Solvency capital requirement and the claims development result

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Abstract

In this paper, four variants of calculating the Solvency Capital Requirement for long-tail liabilities satisfying Solvency II regulations are discussed. The merits of each metric are related to the stated objectives of Solvency II. Assumptions made in the calculations are assessed for suitability for the determination of an appropriate level of Solvency Capital. We show that two methods for calculating Solvency Capital provide insufficient capital to restore the Economic Balance Sheet in the event of distress. The standard formula referencing the Claims Development Result is shown to be too conservative when models are correctly specified.

1. Introduction

Solvency II regulation (European Commission, 2007, 2009b) specifies an insurer should be able to withstand a 1 in 200 years loss and still have sufficient capital for risk to be fairly transferred to a third party. While some example formulae are provided, the onus is on the insurer to demonstrate that a sufficient level of capital is held. Due to the general nature of the specification, there are several measures for calculating the appropriate level of risk capital (Merz & Wüthrich, 2008; England, 2011; Munroe *et al*., 2015; and others).

Loss data arrive each calendar year. When new data come in, there are two possibilities:

- the data fall within tolerance of the loss distributions projected by the fitted model and so parameters can be simply reestimated (see also Ohlsson & Lauzeningks (2009) in respect of algorithm expectations on updating);
- the data call the prior year's parameter-structure into question and the fitted model needs to be revised.

We only consider the first case in this paper since the second requires external input.

Reestimating the fitted model introduces implicit correlations between projected distributions in each subsequent calendar year. Correlations between these projected distributions will be greater than 0 since the data used to estimate any parameter at time $t + 1$ includes all the data used to estimate the parameter at time t. A high loss at time $t + 1$ will increase the parameter estimate at time $t + 1$ relative

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to time t and vice versa. The magnitude of the correlation depends on the level of uncertainty in the parameters. The higher the parameter uncertainty, the higher the correlations between successive distributions.

In this paper, we discuss four of these measures for the Solvency II Capital Requirement (SCR) with increasing degrees of complexity. The complexity of the solutions is directly connected to calendar year correlation assumptions. The first three measures were presented in Munroe *et al.* (2015). The fourth example is a proxy for SCR given in the standard formula (European Commission, 2010, SCR.10.54.) based on the concept of the Claims Development Result (CDR) (Merz & Wüthrich, 2008; England, 2011). This treatment is intended to bring the crucial issues to the fore.

Two naïve solutions from Munroe *et al.* (2015) are demonstrably insufficient to truly restore the Economic Balance Sheet to fair value except under very specific conditions. Real world environments are rarely so simple, and in this paper we demonstrate heuristically that the third measure is the most suitable candidate for achieving the Solvency II objectives.

The CDR, as introduced by Merz & Wüthrich (2008), is commonly used in the Solvency II literature. CDR is the difference between the expected ultimate loss at inception (or at a given time, t) and the new estimate 1 year later. Over a 1 year time horizon, *CDR* is said to represent the movement of the Economic Balance Sheet (Merz & Wüthrich, 2008).

A method for calculating the Solvency II SCR via the CDR has been given as the third standard formula proxy for SCR (European Commission, 2010). Here the standard deviation of the CDR is used to quantify reserve risk. A distribution, such as the log-normal, is assumed where the mean and standard deviation correspond to the expected ultimate loss and standard deviation of the CDR, respectively. The 99.5[th] percentile of this distribution is the estimate of the SCR.

In this formulation, the CDR is not explicitly conditioned on a distress scenario arising in the next calendar year but rather examines the situation such that the adjustment to the mean ultimate in the next calendar year is a 1 in 200 adjustment. We show that, should the distributional assumptions be known, the 99.5[th] percentile of the variation in mean ultimate (VMU) is far too conservative a measure of risk capital and does not properly allow for risk diversification between the calendar years.

The four measures of the SCR are related to each other and compared using a few examples.

2. Solvency II 1-Year Risk Horizon and the Economic Balance Sheet

There are three basic elements to the Solvency II directives issued by the European Commission (2010) in respect to Risk Capital:

- For the purpose of calculating the risk margin (RM) and understanding the hypothetical flow of capital, it is assumed that Risk Capital is raised at the beginning of each year and any unused capital is returned to the capital provider at the end of the year;
- the analyses are conditional on the first (next) calendar year being in distress (99.5%);
- at the end of the first year in distress, the balance sheet can be restored in such a way that the company has sufficient Technical Provisions (TPs) (equivalently the Fair Value of Liabilities) to continue business or to transfer the liabilities to another risk bearing entity.

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The Cost of Capital approach is used to calculate the RM for non-hedgeable risks. The RM represents the premium paid to the notional risk capital providers for providing Risk Capital.

As an insurer must be able to withstand a 1 in 200 year loss and still have sufficient capital for risk to be fairly transferred to a third party (European Commission, 2007, 2009b), it follows that the Best Estimate of Liabilities (BEL) and RM, collectively known as the TP, are the cornerstone of the Solvency II 1-year risk horizon.

$$TP = BEL + RM \qquad (1)$$

The regulation includes provision to discount TP by the risk-free rate. However, for simplicity the risk-free rate (the present value discount) is omitted from all calculations in this paper.

3. SCR: A Naïve Definition

In order to demonstrate that the company has sufficient capital to sustain losses at the 99.5^{th} percentile in the first calendar year the effect of a 1 in 200 year loss in the next calendar year is to be estimated.

The BEL covers the loss up to the best estimate. Only the excess loss, the loss above the mean, is required as risk capital to sustain losses at the 99.5^{th} percentile.

Let Y denote a random variable representing the loss (the reserve).

In the reserving context, the Value-at-Risk ($V@R$) is the capital at risk for a quantile, $\alpha \in (0, 1)$, given the distribution of losses, Y, and the expected loss, $\overline{Y}$.

$$V @ R_\alpha(Y) = F_Y^{-1}(1 - \alpha) - \overline{Y} \qquad (2)$$

where F is the cumulative distribution function.

Consider a loss reserve distribution as displayed in Figure 1. The vertical bar displays the mean, $699M$, of the distribution. The 75th percentile of the distribution is $725M$. The difference between the 75th percentile and the mean, $27M$ (rounding), is the capital that would be lost if the mean reserve
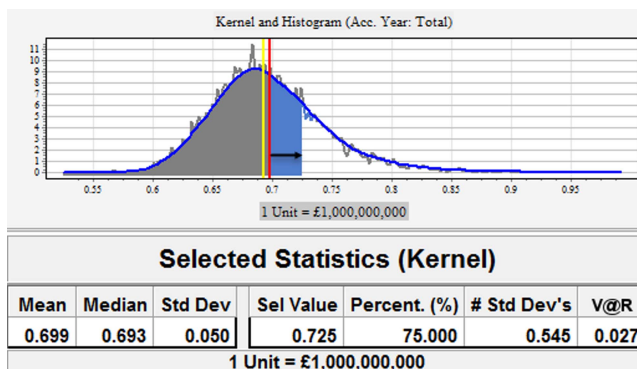

Selected Statistics (Kernel)						
Mean	Median	Std Dev	Sel Value	Percent. (%)	# Std Dev's	V@R
0.699	0.693	0.050	0.725	75.000	0.545	0.027
1 Unit = £1,000,000,000						

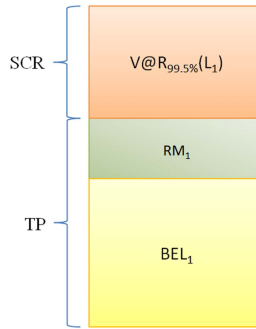**Figure 1.** Value-at-risk at 75th percentile for an example distribution.

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Figure 2. One-year run-off. TP, Technical Provision; SCR, Solvency II Capital Requirement

was booked and the losses actually came in at the 75th percentile of the loss distribution. That is, the V@R is 27M. The probability of losing at least 27M is (1−α), which is 25%.

There are other measures of V@R such as Tail — V@R which calculates the average loss should the loss exceed the percentile, α. These alternatives are not considered in the Solvency II context, however they may be used in other solvency regulation (for instance, Swiss Solvency Test Swiss Federal Office of Private Insurance, 2004).

This simple definition of SCR is

$$SCR^1 = V @ R_{99.5\%}(L_1) \tag{3}$$

where L_1 is a random variable representing the first future calendar year loss and $V@R_{99.5\%}$ is the 99.5th percentile of the V@R.

In this case, the Economic Balance Sheet for 1-year run-off is depicted in Figure 2.

Here, BEL_1 is the BEL for year 1, that is

$$BEL_1 = E(L_1) \tag{4}$$

RM_1 is the RM allocated for year 1, that is

$$RM_1 = s \cdot V @ R_{99.5\%}(L_1) \tag{5}$$

where s is the spread — the return required above the risk free rate as specified in the regulation (European Commission, 2009a), and

$$TP_1 = BEL_1 + RM_1 \tag{6}$$

where TP_1 is the TP (Fair Value) for year 1 ($TP = TP_1$).

In the block diagram above, SCR is the Risk Capital raised from the (hypothetical) risk capital providers and RM_1 is the cost to the insurer for access to this capital.

If the loss exceeds the BEL_1 the Risk Capital (in this case SCR^1) is drawn on. At the end of the year, the risk capital providers receive the SCR^1 less the drawn capital.

RM_1 is paid to the risk capital providers at the end of the year irrespective of the value of the loss.

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3.1. Extending SCR^1 to Multiple Year Run-Off

In the case of multiple years in run-off, equation (6) is generalised to equation (1). Rather than calculating the BEL of the first calendar year, the sum of the mean losses of all future calendar years, years 1, 2, … , *n*, is calculated. Similarly, *RM* is calculated as the sum of the RMs for all future calendar years.

Note SCR^1, which depends only on L_1, is unaffected by broadening the context to include *n* future calendar years.

SCR^1 is naïve because it assumes future calendar year loss estimates are unaffected by an unusually high result in year 1. In other words, it would only satisfy the Solvency II requirement on the assumption that future calendar year correlations are 0. The correlations can only be zero exactly if there is no parameter uncertainty. In this case, the only source of variation would be the process volatility inherent in the data. The unconditional distributions will then be adequate for complete calculation of risk capital and there is no need to allocate additional funds to update either the *BEL* or the *RM*. This is the theoretical situation where there is maximum diversification between calendar years and it forms the lower bound of the SCR^2 and SCR^3 solutions.

Parameter uncertainty induces correlation between future calendar year loss distributions. Further, calendar year correlations between loss distributions are positive when parameter estimates for a fixed model structure are updated. SCR^1 fails the third of the Solvency II requirements, unless the boundary case is applicable.

4. SCR: Including Change in BEL

Incorporating calendar year correlation leads to an updated definition of SCR. If the first calendar year is in distress, the conditional loss distributions for subsequent years will have higher mean losses.

If the first year is in distress, the adjustments in the mean losses for subsequent years should be part of the Risk Capital raised. This way the BEL can be restored to Fair Value at the beginning of the second calendar year. The business can then continue to operate or the liabilities can be transferred to another risk bearing entity.

We denote the measure including these increments as SCR^2. This definition is closely related to the definition of the CDR — see section 7. The change in the mean is equivalent to change in own funds when a distress event occurs.

That is

$$SCR^2 = SCR^1 + \sum_{i=2}^{n} \Delta BEL_i \qquad (7)$$

where

$$\Delta BEL_i = BEL_i \mid (L_1 = \lambda) - BEL_i \qquad (8)$$

BEL_i denotes $E(L_i)$, and $BEL_i|(L_1 = \lambda)$ is the mean of the conditional distribution of L_i (loss in calendar year *i*) given that the first year is in distress, that is $(L_1 = \lambda)$, where

$$\lambda = E(L_1) + V @ R_{99.5\%}(L_1) \qquad (9)$$

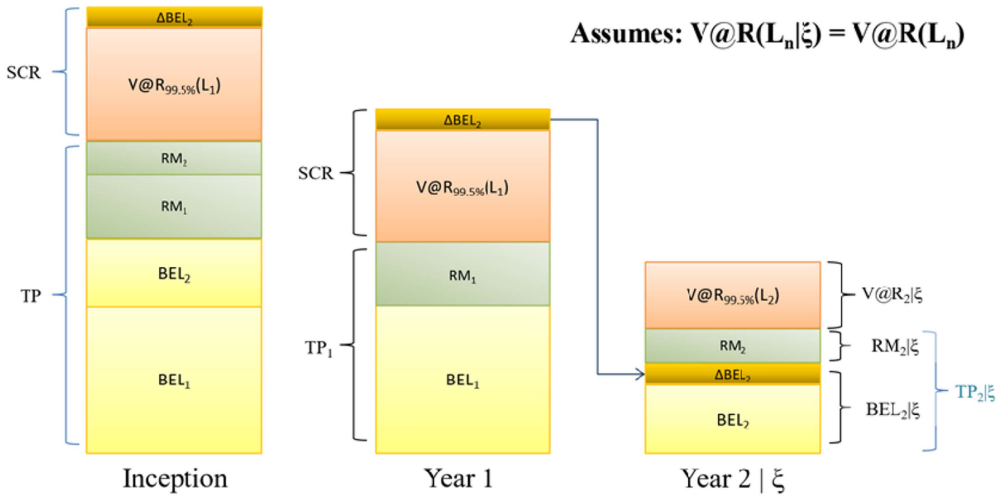and *n* is the number of future calendar years in the calendar year liability stream.

Figure 3. Two-year run-off. TP, Technical Provision; SCR, Solvency II Capital Requirement

The 2 years in run-off case is illustrated in Figure 3. The event that $L_1 = \lambda$ is designated by ξ.

Consider the block diagram in Figure 3.

Given year 1 is in distress, we study year two first.

For the conditional distribution $L_2|\xi$ the increment in mean (above the unconditional distribution of L_2) is ΔBEL_2.

The event ξ results in (1) drawing on the risk capital for year 1 and (2) re-estimation of BEL at year 2 and hence SCR^2 is the sum of the V@R plus the ΔBEL_2.

The RM, RM_1, is now based on an SCR that includes ΔBEL_2.

The balance sheet at inception contains the sum of the BELs, the sum of the RMs, and the SCR^2.

The flow of capital should the first year be in distress is as follows:

- BEL_1 and $V@R_{99.5\%}(L_1)$ are consumed;
- $BEL_2 + \Delta BEL_2$ is allocated to year 2 to restore the mean loss to conditional mean;
- RM_2 is allocated to the RM for year 2;
- RM_1 is returned to the risk capital provider;
- RM_2 is left unchanged.

However, should the first year be in distress, it is likely that the $V@R_{99.5\%}(L_2|\xi)$ will be larger than $V@R_{99.5\%}(L_2)$. Let ΔRM_2 be the difference between the RM RM_2 and the RM $RM_2|\xi$. Additional capital is needed to restore the Economic Balance Sheet to its Fair Value. This is necessary to satisfy the Solvency II objective that the liability risk can be fairly transferred to a third party (European Commission, 2007, 2009b).

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SCR^2 acknowledges the need for recalculation of future years after a distress year but fails to carry it through to every component of the TP. This leads to SCR^3.

5. SCR: Including Change in TP

Figure 4 includes the increment in RM in year 2 assuming the first year is in distress.

Given year 1 is in distress, the increase in $V@R_{99.5\%}(L_2)$ is denoted by $\Delta V@R_{99.5\%}(L_2)$. This leads to an increase in the RM for year 2, ΔRM_2. The ΔRM_2 and ΔBEL_2 are both incorporated in the SCR for year 1 allowing the Economic Balance Sheet to be fully restored to its Fair Value at the beginning of year 2 in the event of distress. The cost of capital for holding SCR, RM_1, is adjusted to include the cost of also holding ΔRM_2 and ΔBEL_2.

The flow of capital should the first year be in distress is as follows:

BEL_1 and $V@R_{99.5\%}(L_1)$ are consumed;

$BEL_2 + \Delta BEL_2$ is allocated to year two to restore the mean loss to conditional mean;

$RM_2 + \Delta RM_2$ is allocated to the RM for year 2;

RM_1 is returned to the risk capital provider.

At inception the two Δ's are part of the risk capital in year 1 but they are allocated to the TP in year 2 in the distress event.

The definition of SCR^3 can be appropriately extended to a multiple year run-off as follows:
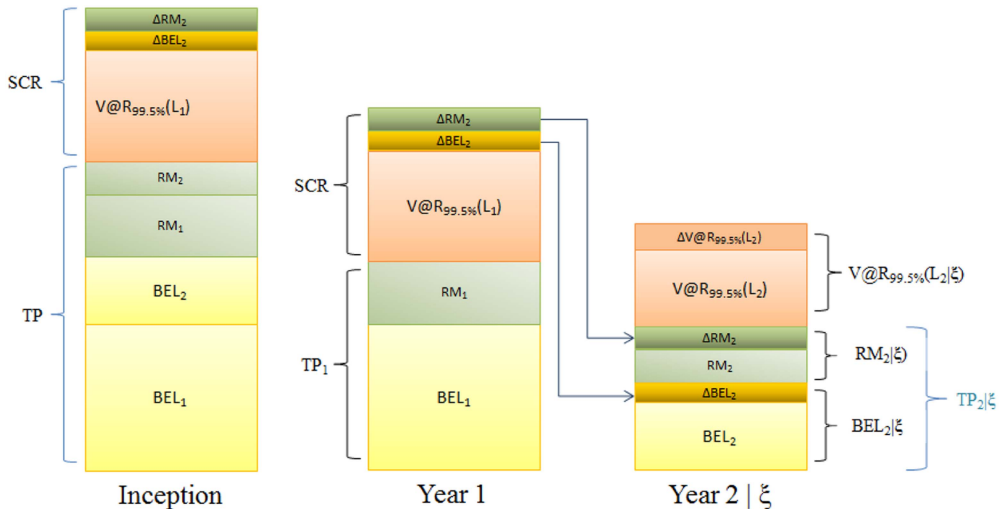
$$SCR^3 = SCR^2 + \sum_{i=2}^{n} \Delta RM_i \qquad (10)$$

Figure 4. Two-year run-off including ΔTP. TP, Technical Provision; SCR, Solvency II Capital Requirement

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If spread is 0%, then $SCR^3 = SCR^2$. However, the Economic Balance Sheet would then be different as the TP is different.

6. Variation in the Mean Ultimate Loss 1 Year Hence and the Claims Development Result

Let's decompose the ultimate loss, L_{Ult}, as the sum of the first future year calendar year loss and the sum of subsequent calendar year losses:

$$L_{Ult} = L_1 + \Sigma_{i=2}^n L_i \tag{11}$$

Then the expectation of ultimate loss conditional on the first future year loss $E(L_{Ult}|L_1)$ is

$$E(L_{Ult}|L_1) = L_1 + \Sigma_{i=2}^n E(L_i|L_1) \tag{12}$$

The quantity

$$\Delta_1 L_{Ult} = E(L_{Ult}|L_1) - E(L_{Ult}) \tag{13}$$

is a random variable dependent on L_1 that represents the variation in the mean ultimate loss. We denote it by VMU(L_1) or simply VMU. In the Solvency II literature, $-\Delta_1 L_{Ult}$ is commonly referred to as the CDR (Merz & Wüthrich, 2008; England, 2011). The VMU, is a more informative descriptor and under a distress scenario it is positive. (See discussion.)

The expected value of the VMU is 0. That is

$$E(\Delta_1 L_{Ult}) = E(E(L_{Ult}|L_1)) - E(L_{Ult}) = 0 \tag{14}$$

Taking the variance of the VMU produces the following:

$$Var(VMU) = Var(E(L_{Ult}|L_1)) \tag{15}$$

which is the same as the variance of the CDR.

This quantity represents the variance of the conditional mean ultimate 1 year hence.

This metric is important for senior management as it contextualises year to year changes in the estimate of mean ultimate. For a model and forecast scenario that are consistent there is statistical variation in the mean ultimate 1 year hence. That is, when a model and forecast scenario are updated with 1 year of new data, forecast scenario assumptions remaining consistent, variation in the mean ultimate is expected. See the example in section 8.

Indeed, by the law of total variance, $Var(L_{Ult})$ can be decomposed as follows:

$$Var(L_{Ult}) = E(Var(L_{Ult}|L_1)) + Var(E(L_{Ult}|L_1)) \tag{16}$$

The variance of the ultimate is decomposed into two parts. $E(Var(L|L_1))$ and the change in the mean ultimate, $Var(E(L|L_1))$. Substituting using equation 15 gives:

$$Var(L_{Ult}) = E(Var(L_{Ult}|L_1)) + Var(VMU) \tag{17}$$

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$$Var(VMU) = E(Var(L_{Ult}) - Var(L_{Ult} \mid L_1)) \tag{18}$$

The information acquired when the next year's results are known will reduce the variance of the ultimate. The variation contributed by the first future calendar year is removed (the losses are known), the subsequent parameter uncertainty is lower, and the forecast horizon is shorter. The mean reduction is: $E(Var(L_{Ult}) - Var(L_{Ult} \mid L_1))$. By (18) this is the Variance of the VMU.

The proportion of the VMU component is indicative of the sensitivity to the next calendar year's losses. If $Var(VMU)$ takes a large proportion of the total variance, then the expected ultimate is very sensitive to the losses in the next year. Conversely, if $Var(VMU)$ takes a low proportion of the total variance then the effects of re-parameterisation after year 1 are swamped by the process volatility. The impact of correlations on the VMU are discussed in section 9.

7. The Relationship Between the VMU and SCR^2

In this section, we show that $VMU(L_1 = \lambda) = SCR^2$. Where:

$$\lambda = E(L_1) + V@R_{99.5\%}(L_1) \tag{19}$$

Now:

$$VMU = E(L_{Ult} \mid L_1) - E(L_{Ult}) \tag{20}$$

$$= L_1 + \sum_{i=2}^{n} E(L_i \mid L_1) - E(L_1) - \sum_{i=2}^{n} E(L_i) \tag{21}$$

$$= L_1 - E(L_1) + \sum_{i=2}^{n} [E(L_i \mid L_1) - E(L_i)] \tag{22}$$

Consider VMU when L_1 is in distress. We can then write:

$$VMU(L_1 = \lambda) = V@R_{99.5\%}(L_1) + \sum_{i=2}^{n} [E(L_i \mid L_1 = \lambda) - E(L_i)] \tag{23}$$

$$= V@R_{99.5\%}(L_1) + \sum_{i=2}^{n} \Delta BEL_i \tag{24}$$

Given that calendar year loss correlations are positive, the ΔBEL_i are typically positive:

$$VMU(L_1 = \lambda) = -CDR(L_1 = \lambda) = SCR^2 \tag{25}$$

In the standard formula, one method to calculate the SCR is to calculate the $V@R_{99.5\%}(VMU)$.

Calculating the 99.5^{th} percentile of VMU is not equivalent to the realisation of $VMU(L_1 = \lambda)$.

8. The Distribution of VMU (-CDR)

In order to find the theoretical distribution of VMU it is necessary to have loss distributions projected by calendar year and their correlations because the measure is inherently linked with calendar time.

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The distributions going forward will depend on the future process volatility, the parameter estimates and their associated uncertainties. Correlations between the calendar year loss distributions are driven by parameter uncertainty. The magnitude of the correlations will also depend on process volatility.

The distribution of VMU is the sum of the conditional means (conditional on the first calendar year) minus the unconditional means: $\Sigma_{i=2}^{n}(E(L_i|L_1) - E(L_i))$.

The unconditional means, $E(L_i)$, are the mean losses based on the original model. In order to estimate the distribution of VMU, the following steps apply:

1. Use the model to simulate the next calendar year (L_1).

2. Re-estimate the same model, including the data for L_1, to obtain conditional means ($E(L_i|L_1):i>1$).

For the first calendar year, the difference between the simulated values and the unconditional mean of this year is ($L_1-E(L_1)$). For subsequent years, $i=2$ to n, the ith difference between the conditional means and the unconditional means is ($E(L_i|L_1) - E(L_i)$).

The sums of these differences are realisations of VMU.

To illustrate, assume a model specifies the loss distributions by calendar year as in Table 1. The total mean outstanding is 7, 138, 147 total losses paid to date of 10, 221, 194. The corresponding mean ultimate is 17, 138, 147.

If we simulate the next calendar year using the model, the distribution of the simulations for that year will have a mean of 1, 864, 457 and a SD of 75, 213. For each simulation, the model parameters are updated and used to obtain mean projections of the years $E(L_i|L_1)$ where $i>1$.

This leads to a distribution for the VMU, $\Sigma_{i=2}^{n}(E(L_i|L_1) - E(L_i))$. One such simulation is shown in Figure 5.

The 99.5[th] percentile of the VMU, 581, 656, is highlighted. The distribution of the VMU shows that there is a 1 in 200 probability the current estimate of the mean ultimate, 17, 138, 147, will be

Table 1. Example future calendar year distributions.

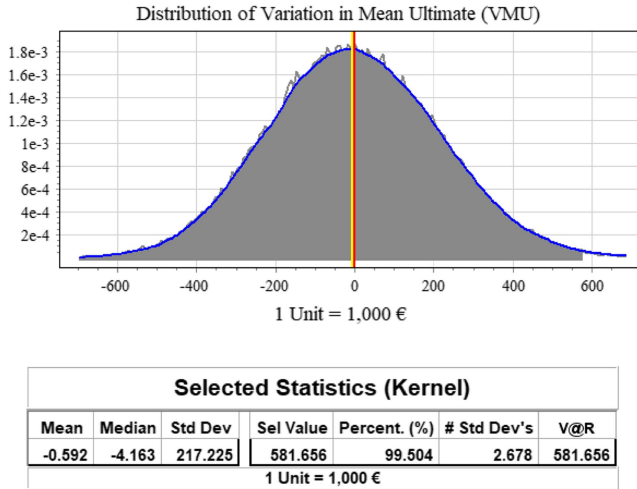| Calendar Year Summary		
Calendar Year	Mean Outstanding	SD
1988	1,864,457	75,213
1989	1,426,922	65,240
1990	1,063,255	55,376
1991	796,935	47,459
1992	604,496	40,873
1993	470,073	35,793
1994	363,510	30,881
1995	273,885	25,831
1996	182,833	19,397
1997	91,781	11,442
Total	7,138,147	309,956
1 Unit = 1 €		

Figure 5. Distribution of Variation in Mean Ultimate

increased by at least $581,656$, after next year's losses are realised. See section 10 for the relationship between the VMU at the 99.5^{th} percentile and SCR^2.

9. Decomposing the Total Variation of L_{Ult} into L_1 and $L_2 \dots L_n$ Given L_1

Consider two hypothetical cases where we assess the value of information about the next calendar year's losses. This is discussed in the context of the VMU.

High correlations imply a lot of value in knowing the next calendar year's loss. Low correlations imply knowledge of the next calendar year does not result in knowledge about future calendar years.

$$Var(E[L_{Ult}|L_1]) = Var(L_1 + \sum_{i=2}^{n} E(L_i \mid L_1)) \tag{26}$$

9.1. Case 1: Low Correlation Between Future Calendar Years

Assume there is very little correlation in the calendar year loss distributions (Table 2).

Since the correlation between L_i and L_1 is close to 0 (Table 2), then $E(L_i|L_1) \approx E(L_i)$. It therefore follows that:

$$Var(E[L_{ULt}|L_1]) \approx Var(L_1) \tag{27}$$

The forecast table with means, standard deviations, and expected variation conditional on the next calendar year's data are shown in Table 3.

In Table 3, the variation in the next calendar year's loss, $796,830$, is close to the total variation in the expected ultimate, conditional on the next calendar year's data, $807,822$.

When the correlation between calendar years is close to 0, the variance of the sum of the conditional expectations, $Var(\sum_{(i=2)}^{n} E(L_i|L_1))$, is close to 0.

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Table 2. Case 1: low future calendar year correlations

Calendar Year	Calendar Year Total Reserve Distribution Correlations									
	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
1988	1.0000	0.0031	0.0039	0.0046	0.0053	0.0058	0.0061	0.0061	0.0056	0.0044
1989	0.0031	1.0000	0.0056	0.0067	0.0078	0.0086	0.0091	0.0090	0.0083	0.0065
1990	0.0039	0.0056	1.0000	0.0090	0.0105	0.0115	0.0122	0.0122	0.0112	0.0088
1991	0.0046	0.0067	0.0090	1.0000	0.0131	0.0144	0.0153	0.0153	0.0140	0.0110
1992	0.005 3	0.0078	0.0105	0.0131	1.0000	0.0171	0.0182	0.0181	0.0167	0.0130
1993	0.005 8	0.0086	0.0115	0.0144	0.0171	1.0000	0.0205	0.0203	0.0186	0.0146
1994	0.0061	0.0091	0.0122	0.0153	0.0182	0.0205	1.0000	0.0219	0.0201	0.0158
1995	0.0061	0.0090	0.0122	0.0153	0.0181	0.0203	0.0219	1.0000	0.0205	0.0162
1996	0.005 6	0.0083	0.0112	0.0140	0.0167	0.0186	0.0201	0.0205	1.0000	0.0152
1997	0.0044	0.0065	0.0088	0.0110	0.0130	0.0146	0.0158	0.0162	0.0152	1.0000

Table 3. Case 1: forecast table and contribution to variation in mean ultimate (VMU)

Calendar Year	Mean Outstanding	SD	Conditional on Next Calendar Periods		Contribution to Variation in VMU
			VE[Var[Outs|Data]]]	SD[E[Outs|Data]]	
1988	2,271,475	796,830	0	796,830	▬▬▬▬▬▬▬
1989	1,738,278	618,518	618,511	2,935	▪
1990	1,295,092	460,371	460,362	2,887	▪
1991	970,534	345,194	345,184	2,683	▪
1992	736,019	264,090	264,079	2,414	▪
1993	572,207	212,380	212,370	2,147	▪
1994	442,367	172,743	172,733	1,863	▪
1995	333,198	143,302	143,294	1,548	▪
1996	222,351	113,518	113,513	1,132	▪
1997	111,577	78,963	78,961	620	▪
Total	8,693,097	1,266,196	975,026	807,822	

Forecast scenario: Case-l

The contribution of the variation in the mean losses, $L_2 \ldots L_n$ conditional on L_1, to the VMU is negligible compared to the variation in L_1 (Table 3).

9.2. Case 2: High Correlation Between Future Calendar Years

Consider high positive correlation between calendar year loss distributions (Table 4).

The importance of the covariance terms as contributors to the total variation increases as the correlation increases.

$$Var(E[L|L_1]) = Var(L_1 + \sum_{i=2}^{n} E(L_i \mid L_1)) \tag{28}$$

$$= Var(L_1) + 2 \cdot cov(L_1, \sum_{i=2}^{n} E(L_i|L_1)) + Var(\sum_{i=2}^{n} E(L_i|L_1)) \tag{29}$$

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Table 4. Case 2: high future calendar year correlations

	Calendar Year Total Reserve Distribution Correlations									
Calendar Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
1988	1.0000	0.9599	0.9595	0.9560	0.9514	0.9456	0.9389	0.9313	0.9225	0.9118
1989	0.9599	1.0000	0.9862	0.9849	0.9320	0.9776	0.9721	0.9654	0.9574	0.9476
1990	0.9595	0.9862	1.0000	0.9930	0.9915	0.9884	0.9840	0.9785	0.9715	0.9627
1991	0.9560	0.9849	0.9930	1.0000	0.9955	0.9936	0.9905	0.9861	0.9803	0.9726
1992	0.9514	0.9820	0.9915	0.9955	1.0000	0.9955	0.9947	0.9915	0.9868	0.9803
1993	0.9456	0.9776	0.9884	0.9936	0.9965	1.0000	0.9972	0.9951	0.9916	0.9362
1994	0.9389	0.9721	0.9840	0.9905	0.9947	0.9972	1.0000	0.9973	0.9950	0.9910
1995	0.9313	0.9654	0.9785	0.9861	0.9915	0.9951	0.9973	1.0000	0.9973	0.9946
1996	0.9225	0.9574	0.9715	0.9803	0.9868	0.9916	0.9950	0.9973	1.0000	0.9965
1997	0.9118	0.9476	0.9627	0.9726	0.9303	0.9862	0.9910	0.9946	0.9965	1.0000

Table 5. Case 2: forecast table and contribution to variation in mean ultimate (VMU)

			Conditional on Next Calendar Periods		Contribution to
Calendar Year	Mean Outstanding	SD	VE[Var[Outs|Data]]	SD[E[Outs|Data]]	Variation in VMU
1988	1,863,279	110,997	0	110,997	
1989	1,431,551	155,420	33,443	151,779	
1990	1,073,424	170,458	32,943	167,244	
1991	811,554	170,229	31,865	167,221	
1992	622,395	162,661	30,183	159,836	
1993	490,446	153,724	28,652	151,030	
1994	385,174	141,102	26,450	138,601	
1995	295,338	124,002	23,377	121,778	
1996	201,097	95,579	18,311	93,809	
1997	103,,216	55,039	10,868	53,955	
Total	7,277,474	1,326,688	217,202	1,308,787	

Forecast scenario: Case 2

In this case, the contribution of $2 \cdot cov(L_1, \sum_{i=2}^{n} E(L_i|L_1))$ and $Var(\sum_{i=2}^{n} E(L_i|L_1))$ dominate the contribution of $Var(L_1)$ (Table 5) to the VMU.

In Table 5, the variation in the next calendar year's loss, 110, 997, is a small fraction of the total variation in the expected ultimate conditional on the next calendar year's data, 1, 308, 787. Taking into consideration the correlation between the distributions, the losses in L_1 contribute to about 15% of the variation in the VMU compared to 99.99% of the variation in Case 1.

10. $VMU_{99.5\%}$ Versus SCR^2

The 99.5[th] percentile of the VMU is usually not the same as the value of SCR^2. These measures will only be similar if the correlations between future calendar year loss distributions approach 1 (see section 9.2).

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If the correlations between future calendar years are close to one, then:

$$SCR^2 \approx VMU_{99.5} \tag{30}$$

SCR^2 and $VMU_{99.5}$ are approximately the same.

When the correlations are high, looking at the percentile (99.5) of the sum of the changes is close to taking the sum of the 99.5[th] percentiles for each year. That is, it is closer to taking the $V@R_{99.5}$ of L_{Ult}.

If there is no parameter uncertainty then the correlation between calendar periods is 0. That is, $E[L_i|L_1] = E[L_i]$ and:

$$SCR^2 << VMU_{99.5} \tag{31}$$

In the case of SCR^2, only the first year is in distress and subsequent $\Delta BELs$ are not likely to be at the same percentile at the same time.

11. Solvency II Standard Formula and SCR

In the Solvency II context, the QIS5 SCR standard formula is based on E(L) and Var(CDR) (=Var(VMU)) (European Commission, 2010).

The $V@R_{99.5\%}(VMU)$ is one of the approximations for SCR based on the standard formula (European Commission, 2010). We denote this SCR as SCR^4. Since the distress in the next calendar year is not explicitly calculated, it is effectively assumed that $V@R_{99.5\%}(VMU)$ corresponds to being in distress in the first year and having sufficient capital to restore the means of the Economic Balance Sheet (RMs are ignored).

However, the recommended approach in the standard formula is to fit a log-normal to the mean (Provision of Claims Outstanding = E(L)) and Variance (Var(CDR)). The log-normal distribution is not necessarily a good proxy for the distribution of VMU. (See QIS5 technical specifications European Commission (2010), paragraph SCR.9.18.) This approximation to SCR^4 we denote SCR^{4*}.

SCR^{4*} is not related to SCR^1, SCR^2, SCR^3 nor even to the distribution of VMU and therefore to SCR^4. The differences between SCR^4 and any of the other SCR calculations are model dependent (and data dependent).

Consider the two cases from section 9. In the situation where future calendar year correlations are close to 0 (Case 1), SCR^1, SCR^2, SCR^3, and SCR^4 are about the same (Table 6) — they are within expected statistical variation. The lognormal distribution approximation, SCR^{4*}, produces a much lower estimate — in fact lower than the 99.5[th] percentile of the first calendar year (SCR^1). The company would not even be able to pay for the losses up to the 99.5[th] percentile even if the model was completely correct.

Table 6. SCR comparison for Cases 1 and 2

Case	SCR^1	SCR^2	SCR^3	SCR^4	SCR^{4*}
1	3.18M	3.20M	3.24M	3.21M	2.29M
2	0.30M	3.69M	3.56M	3.78M	4.05M

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Table 7. Solvency II Capital Requirement (SCR) comparison for Cases 1 and 2

SCR Estimate	Key Assumptions	Properties
SCR^1	No parameter uncertainty; implies correlations between future calendar year loss distributions are 0	Process volatility inclusion ensures sufficient capital to cover losses in the first calendar year
SCR^2	Receiver of Risk if liabilities transferred does not have to subscribe to Solvency II regulation	Process volatility and parameter uncertainty included. Fair value obtained for restoration of mean estimates but not the Risk Margin. $SCR^2 \geq SCR^1$
SCR^3	Receiver of liabilities subscribes to Solvency II regulation	Economic Balance Sheet can be fully restored after a distress event. Usually $SCR^3 \geq SCR^2$
SCR^4	All future loss distributions have a correlation of 1 with the losses in the first calendar year	The future correlation assumption almost guarantees $SCR^4 \geq SCR^3$
SCR^{4*}	Distribution of SCR^4 is log-normal with mean $E(L)$ and variance $(Var(CDR))$	The distribution of SCR^4 is not likely to be log-normal and the value of the estimator is unknown

However, when the parameter uncertainty and correlations between the loss distributions by calendar year are high (Case 2), SCR^{4*} well exceeds $SCR^4 > SCR^2 > SCR^3 >> SCR^1$ (Table 6).

Note that in the second example, $SCR^2 > SCR^3$ as the $V@R_{99.5}$ requirements decrease, even in the case of distress, as improved knowledge of the parameter actually reduces the $V@R$ in subsequent calendar years. The calculation of SCR^3 takes advantage of this knowledge whereas SCR^2 is oblivious.

12. Summary

Four methods of estimating the appropriate SCR were compared along with an approximation. The calculation method most consistent with the Solvency II directives is the SCR^3 estimate as presented in Munroe *et al.* (2015). This calculation is consistent with the Solvency II directives and also ties in with the way one would calculate RMs.

The simpler calculations, SCR^1 and SCR^2, usually do not provide sufficient capital to restore the Economic Balance Sheet should a distress event occur. Estimation of required risk capital is reasonable only in very particular conditions.

All measures of SCR in Table 7 assume the model used to estimate the loss distributions is correct.

The standard formula calculation for SCR based on the distribution of the VMU, SCR^4, is too conservative when correlations are high. The approximation to the distribution of VMU using a lognormal distribution assumption to $E(L)$ and Var(VMU) was shown to be too conservative (section 11) when correlations were high, and far too optimistic when correlations were low. The lognormal distribution approximation for SCR^4, SCR^{4*}, is a very poor estimator of SCR^4.

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