement of heavy elements content in the Novae ejecta which was first noted by Mustel and Boyarchuk (1959) in DQHer ejectum, is a matter of fact. In current thinking it may be achieved in two ways. First, as a result of shear instability mixing at the interface of the envelope and core (Kippenhahn and Thomas, 1978). Second, it may result from diffusion of hydrogen into the core, and heavier elements into the envelope (Prialnik and Kovetz, 1984). However, the enrichment in CNO-elements is not a necessary requirement. The envelope expands to the dimensions of the orbit and engulfs the whole system for normal Z and any $\dot{M} \lesssim 10^{-8} M_{\odot}/yr$ (Fujimoto, 1986b).

According to Iben and Tutukov (1989) the burning of hydrogen is stable if $6.25 \cdot 10^{-7}(M_d/M_{\odot}-0.6) \lesssim \dot{M}(M_{\odot}/yr) \lesssim 8 \cdot 10^{-7}(M_d/M_{\odot}-0.4)$. For helium burning the limits are $5.2 \cdot 10^{-6}(Md/M_{\odot}-0.6) \lesssim \dot{M}(M_{\odot}/yr) \lesssim 6.6 \cdot 10^{-6}(M_d/M_{\odot}-0.5)$. The burning shells for these $\dot{M}$ are nondegenerate. For lower $\dot{M}$ the burning is possible in quasiperiodical outbursts (Iben, 1982). As Fig. 5 shows for $M_d = 1.3\ M_{\odot}$ and $10^{-7} \lesssim \dot{M}(M_{\odot}/yr) \lesssim 5 \cdot 10^{-7}$ the outburst occurs in a nondegenerate shell. The recurrence time may be as short as $\sim 0.1$ year. Our simple estimate of it is in a good agreement with numerical results for one-zone model of Paczyński (1983) who had shown, that the minimal time span between two successful hydrogen thermonuclear runaways is about a month. However, the runaways in non-degenerate matter do not cause significant expansion of the shell and are not followed by a considerable mass loss. They will probably manifest themselves in UV part of spectrum or by quasiperiodical increases of luminosity of the donor.

The analytical investigation of the thermal phase of runaway in one-zone approximation which is illustrated in Fig. 5 allows to estimate critical masses of envelopes $M_H$ and $M_{He}$ and interflash periods $\tau_H$ and $\tau_{He}$ :

$$(M_H/M_{\odot}) \approx 4 \cdot 10^{-5} \dot{M}_{-8}^{-0.65} (M_d/M_{\odot})^{-10}; \quad (M_{He}/M_{\odot}) \approx 7 \cdot 10^{-2} \dot{M}_{-8}^{-0.56} (M_d/M_{\odot})^{-10}; \quad (8)$$

$$\tau_H \approx 4000\, M_{-8}^{-1.65} (M_d/M_{\odot})^{-10} \text{ yrs}; \quad \tau_{He} \approx 7 \cdot 10^{6} \dot{M}_{-8}^{-1.65} (M_d/M_{\odot})^{-10} \text{ yrs}.$$

We note that the tracks in $\lg \rho - \lg T$ diagram only weakly depend on the mass of the dwarf, only the hydrogen-ignition line is slightly shifted. Our simple estimates are in reasonable agreement with numerical results of Iben (1982) and Sion and Starrfield (1986) for hydrogen flashes and Kawai et al. (1987) for helium ones.

Let us now discuss several factors that can significantly influence our estimates of the critical mass of the shell. (i) The one-zone approximation of a shell with variable opacity can give only general outline of thermal runaway properties. (ii) We had assumed that the heating of the envelope occurs due to addition of mass which is itself cold. In reality, the matter infalls from the disk and has considerable energy related to its Keplerian motion. The heating by boundary layer increases the temperature of the envelope and decreases its mass (Fig. 5). (iii) The estimates (8) are obtained for normal $X_{CNO} \approx 0.004$. The increase of $X_{CNO}$ to 0.4 significantly reduces the hydrogen ignition temperature (Fig. 5). The same is valid for enhanced $He^3$ content in accreted matter. Shara (1980) has suggested that $He^3$ ignition may trigger the thermonuclear runaway. He has assumed this possibility for red giant donors, where $X^3$ reaches $(1-2)\cdot 10^{-3}$. Tutukov et al. (1985) has shown that for very low-mass donors $X^3$ reaches $\sim 5\cdot 10^{-3}$. The growth of $X_{CNO}$ to 0.4 or $X^3$ to $3\cdot 10^{-3}$ as indicated in Fig. 5 can reduce $\tau_H$ about twofold. (iv) In our estimates we assumed that the temperature of the core is much lower than that of the hydrogen layer. The reduction of this difference also decreases $\tau_H$ (or $\tau_{He}$ ). (v) One cannot exclude that the envelope is not spherically-symmetrical due to e.g. rotation or presence of magnetic field (Tutukov and Yungelson, 1974). Relatively low rotation velocities and/or moderate magnetic fields are sufficient to produce conditions under which the thermonuclear runaway will be initialized in the polar regions of the dwarf with much lower $M_H$.

All above mentioned factors together with uncertainty with distribution of binaries over initial mass ratios of components enter the estimates of number of Novae in the Galaxy.

Let us assume that all systems in $P_{orb} = 3^h - 10^h$ range are able to produce Novae. If for $P_{orb} \approx 3^h - 10^h$ we roughly take $<\dot{M}> \approx 5\cdot 10^{-9}$ $M_\odot$/yr and assume that the donor in this period interval loses on average $\sim 0.3 M_\odot$ , then for the estimated in Section 2 birthrate of systems with CO or ONeMg dwarfs $\nu \approx 5\cdot 10^{-3}$/yr, we obtain that the total number of Novae in the Galaxy is $\sim 3\cdot 10^5$. This estimate does not severely contradict the empirical estimates by Patterson (1984) - $10^5$ and Downes (1986) - $5\cdot 10^4$. The recurrence time of outbursts for $M_d = 1 M_\odot$ and $\dot{M} = 5\cdot 10^{-9} M_\odot$/yr is $\tau_H \approx 10^{-4} M_\odot$/yr. This results in the rate of Novae $\sim 30$/yr, quite comparable with empirical estimate $\sim 70$/yr (Liller, 1987). However, having in mind all uncertaintie invlolved in the estimates, the coincidence still may be simply casual.

All known Novae (with possible exclusion of CP Pup) have orbital periods exceeding $3^h$ (Ritter, 1987). The preceeding discussion shows that it is a consequence of high accretion rates in systems with $P \gtrsim 3^h$. But more rare and more violent outbursts have to occur also in systems with $P_{orb} = 80^m - 2^h$.

The eqs. (8) reveal a strong dependence of frequency of outbursts on the mass of accretor. E.g. the increase from 1.0 to $1.4M_\odot$ increases this frequency about thirty times. Therefore, despite the birthrate of ONeMg dwarfs is two orders of magnitude lower than that of COdwarfs, the frequencies of occurence of both types of dwarfs in the observed Novae have to be comparable, as was indeed found by Truran and Livio (1986).

The loss of common envelope due to thermonuclear runaway and braking of double core inside the envelope changes the semimajor axis of the orbit :

$$\frac{\Delta a}{a} \approx \frac{M_H}{M_t}\left(1 - \frac{M_t^2}{M_1 M_2}\,\frac{a}{R}\right). \qquad (9)$$

Here $M_1$ and $M_2$ are masses of accretor and donor, $M_t = M_1 + M_2$, $R \approx 30R_\odot$ is the radius of the envelope in the maximum of brightness of Nova. Evidently, for $M_1 M_2 / M_t^2 \gtrsim a/R$ the semimajor axis increases after the outburst. Such an increase was discovered indeed for BTMon as an increase of orbital period (Schaefer and Patterson, 1983). The growth of a has to be followed by a decrease of mass exchange rate or even by an interruption of the RLOF. However, it remains high ($10^{-9}$-$10^{-8}M_\odot$ /yr) for several centuries after the outburst because of irradiation of red dwarf by highly luminous white dwarf (Osaki, 1985; Kovetz et al., 1988; Sarna ,1989).

It follows from Eq.(9) that for $M_1 M_2 / M_t^2 \lesssim a/R$ the system after eruption becomes more tight. This can cause a runaway mass loss, formation of common envelope and complete disruption of the red dwarf if $M_2 \lesssim 0.03M_\odot$ , as suggested by Tutukov and Yungelson (1987).

## CONCLUSION

The modern theory of stellar evolution allows to estimate the range of systems producing white dwarfs accompanied by Roche lobe filling red dwarfs. Practically in all of them accreting white dwarfs can experience thermonuclear runaways. But it is easily seen by means of Eqs.(8), that most outbursts occur in systems with comparatively large mass exchange rates and most massive white dwarfs:

$$N \propto \frac{M_2}{\dot{M}_2} \cdot \dot{M}_2^{1.65} M_1^{10} \propto M_2 \dot{M}_2^{0.65} M_1^{10} . \qquad (10)$$

The thermonuclear runaways are therefore most frequent in the beginning of semidetached phase of evolution when $M_1$, $M_2$ and $\dot{M}$ are highest (Fig. 3). In Novae with the observed CNO elements enhancement in their ejecta the masses of accretors secularly decrease.

Let us discuss now several types of systems related to Novae. As we have already mentioned, some outbursts with energy up to $\sim 10^{48}$ ergs and masses of ejected envelopes up to $\sim 10^{-3} M_{\odot}$ may occur in systems with helium nondegenerate donors. The birthrate of them is close to the birthrate of usual CV - 0.007/yr (Tutukov and Fedorova, 1989), but the frequency of outbursts under comparable mass exchange rates is about three orders of magnitude lower than for hydrogen flashes. Therefore, there will be only one helium flash per thousand hydrogen ones. However, their high brightness which is intermediate between Novae and Supernovae can make them relatively more abundant in the observed ensemble of erupting stars. Such events can be detected if one studies the distribution over the light curves in a complete sample of Novae in e.g. a rich cluster of galaxies.

Novae can erupt also in wide semidetached binary systems which we observe as symbiotic stars. The necessary conditions are a very massive dwarf-(1.2-1.4)$M_{\odot}$ and very narrow range of donor mass - (0.8-0.9) $M_{\odot}$ (Iben and Tutukov, 1984). The mass exchange rate in such systems is $\dot{M} \approx 1.4 \cdot 10^{-10} (a/R_{\odot})^{1.4} M_{\odot}$ /yr. In systems that have semimajor axes of orbits $4 \lesssim a \lesssim 460\ R_{\odot}$ the $\dot{M}$ value is in the range that is able to produce thermonuclear runaways: $10^{-9}$-$10^{-7} M_{\odot}$/yr. The degree of violence of eruption can be comparable to Nova due to a high mass of accretor. But the low birthrate of such systems - $2 \cdot 10^{-5}$/yr makes this channel of Novae production respectively noneffective.

The condition of RLOF is not necessary for symbiotic stars. If the distance between white dwarf and its giant companion does not exceed 3-10 radii of the latter, a considerable proportion of the stellar wind of the giant would be intercepted by the dwarf. This is the original model of the symbiotic stars suggested by Tutukov and Yungelson (1976). About one Nova per year is possible in this kind of binaries (Iben and Tutukov, 1989).

The evolution of close binaries with $M \sim M_{\odot}$, $10 \lesssim a/R_{\odot} \lesssim 500$

leads to formation of degenerate helium dwarfs. Such dwarfs may be possibly formed also in wide binaries , where stars evolve like single ones (Harpaz et al., 1987). During the cooling stage thèse dwarfs experience one or two thermal hydrogen shell flashes accompanied by a considerable expansion of the envelope. The interaction of a close companion with the envelope can cause the ejection of the latter on the dynamical friction time scale (5). Even if such events would be comparable to Novae, their frequency would not exceed one per year, because it is close to the birthrate of helium degenerate dwarfs.

Quite new prospects for study of the physics of Novae are opened by gamma-ray astronomy. The hydrogen shell burning produces a number of short-living isotopes: $^{18}$F (110min), $^{14}$O(71s), $^{15}$O(123s), $^{13}$N(10 min), $^{22}$Na(202 yr), $^{26}$Al($10^6$yrs), $^{30}$P(150s), $^{34}$Clm(32min), $^{37}$Ar(35 days). Their abundances vary from $10^{-3}$ to $10^{-2}$ (Kudryashev and Tutukov, 1989). Their decay is accompanied by $\gamma$-radiation (Leising, 1987; Leising and Clayton, 1987). The isotopes with shortest lifetimes produce the most intense radiation, but the latter is mostly absorbed in dense envelope on the early stage of expansion. Long-living isotopes decay in practically trasparent envelope, but the intensity of their $\gamma$-radiation is low. However, the systematical $\gamma$-ray sky surveys would be helpful in discovery of $\gamma$-radiation of nearby Novae. This will provide new information for constructing more accurate picture of the Novae phenomenon.

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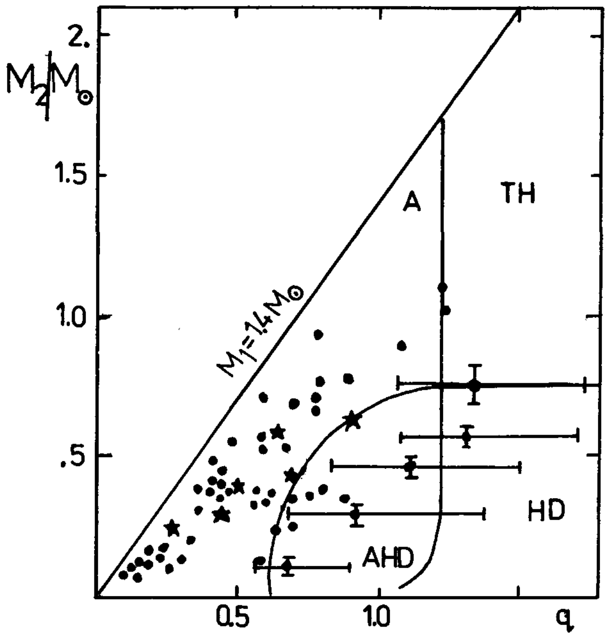

Fig. 1. The position of cataclysmic variables in the mass of the donor - mass ratio of components diagram. Novae are shown by asterisks. For systems falling into HD and AHD regions the errors of parameters are indicated. The data on CV is from Ritter (1987), the borderlines in the diagram are after Tutukov et al. (1982)

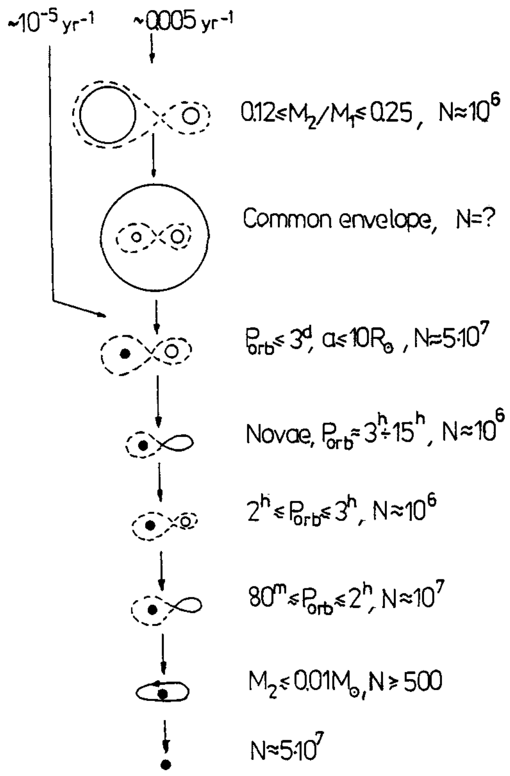

Fig. 2. The scenario of formation of CV with CO dwarfs. The estimates of orbital periods and numbers in the Galaxy are given.

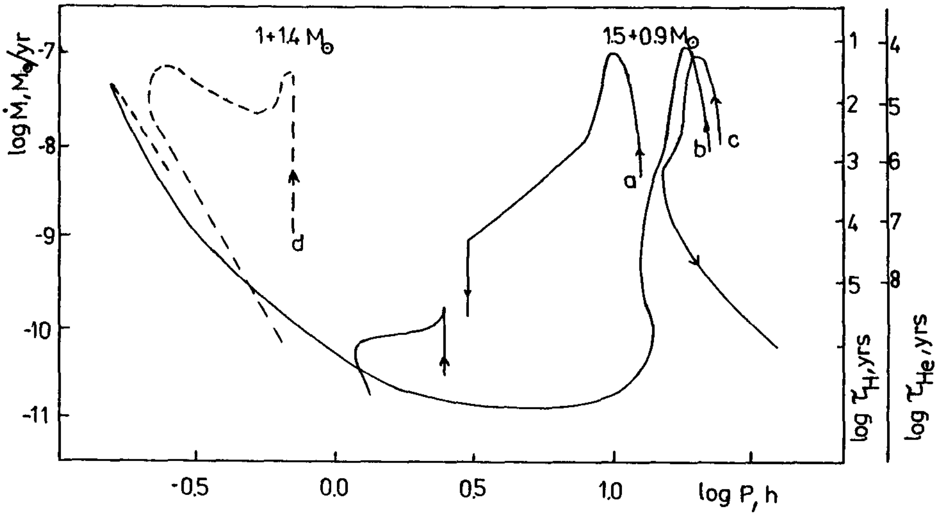

Fig. 3. Evolutionary tracks of $1.5M_\odot$ hydrogen and $1.4M_\odot$ helium donors in the lgPorb - lgM diagram. The initial masses of accretors are 0.9 and 1.4 $M_\odot$, respectively. The dashed part of track of $1.5M_\odot$ star corresponds to the stage when donor becomes a non-degenerate helium star. a - initially homogeneous hydrogen donor, b - donor with $X_c \approx 0$ at the instant of RLOF, c - donor with small helium core ($M_{He} \simeq 0.03M_\odot$), d - initially homogeneous helium donor.

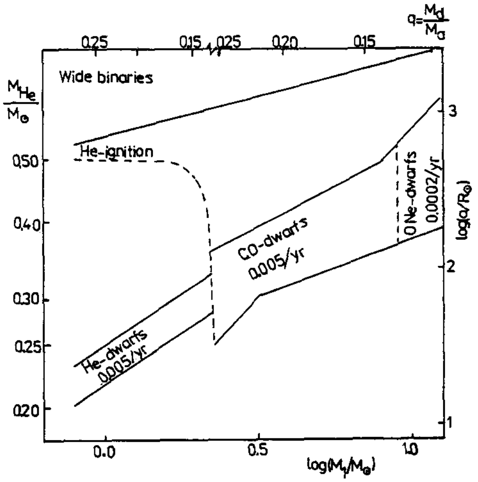

Fig. 4. The position of protocataclysmic binaries in the initial mass of the primary - initial semimajor axis of the orbit diagram. The lower continuous lines delineate the border of protocataclysmic binaries region, the upper one is the borderline between close and wide binaries. The upper scale shows the range of upper limits of mass ratios of components of initial systems. The left-hand scale shows the mass of helium dwarf corresponding to given initial separation of components. Indicated are the nature of white dwarfs - outcomes of evolution of respective system and the estimate of their birthrates according to Eq.(2). After Tutukov and Yungelson (1987).

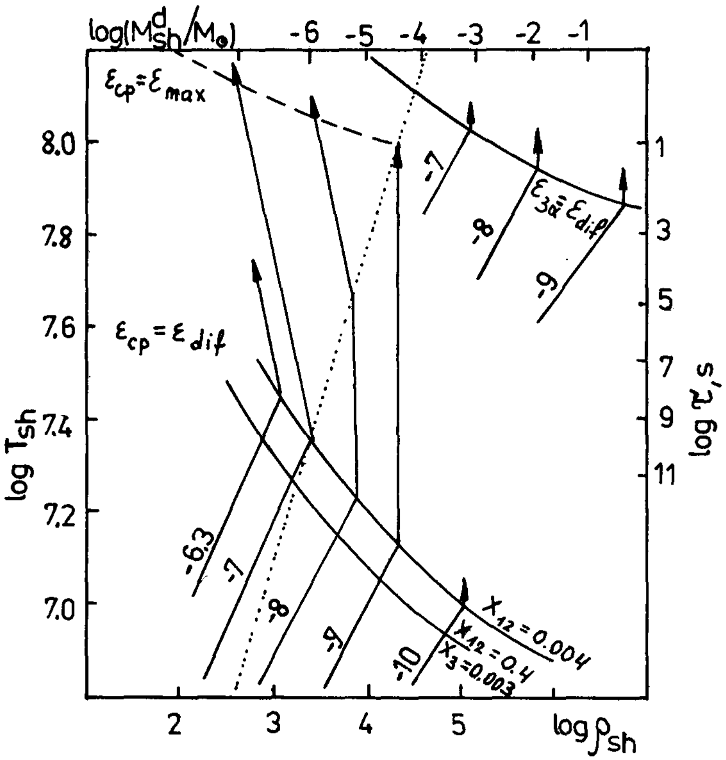

Fig.5. Evolution of density and temperature of thin accreting hydrogen and helium shells (after Tutukov and Ergma, 1979). The upper scale shows the masses of accumulated degenerate envelopes at the beginning of thermonuclear runaway. The right-hand scale shows the characteristic thermal time scales of accreted shells. The dotted line is the border of the region of degenerate matter. At the $\varepsilon_{cp}=\varepsilon_{dif}$ and $\varepsilon_{3\alpha}=\varepsilon_{dif}$ curves the energy release by nuclear burning equals the energy losses. The former curve is shown for two $C^{12}$ abundances; the curve for enhanced $He^3$ abundance coincides with the curve for $X^{12}=0.4$. The dashed line shows the limit of steady state hydrogen burning, which is limited by $\beta$-decays. The numbers along evolutionary tracks indicate the logarithms of accretion rates (in $M_\odot$/yr).