DU VAL, PATRICK, *Homographies Quaternions and Rotations* (Clarendon Press: Oxford University Press, 1964), ix + 116 pp., 35s.

This delightful book is one of the series of Oxford Mathematical Monographs.

Homographies and anti-homographies, at first defined in the complex plane, are transferred by stereographic projection to the sphere, on which they become projective self-transformations. Rotations and reflections appear as special cases. Consideration of finite groups of rotations follows, and naturally introduces the regular polyhedra. Quaternions enter the story because of their application to the representation of rotations of three- and four-dimensional Euclidean space. The elements of certain finite groups of unit quaternions correspond to the vertices of regular polytopes, which are discussed in Chapter 4. The final chapter deals with invariant forms of finite groups of homographic transformations. A connection with the theory of singularities of algebraic surfaces is only briefly mentioned. However the author explains in his Preface, which forms a succinct historical introduction to the book, that it was this connection which prompted him to undertake the work.

The monograph will appeal strongly to those readers for whom the fascination of mathematics lies in the exploration of elegant relationships between its different branches. The printing is first-class and the excellent illustrations include a colour plate.

D. MONK

WIELANDT, HELMUT, *Finite Permutation Groups* (translated from the German by R. Bercov) (Academic Press, New York and London, 1964), x + 114 pp., $2.45.

Although there are plenty of textbooks on abstract group theory and representation theory, none on permutation groups alone has appeared for over forty years, although the subject is very much alive. A monograph on this topic is therefore to be welcomed, especially when written by an authority like Professor Wielandt.

In the author's words, the aim is " to bring together some rather elementary theorems on permutation groups which either no longer appear in current textbooks or have not yet appeared in textbooks at all ".

In Chapter I we meet the fundamental concepts of a constituent of G (where G is a finite permutation group on a set Ω), an orbit of G, the subgroup G_Δ of G consisting of those permutations in G leaving each element of Δ fixed ($\Delta \subseteq \Omega$), transitivity, regularity, primitivity, a Frobenius group. Chapter II consists of 24 pages on k-fold transitivity, k-fold primitivity and $(k+\frac{1}{2})$-fold transitivity. (Some of this is hard going.) Chapter III deals with the transitive constituents of $G_\alpha(\alpha \in \Omega)$. Chapter IV deals with the situation where G, H are permutation groups on Ω and H is regular; usually H is a subgroup of G. In a natural way G can be regarded as a permutation group on H, and by working in the group ring of H over the integers or complex numbers one can obtain information about G. An interesting and valuable part of the book is the 13 pages of applications and discussion which follow the basic theory. A permutation group on a set of n elements can be regarded as a group of $n \times n$ matrices, so the methods of representation theory may be applied; this is discussed in the last chapter. The last section of this chapter is devoted to proving the author's theorem that a primitive group of degree $2p$ is doubly transitive if $2p$ is not of the form a^2+1.

J. L. BRITTON

S. G. MIKHLIN, *Variational Methods in Mathematical Physics*. Translated by T. Boddington (Pergamon Press, Oxford, 1964), xxxii + 584 pp., £5.

The variational method which forms the subject of this book consists in substituting for the problem of solving a given differential equation with boundary conditions an equivalent problem of finding a function which minimises a certain integral or functional associated with the equation. A familiar example occurs when the differential equation is that governing the equilibrium of an elastic medium, in which case the integral to be minimised represents the potential energy of the medium. The eigenvalue problem for a given operator can also be transformed into an equivalent problem of minimising a certain functional; this was recognised nearly a century ago, in relation to vibrating systems, by Rayleigh, who enunciated the principle that " the period of a conservative system vibrating in a constrained type about a position of stable equilibrium is stationary in value when the type is normal ", the period being of course a functional depending on the assumed type. Various practical methods for finding approximate solutions to certain problems, including many of the problems of classical mathematical physics, are based on the variational method. They usually depend (e.g. in Ritz's method) on the construction of a " minimising sequence " of functions which under certain conditions converges in some well-defined sense to the function which minimises the appropriate functional, so that the successive terms of the sequence constitute successive approximations to the solution.

The first half of the book under review is theoretical, and gives a lucid account, with a minimum of mathematical sophistication but an adequate degree of precision, of the relevant properties of the operators commonly occurring in mathematical physics, with a careful discussion of the conditions they must satisfy for the variational method to be applicable. The construction of minimising sequences is also treated along with the nature of their convergence (usually a generalisation of convergence in the mean). The illustrations are chosen mainly from the theory of elasticity.

The rest of the book uses more advanced mathematics. In it the results of the first half are re-formulated and generalised in terms of Hilbert Space, using the Lebesgue integral; a concise outline of both of these topics is included, reinforced by references to the standard literature. A subsequent chapter is devoted to the important question of the theoretical estimation of the error of the approximate solution, and there is an illuminating chapter consisting of numerical examples of the methods developed in the text. Variants of the method associated with the names of Trefftz, Dubnov, Galerkin, Kryloff and others including the author himself are treated. A final short chapter discusses finite difference methods, widely used in engineering practice, in which a differential equation is replaced by a set of difference equations. This chapter has no real connection with the rest of the book, but has apparently been retained from an earlier version of the work under the wider title " Direct methods in Mathematical Physics ".

The bibliography will be of limited use to non-Russian readers since references are, perhaps not unexpectedly, almost exclusively to Russian work, of which it is clear that a considerable amount exists, much of it probably little known outside U.S.S.R.

The English version reads well, apart from an occasional Muscovism—such as the use of the dash in place of the copula—which seems to have survived the translation. It is a pity that such an expensive book should be marred by an undistinguished layout and typography which can only be described as slipshod; but these superficial defects should not impair its usefulness. It is a most valuable work which brings together in a systematic treatment a wide range of material not readily accessible elsewhere, and can be studied with great advantage by anyone interested in the theory and practice of approximate solutions in Mathematical Physics.

R. SCHLAPP

MEINARDUS, GÜNTER, *Approximation von Funktionen und ihre numerische Behandlung* (Berlin, Springer-Verlag, 1964), 180 pp.

Until recent years, most of the theory of function approximation has remained in original papers, and there have been few, if any, satisfactory textbooks on the subject. The advent of high-speed computers, however, has made it essential that the known facts regarding this topic be gathered together and presented in a manner suitable for

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