Stellar Rotation
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Solar Differential Rotation Revealed by Helioseismology and Simulations of Deep Shells of Turbulent Convection

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Abstract. The sun is supposedly a very simple star, halfway along its long and possibly boring life on the main sequence. Yet it has some distinguishing features. Since we live close to it, our existence is probably blessed by this star having only modest cycles of magnetic activity and weak winds. By being so close, we can observe many aspects of the diverse range of motions and magnetic fields linked to turbulent convection in its convection zone. And this turns out to be anything but simple as we consider the dynamical coupling of convection, rotation and magnetism within this modest star. The lessons that have emerged from recent helioseismic probing of the solar interior and from 3–D numerical simulations of turbulent convection have bearing on differential rotation and magnetic dynamo action also occurring within more complex stars. We consider recent findings from both helioseismology and theoretical modelling on the operation of the deep shell of vigorous convection within our nearest star.

1. Interaction of Convection, Rotation and Magnetism

The sun is a magnetic star whose variable activity has a profound effect on our technological society on Earth. The high speed solar wind and its energetic particles, mass ejections, and flares that affect the solar-terrestrial interaction all stem from the variability of the underlying solar magnetic fields. We are in an era of fundamental discovery about the overall dynamics of the solar interior and its ability to generate magnetic fields through dynamo action. This has come about partly through guidance and challenges to theory being provided by helioseismology as we now observationally probe the interior of this star. It also rests on our increasing ability to conduct simulations of the crucial solar turbulent processes by using the latest generation of supercomputers.

Helioseismology has established that the latitudinal variation of angular velocity Ω observed near the surface, where the rotation is considerably faster at the equator than near the poles, extends through much of the deep convection envelope with relatively little radial dependence (see review here by Christensen-Dalsgaard 2003). A region of strong shear in the differential rotation, now known as the *tachocline* (Spiegel & Zahn 1992), exists at the base of the convection zone where Ω adjusts to apparent solid body rotation in the deeper radiative interior (Fig. 1*a*). Also present near the surface is a thin shearing boundary layer in which Ω increases rapidly with depth at intermediate and low latitudes. This *near-surface shear layer* exhibits complex meandering large-scale flows (Fig. 4) over a range of depths that are not unlike jet-streams, evolving meridional circulations that differ in the two hemispheres, and propagating bands of faster

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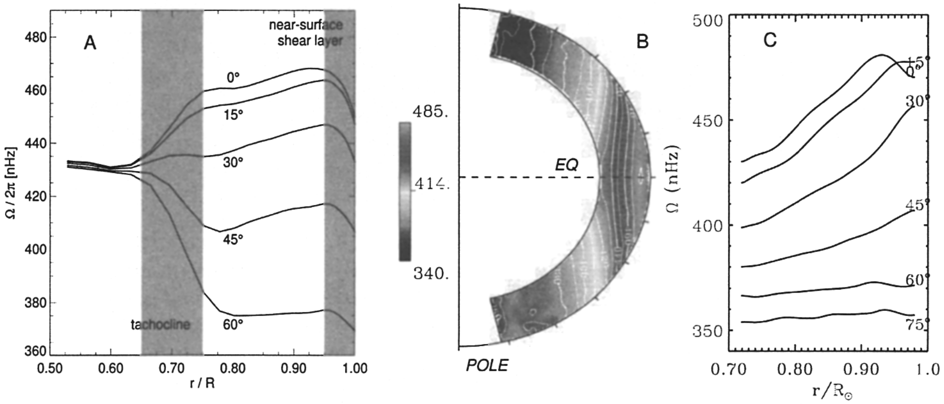

Figure 1. (*a*) Angular velocity profiles $\Omega/2\pi$ as deduced from inversion of five yr of GONG helioseismic frequency-splitting data, plotted against radius r/R at indicated latitudes. The variable shear just below the surface is clearly evident, as is the tachocline near the base of the convection zone, determined to be at radius 0.713 R [Howe et al. 2000]. Time-averaged angular velocity profiles with radius and latitude obtained from the 3–D deep spherical shell convection simulation (Case E, Fig. 3), displayed both (*b*) as contours involving a fast equator and slow poles, and (*c*) as cuts with radius at the indicated latitudes.

zonal flow. Such *Solar Subsurface Weather* (SSW, Haber et al. 2002, Toomre 2002) coexists with the highly turbulent convection visible as granulation and supergranulation.

A major dynamical question about our nearest star is the origin of its varying magnetic fields. The fields, like the underlying turbulence, can be both orderly on some scales and chaotic on others. Most striking is that the sun exhibits 22-year cycles of global magnetic activity, involving sunspot eruptions with very well defined rules for field parity and emergence latitudes as the cycle evolves. Coexisting with these large-scale ordered magnetic structures are small-scale but intense magnetic fluctuations that emerge over much of the solar surface, with little regard for the solar cycle. This diverse range of activity is most likely generated by two conceptually distinct magnetic dynamos: a *small-scale dynamo*, functioning within the intense turbulence of the convection zone, that builds the chaotic magnetic fluctuations, and a *global dynamo*, seated within the strong rotational shear of the tachocline at the base of the convection zone, that builds the more ordered fields.

2. Spherical Shells of Turbulent Compressible Convection

We have conducted extensive 3–D simulations of compressible convection in rotating spherical shells to study the coupling between global-scale convection and rotation in seeking to understand how the solar differential rotation is established. Such simulations capable of studying fairly turbulent convection have been enabled through recent advances in massively parallel supercomputing. We have been using our Anelastic Spherical Harmonic (ASH) code to study full spherical shells of rotating turbulent convection, using spherical harmonic Y_ℓ^m

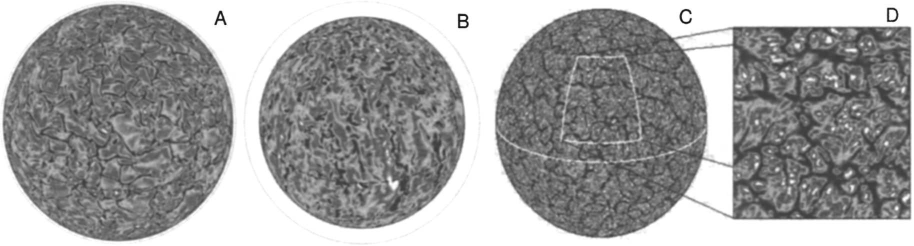

Figure 2. Snapshots of radial velocity on spherical surfaces near (*a*) the top and (*b*) the middle of the deep shell of rotating turbulent convection studied with ASH. Dark tones denote downflows, lighter shades upflows; dashed curve indicates the equator. The flow fields are dominated by intermittent plumes of upflow and stronger downflow extending over much of the shell depth, some possessing distinctive cyclonic swirl [Brun & Toomre 2002]. (*c*) More complex patterns in radial velocities, including (*d*) a close-up view, realized in even higher resolution simulations carried out in thinner shells used to study the formation of near-surface shear layer [DeRosa, Gilman & Toomre 2002].

expansions up to degree $\ell \sim 680$ (Elliott et al. 2000; Miesch et al. 2000; Miesch 2000; Brun & Toomre 2001, 2002). These 3–D models are intended to be a faithful if highly simplified description of the solar convection zone. Solar values are taken for the heat flux, rotation rate, mass and radius, and a perfect gas is assumed since the upper boundary of the shell lies well below the H and He ionization zones; contact is made with a realistic solar structure model for the radial stratification being considered. The anelastic approximation is used to capture the effects of density stratification without having to resolve sound waves. The computational domain extends from about 0.63 R to 0.98 R, where R is the solar radius, thereby including in some of our studies a region of stable stratification of thickness 0.07 R below the primary unstable zone in which effects of penetrative convection can be studied. Such shells have an overall density contrast in radius up to about 90, and thus compressibility effects are substantial. We have softened the effects of the very steep entropy gradient close to the surface that would otherwise favor the driving of very small granular and mesogranular scales of convection, since these require a spatial resolution at least ten times greater than presently available. The flux of enthalpy by the unresolved eddies near the surface is explicitly taken into account with subgrid-scale (SGS) terms.

Complex and Evolving Flow Fields The resulting convection in such spherical shells is highly time dependent and the flows are intricate. As shown in radial velocity images (Figs. 2*a*, *b*, 3*a*), the convection is dominated by intermittent plumes of upflow and stronger downflow, some possessing a distinctive cyclonic swirl. The dominant role of coherent plumes, first revealed in planar geometry, has now become apparent as we study more turbulent flows. Many downflows extend over the full depth of the zone, changing from wavy downflowing sheets to distinct plumes at greater depths. The convective patterns also evolve over fairly short time scales compared to the solar rotation period, and are advected and sheared by the strong differential rotation that they drive. This suggests that the largest global scales of convection are unlikely to be recognizable after one full rotation, and thus may explain much of the large-scale flow evolution seen in helioseismic probing of SSW.

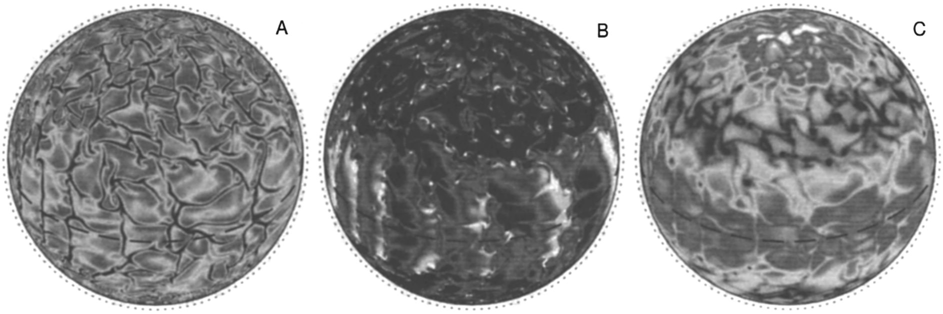

Figure 3. Snapshots in simulation with ASH (Case E) of (a) the radial velocity (with dark tones being downflows), (b) the enstrophy, revealing strong fronts (bright tones) at the lower latitudes that exhibit prograde propagation and more isotropic swirling structures at higher latitudes, and (c) the fluctuating temperature field involving a warm band near the equator, a relatively cool band at mid latitudes, and the warmest region near the poles.

The range of convective structures present is particularly evident as viewed in terms of the flow enstrophy (vorticity squared) shown in Figure 3*b*. Here the prograde propagating fronts of strong enstrophy (aligned roughly with the rotation axis, or north-south) are strikingly visible at low latitudes. They are replaced at higher latitudes by more isotropic, swirling cyclonic structures (of smaller scale) that are retrograde propagating. This emphasizes that the large-scale convection has a different intrinsic character at low and high latitudes (with a delineation roughly at 40°). One might well expect SSW to likewise show such behavior, and this prediction is being pursued now with helioseismic analyses. Global-scale variations in temperature are evident as well, with Figure 3*c* showing prominent zonal banding, involving overall variations of about 6K near the top of the convection zone. Such banding arises from systematic variations with latitude in both the radial and latitudinal enthalpy (heat) fluxes carried by the convection, and through heat transported by its associated meridional circulations. The presence of such latitudinal temperature contrasts contributes to a thermal wind component (baroclinicity) to explain some of the differential rotation that results, as analyzed in detail in Brun & Toomre (2002).

Maintaining a Strong Differential Rotation These 3–D simulations of solar convection with ASH are making serious contact with helioseismic findings about differential rotation within the deep interior of the sun. The time-averaged angular velocity Ω in one of our turbulent simulations (Fig. 1*b*) is nearly constant on radial lines throughout much of the convection zone at mid-latitudes, and there is a systematic decrease of rotation rate with latitude from the equator to the poles (Brun & Toomre 2001, 2002). The equator to pole contrast in Ω is of order 30%, much like in the sun. A surprising degree of coherent structure involving downflowing plumes can be embedded in otherwise chaotic flow fields. These structures play a significant role in yielding Reynolds stresses that serve to redistribute angular momentum. We can understand many aspects of the resulting Ω profiles in terms of the angular momentum transport achieved by the modified mean Reynolds stresses established by the turbulent convection, coupled with the effects of the fairly complex meridional circulations that are present. The meridional circulations typically involve several cells in latitude

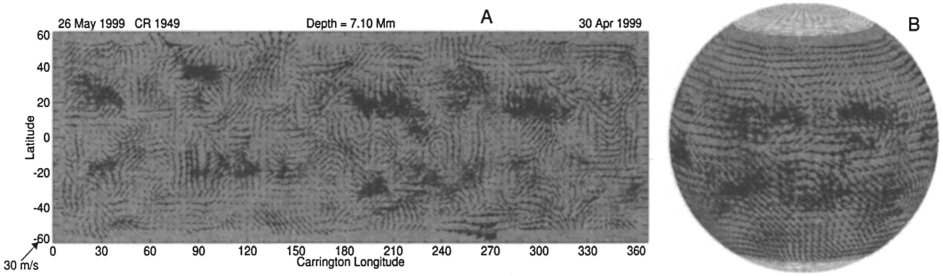

Figure 4. (*a*) Synopotic maps of horizontal flows of SSW with longitude and latitude deduced from helioseismic ring-diagram probing at a depth of 7 Mm below the solar surface for a rotation in 1999 when the sun was nearing the peak of magnetic activity. Shown also are surface magnetic field patterns (dark and light tones indicate field polarity). [Haber et al. 2002] (*b*) Projection of SSW flows onto spherical surface for flows in April 2001 near long-lived active region complexes exhibiting major flaring and coronal mass ejections.

within a given hemisphere, and often two layers of cells in depth across the convection zone. The latter appears to be crucial in obtaining the particularly slow rotation at high latitudes now being realized in our simulations (Fig. 1*b*); this is a prominent feature in the helioseismic findings for Ω (Fig. 1*a*) that has previously been difficult to replicate. Further, the two hemispheres in our simulations can exhibit differing meridional circulation patterns as deduced from averages formed over several rotations, emphasizing that such broken symmetries are typical rather than an exception. There is also noticeable evolution in what is deduced to be the meridional circulation from such longitudinal and temporal averaging. Something similar is seen in the latest helioseismic probing of SSW, with reversing circulations apparent in one hemisphere and not in the other (Haber et al. 2002). It should be emphasized that the ASH simulations have not yielded single meridional circulation cells that span all latitudes and depths within a hemisphere, as is sometimes favored in estimates of transport that might be achieved by such circulations. We see our simulations with ASH now providing subtle tools, previously unavailable, for interpreting the complex interior dynamics being revealed by helioseismology.

Near-Surface Shear Layer To examine some of the convective dynamics that may yield the near-surface rotational shear layer (Fig. 1*a*), we have conducted preliminary simulations with ASH within thin spherical shells, such as positioned between 0.90 R and 0.98 R (DeRosa, Gilman & Toomre 2002). Figures 2*c*, *d* show the level of complexity in the turbulent convection of many scales realized in those simulations, with the smallest resolvable cellular flows now close to that of supergranulation. These small cells (15 to 30 Mm across) tend to be advected laterally by the horizontal outflow motions associated with the broader cells (about 200 Mm across) possessing a connected network of downflow lanes near the top of the layer. These shells are the first to exhibit a decrease in angular velocity Ω with radius at low and mid latitudes that are in the spirit of the helioseismic findings, strongly encouraging the deep shell simulations in which we now are engaged that are capable of both resolving supergranulation and of dealing with angular momentum redistribution within the full shell.

3. Contact with Helioseismic Probing of SSW

The complex flows of SSW shown in Figure 4 are being determined with helioseismic probing of wave fields within localized domains, using techniques known as ring-diagram dense-pack analyses (Haber et al. 2000) and time-distance tomography (Kosovichev, Duvall & Scherrer 2000). The prominent evolution and propagation of the large-scale convection patterns in our modelling with ASH are beginning to provide the means to understand many of the flow changes seen within SSW. This includes the symmetry breaking and evolving multi-cell character exhibited by the meridional circulations deduced from helioseismology. Magnetism appears to play an important role in some of the deflected or converging flows seen near active complexes. The inclusion in a self-consistent manner of large-scale magnetism into the convection simulations is now becoming feasible. Thus we foresee many improvements in how the theoretical models will be used in the interpretation of SSW. The likely advances in such theoretical approaches, combined with refined helioseismic probing forthcoming from GONG+ and from HMI on the Solar Dynamics Observatory, holds out a promising future for studying the very rich dynamics of the solar convection zone.

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