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Families classification including multiopposition asteroids

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Abstract. In this paper we present the results of our new classification of asteroid families, upgraded by using catalog with $> 500,000$ asteroids. We discuss the outcome of the most recent update of the family list and of their membership. We found enough evidence to perform 9 mergers of the previously independent families. By introducing an improved method of estimation of the expected family growth in the less populous regions (e.g. at high inclination) we were able to reliably decide on rejection of one tiny group as a probable statistical fluke. Thus we reduced our current list to 115 families. We also present newly determined ages for 6 families, including complex 135 and 221, improving also our understanding of the dynamical vs. collisional families relationship. We conclude with some recommendations for the future work and for the family name problem.

Keywords. Asteroids: dynamics, Collisional evolution, Non-gravitational perturbations.

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1. Introduction

In our previous work (Milani *et al.* 2014, hereinafter referred to as Paper I), we have established methods for the classification of asteroids into families, capable of handling very large data sets of proper elements (for $336,319$ numbered main belt asteroid) and of updating the classification in a semiautomatic way whenever the data set is expanded. Indeed in Knežević *et al.* (2014), hereinafter referred to as Paper II, we have already expanded the data set to proper elements for $384,336$ numbered asteroids; the number of family members increased from $87,095$ to $97,440$. Now we have further expanded the data set to proper elements for $406,251$ numbered and $99,475$ multiopposition asteroids, thus finding $121,448$ family members (see Section 2.1).

Such large numbers may be considered a technical achievement, but the large amount of information in the data we use is not a scientific goal in itself: it is a tool to be used to increase our level of understanding of the dynamical and collisional evolution of the main asteroid belt. What matters is not the large number of families, but the number of those for which we have achieved a rational model of the collisional processes involved in the family formation and of the successive dynamical evolution. Of course a high confidence level understanding is possible only for families large enough, in terms of the number of members, to allow for statistically significant derivation of basic quantities, including the minimum number of collisions and the age of each one of them, and in this the large data set plays an important role. As an example in Spoto *et al.* (2015), hereinafter referred to as Paper III, we have computed 37 ages for 34 families, and we have identified at least

Table 1. Increase of the proper elements sets between those used in Paper I and the present paper: for each zone, the range of values in proper a, the constraints in proper $\sin I$, the number in Paper I (numbered asteroids only), the numbered asteroids in this paper, the multiopposition added in this paper, the percentage of increase between the Paper I and the total used in this paper.

Zone	Range a, au	$\sin I$	Paper I	Numbered	Multiopp	% inc.
1	$1.600 \div 2.000$	> 0.3	4,249	5,306	3,017	96
2	$2.065 \div 2.501$	< 0.3	115,004	135,807	28,334	43
2h	$2.065 \div 2.501$	> 0.3	2,627	3,221	973	60
3	$2.501 \div 2.825$	< 0.3	114,510	139,450	34,421	52
3h	$2.501 \div 2.825$	> 0.3	3,994	4,586	1,760	59
4	$2.825 \div 3.278$	< 0.3	85,221	105,284	26,316	54
4h	$2.825 \div 3.278$	> 0.3	7,954	9,097	3,569	59
5	$3.278 \div 3.700$	all	991	1,346	284	64
6	$3.700 \div 4.000$	all	1,420	1,763	547	63
Tot			336,319	406,251	99,475	50

7 cases in which the correspondence between *dynamical families*, identified by number density contrast in the space of proper elements, and the *collisional families*, formed by a single collision at a single time, is not one to one.

In this paper we exploit the larger data set to push forward our understanding of the asteroid collisional and dynamical history. First, we solve a number of ambiguities due to *intersections* of two families: in most cases, the solution we adopt is to consider a smaller family as a *satellite family* to be *merged* into a larger one; the terms in italic are defined in Section 2.2. Then we use the increase in number of members of the families, as a consequence of the larger data set of proper elements, as a tool to assess the reliability of the smallest families (Section 2.3). In Section 3 we compute several new ages, in particular for families enlarged beyond a critical number of members, and for complex families, for which only some of the generating events can be dated. In Section 4 we draw some conclusions, in particular on the efficiency of our methods to detetct and date families of different ages, and on the problem of comparing different family classifications.

2. A large classification update

2.1. *The extended data set and the automatic update*

Because of observational selection effects, by adding recently numbered asteroids and others with good orbits but not yet numbered, we are increasing the proportion of smaller, darker, more distant, for whatever reason less observable objects. Main belt asteroids are comparatively less observed either at high inclination or at larger distance from the Sun (the latter also are on average darker) Before applying the *Hierachical Clustering Method* (HCM) Zappala *et al.* (1990) we subdivide the main belt asteroids in zones (see Table 1), finding that the higher percentage of relative increase due to adding recently numbered and multiopposition asteroids, between the data set of proper elements used in Paper I and in this paper, occurs especially in zone 1, but also in the zones 5 and 6 and in the high inclination zones 2h, 3h, and 4h.

Given this increased data set of proper elements, we have used the same core families and small but independent families, for a total of 125, obtained in Paper II, and attached to them the asteroids with distance in the proper elements space up to the maximum used in the previous classification. This procedure is entirely automatic, and thanks to

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an advanced algorithm (with less than quadratic complexity) takes just an hour of CPU time on a standard workstation. The output is a new classification with still 125 families but $121,480$ members.

2.2. *Intersections and families to be merged*

In the new classification, after the automatic attachment step described in the previous subsection, the attachement of new members may result in a single asteroid belonging to two families: there are 36 such *intersections* between different families, up from 22 that remained unsolved after the previous run (Paper II). The question is whether these additional cases of asteroids classified in two different families indicate that the list of families should be changed: this procedure is not automatic, but requires a careful discussion. The outcome of such a procedure may be to decide that a larger family has a *satellite family*, formed by the statistical fluctuations of the number density but not requiring a separate collisional event. In this case the intersection lead to a *merge*, in which the smaller family disappears from the list of families and the larger one incorporates all the members of the smaller one.

We have checked each single case by using the graphic visualizer of asteroid families provided at the AstDyS site†, as well as other graphical tools, in particular to establish whether the two families with intersections have been formed by a single or separate collisional events. This also involves inspecting the same kind of V-shape plot used to estimate family ages, as in Paper III, but formed by a hypothetical merger of the two families. After this analysis, we propose the following modifications to the list of families; we next discuss mergers we have not accepted and dubious cases.

2.2.1. *Proposed family mergers*

We propose performing 9 family mergers.

• Family of (480) Hansa with 34052 by 2 intersections, box‡ of 34052 included 100% in the one of 480; now this family has $1,327$ members, and has been promoted to *big family* (by which we mean with $> 1,000$ members).

• Family of (1040) Klumpkea with 29185 by 1 intersection. 29185 is mostly included in the box of 1040, but for $\sin I$ it extends to values slightly larger than 0.3; family 29185 was found in an HCM run limited to zone 4h. Thus the separation of these two families could be an artifact of the boundary splitting zone 4 at $\sin I = 0.3$.

• Family (135) Hertha with 7220, 1 intersection; 7220 is not a family, but the path of transport away from family 135 along the 1/2 M resonance. The family 7220 was found by HCM due to a chaining effect.

• Family of (4) Vesta with 63 and 3489, 1 intersection each, 100% in the box. 63 is somewhat higher in proper e than the "high eccentricity" Vesta family (as defined in Paper I), but this could be due to transport along the 3-body resonances 2J+3S-1 and 4J-2S-1. Note that this merge implies that (63) Ausonia is an interloper, because it is too big to belong to a cratering family.

• Family of (15) Eunomia with 173 by 6 intersections; however (173) Ino is a large interloper, with IRAS albedo 0.0642. The members of family 173 extend the range of proper a of the family 15 to higher values beyond the 8/3 J resonance, but they do not change significantly the age computation from the OUT side as done in Paper III. The proper $e, \sin I$ of the members of family 173 are affected by passage, during the Yarkovsky secular drift towards larger proper a, through the strong resonance 8/3 J;

† Available at http://hamilton.dm.unipi.it/astdys2/Plot/

‡ By box, as in Paper I, we mean the minimum parallelopipedon in the proper $a, e, \sin i$ space containing all the family members.

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this is consistent with the small number of members of family 15 for a larger than the resonant value.

• Family of (10) Hygiea with 22241 by 2 intersections; 22241 100% in the box of 10.

• Family of (10955) Harig with 19466 by 1 intersection, also single V-shape with compatible ages (see paper III). The V-shape is asymmetric in that the OUT side is much shorter, possibly terminated by the $3J - 1S - 1$ resonance.

• Family of (4203) Brucato with 20494 by 2 intersections. There is a group of small/tiny high I families (4203+20494, 10369, 23255) close in proper elements space: they could, with more data, eventually merge into a single, widely dispersed (thus very old) family.

2.2.2. *Rejected family mergers*

We believe the following case can not be solved by merging the intersecting families.

The family of (1726) Hoffmeister has 1 intersection with the family of (110) Lydia. Such a merge would assemble a family with a shape in proper elements space very difficult to be modeled as the outcome of a single collision, and with no recognizable V-shape in the $(a, 1/D)$ plane. (110) Lydia and (1726) Hoffmeister also have very different albedos and taxonomy. Novaković *et al.* (2015) found that a portion of the Hoffmeister family is affected by the linear secular resonance $s - s_c$ with (1) Ceres. This affects the proper $\sin I$ of the members of family 1726 with lower values of proper a, overlapping the range in a of the family 110, thus creating a bridge between the two families.

2.2.3. *Dubious cases*

In the following 8 cases of intersection the available evidence is not enough to decide a merger, thus it is necessary to wait for more data.

• Family of (135) Hertha with 6769 by 2 intersections; difference in $\sin I$ cannot be explained by the 1/2 M resonance.

• Family of (10) Hygiea with 1298 by 2 intersections: 1298 is too far in proper $\sin I$.

• Family of (5) Astraea with 4945 by 1 intersection: we plan to address this case in our future work.

• Family of (2076) Levin with 298, 2 intersections, and 883, 1 intersection. The age computed in Paper III refers to the family of (2076) Levin, but merging the family of (298) Batistina would not change the V-shape, thus the age estimate. To the contrary, the slope of the 883 family, although weakly determined, is not consistent with the same age. Merging these three families would not remove the difficulty of building a consistent model for the collisional formation of the whole complex.

• The family of (221) Eos has 8 intersections with family 507, which extends to much larger values of proper a; also 1 intersection with each of 21885 and 31811, forming a protuberance in the direction of increasing a. However, the V-shape resulting from these mergers would give an extremely large age from the OUT side, based upon a too small number of fit points. See further discussion on this in Section 3.2. Unfortunately the physical observations of family 507 members are too few and do not clearly indicate a prevalent taxonomy, thus they do not help in solving this problem.

In conclusion, we apply 9 mergers, thus removing 17 intersections. We leave 19 intersections (in part different from the ones of the previous update) as problems to be solved in the future. Note that the complex families of Eos and Baptistina are responsible for the majority, namely 12, of the remaining intersections.

2.3. *Significance of the smallest families*

Another non-automatic step in the classification update procedure is to monitor the increase in membership of the families. If the increase in the size of the catalog of proper

Table 2. The families with < 50 members in the current classification, and comparison with the number of members in a previous classification. For each family we provide no. of asteroids in the box (enlarged by a factor 3 in all directions) surrounding the family from the catalog of Paper I, and from the catalog of the current paper, total no. of members in the classification of Paper I, and of this paper, the expected number for the new membership, and a comment (see text for the codes).

Family	Box I	Box II	Tot. I	Tot. II	Exp	Comm
7468	76	100	45	48	59	-!
2	88	112	38	47	48	
3438	106	158	34	42	51	-
4203	201	296	34	47	50	m
116763	24	58	13	33	31	+
909	26	34	24	32	31	+
10369	113	181	17	28	27	+
7605	21	45	12	26	26	
5931	263	373	19	25	27	
260	27	44	15	25	24	+
58892	215	317	14	21	21	
22805	126	200	17	21	27	-
45637	42	54	15	19	19	
3561	40	52	15	19	20	
1101	78	86	17	18	19	-
69559	27	44	12	17	20	
40134	32	46	13	17	19	
3025	117	136	16	17	19	-!
14916	50	59	16	17	19	-!
6355	75	95	13	14	16	-!
10000	53	93	13	14	23	r
23255	67	92	10	13	14	
10654	46	53	13	13	15	-!

elements is large, as in this paper with 50% increase (mostly due to addition of multi-opposition asteroids), then we would expect all families to increase their membership, on average by roughly the same percentage. Table 1 shows that this increase should be somewhat larger in some regions, especially the ones at high inclination and beyond the 2/1 J resonance. On the other hand, the high inclination regions have lower and quite non homogeneous number densities, and they also contain most of the families we rated as *tiny* (with less than 30 members) in Paper I, which are marginally significant with respect to the Montecarlo based acceptance criterion of the HCM. Thus we must use the local rates of expected increase of added members instead of the regional ones, as a criterion for confirmation of the significance for these marginal cases.

The local expected number of family members, given in the last column of Table 2, is thus obtained by estimating the increase in number of asteroids in an enlarged area around a family. More precisely, the family box defined in Table 6 of Paper I, has been expanded by 100%, in all six directions in the 3D space of proper elements. Thus, the length of every box side defined in this way is three times bigger in the semi-major axis, eccentricity and sine of inclination, than that of the original family box, and the total volume of the enlarged box is 27 times larger than for the family box. This choice of the size for the enlarged box is arbitrary, but we consider it to be a plausible compromise between local and global change in asteroid number density. The expected number of family members in the updated classification is then determined as an increase of the number of members in the family from the old catalog, proportional to the corresponding

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increase of the number of asteroids in the enlarged box from the old to the new catalog of proper elements. This has to be compared to the actual increase in family membership between the Paper I classification and the present one.

In Table 2 we show the increase of membership for all the families with < 50 members in the new classification; 50 is an arbitrary boundary, large enough to show the increase of the former tiny families†. As expected this Table shows very different behaviors: 4 tiny families (marked with + in the comment column) have increased even more than expected, 9 have grown in an acceptable way ($> 1/2$ of expected): all these are likely to be real families (blank in the comment column). Three of them increased between 1/3 and 1/2 of expected: these are suspect flukes to be monitored in the future updates (minus in the comment column). Other tiny families proposed in 2014 have been merged: 3 to larger families (thus they are not in the table) and 2 together (into family 4203, marked m, which has also grown satisfactorily).

There are cases in which we prefer to suspend judgment, as an example for family 14916 which has grown negligibly. It is located at high proper e and sin I, very close to the family of (2) Pallas. In this region families are subject to much stronger perturbations, thus it is expected that any family would be very strongly depleted by dynamical instabilities, especially those resulting from cratering like (2) Pallas. Still, the Pallas family has grown as expected. Anyway, the increase by 1 member when the statistical prediction is 3 is affected by small number statistics and does not allow to propose removal of 14916 from the list of tiny families. Three similar cases with small growth $< 1/3$, but with too small numbers, are 3025, 6355 and 10654‡. All four cases in this class are marked by a minus with exclamation mark: we expect that later, if the the lack of growth is confirmed, some of these may be removed too.

This leaves a single case formerly proposed as tiny family with statistically significant lack of increase, namely family 10000, which has grown by 1 member only, while the expected number was 10. The most likely interpretation is that this case was a statistical fluke, which has passed the Montecarlo significance test by few members: this is consistent with the statistical nature of the HCM method. Thus we are removing this one, marked r in the comment column, from the classification.

The formerly proposed family 7468 has grown by only 3 members, while the expected increase was 14. However, this family is bounded in proper a by the 9/4 resonance with Jupiter on the IN side, by the 11/5 resonance on the OUT side. Thus the growth of the membership might be limited to a rate lower that expected from the statistical argument.

We have also checked among all the other families with between 30 and 50 members in Paper I finding that most have grown in number of members as expected or at least $> 1/2$ of what was expected, in only 3 cases by between 1/3 and 1/2. We have found only one critical case, family 3811 which has grown by 9 out of 30 expected: this case is left as dubious, because it also belongs to an especially unstable region just outside of the 3/1 resonance with Jupiter. In conclusion, we propose for now to remove only one of the tiny families, but there are 3 to 5 other suspect cases which have to be monitored in the next updates with the goal of reaching a reliable conclusion.

Of the 25 tiny families listed in Paper I, 3 have increased their membership above 30, 2 have been merged together, 3 have been merged in larger families, and 1 has been removed: only 16 tiny families (with $\leqslant 30$ members) are left, and by using larger catalogs of proper elements we might be able to further reduce this number. Taking into account

† Family 895 has grown from 25 to 64 members, thus is the only former tiny family not shown in the table.

‡ These three families, together with 1101, form a cluster of neighboring families at high proper I separated only in proper e. They could hint at a depleted ancient family.

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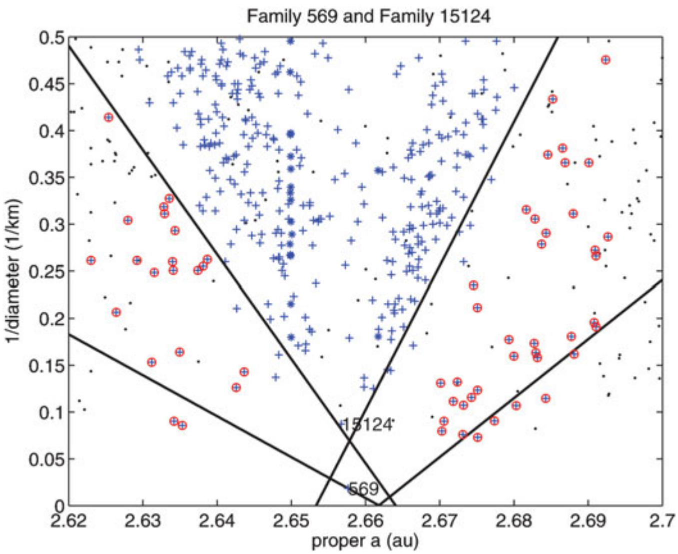

Figure 1. The double V-shape for the family of (569) Misa reveals the subfamily of (15124) 2000 EZ39. The ⊕ signs indicate outliers removed from the fits for the two central slopes, that is asteroids belonging to the larger family but not to the subfamily.

all the mergers and the removals, we have decreased our list of families from 128 in Paper I to the current 115.

3. Family ages

To estimate family ages, we use the method introduced in Paper I and improved in Paper III, based on V-shape slopes in the proper a, $1/D$ plane (D diameter, in km). This procedure is summarized at the beginning of Section 3.2.

3.1. *Multiple collisions, multiple V-shapes*

It is a fact that the V-shape method, when applied without the prejudice that each *dynamic family* (found as density contrast in the space of proper elements) must correspond to one and only one *collisional family* (that is, a single originating collision, with a single age), results in many cases with two ages. In the discussions we had at the symposium, many were puzzled by the fact that two collisions do not result in a W-shape.

The answer is that in some cases a W-shape is actually visible, and all 4 sides are used in the fit for slopes, in other cases some of the sides are either partially or totally obliterated by the superposition of the substructures. In the case of the family of (569) Misa there are indeed 4 slopes, although the one with lowest a results in a poor accuracy fit because of too few data points (Figure 1). A similar case occurs for the family of (847) Agnia with the (3395) Jitka subfamily, see Paper I [Figure 4].

For the family of (15) Eunomia (Figure 2) the ratio of the inverse slopes on the two sides is 0.57 ± 0.05 (that is the OUT side at higher a corresponds to a much younger age that the IN side at lower a), thus the difference in age is statistically very significant. Nevertheless, only 2 slopes have been fit, although the OUT one is based essentially only on data points with proper $a > 2.67$ au. For $2.62 < a < 2.67$ it would be possible to fit a third slope (as shown by an additional dark line in the plot) which can be interpreted as the OUT slope for the family with older age, and would be consistent with the age from

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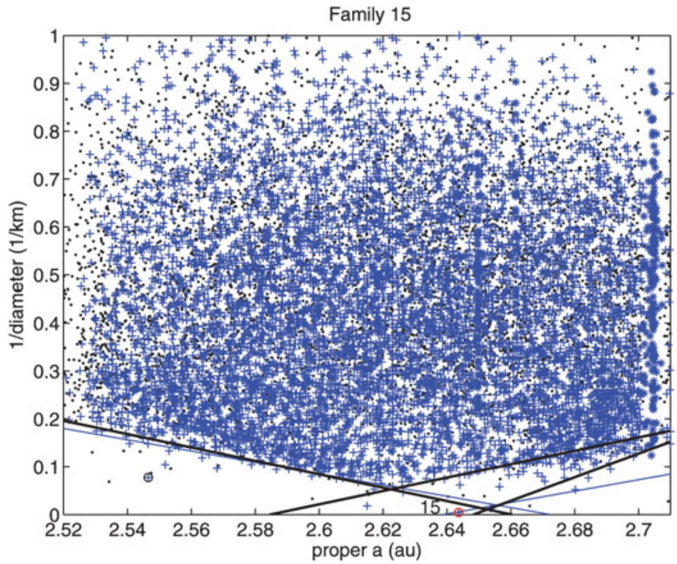

Figure 2. The asymmetric V-shape for the family of (15) Eunomia reveals the presence of a younger subfamily in the far OUT side. However, the region with $2.62 < a < 2.67$ au shows a segment of the OUT side of the V-shape for the older cratering event. Lighter lines are the V-shape obtained in iterations of the procedure, the darker ones are the final result after outlier removal; outliers are marked with a $\oplus$ symbol.

the IN slope. The fourth side of the W, the IN side for the younger family, is obliterated by the superposition of the two V-shapes. Another example is the family of (4) Vesta, see Paper I [Figure 5], in which the OUT side of the younger family (with low a) is partially obliterated and the IN side of the older one is fully obliterated by the strong superposition of the two families, as shown in Paper I [Figure 12].

3.2. *Additional family ages*

We have been able to compute 6 new ages. For each one of them, we have followed the same steps of the procedure as in Paper III.

• Delimitation of the fit region in the $(a, 1/D)$ plane, Table 3; this step is necessary because most families are truncated in the proper a range by mean motion resonances, and the usable V-shape is only the portion with $1/D$ below the truncation level. The "cause" of the truncation, as given in Table 3, thus contains the integer coefficients of the relevant mean motion resonance; when no truncation is found down to very small sizes (below the completeness limit for discovery) we use the acronym FB.

• Estimation of the average and standard deviation (STD) for the family albedo, Table 4; the average is used to compute D from the absolute magnitude H, the STD appears in the error model for the fit of slopes.

• Determination of inverse slopes of the V-shapes, with standard deviation from the fit, Table 5; also the ratio of the two results for the two sides, when available, used to assess the presence of two collisional families.

• Selecting data for the Yarkovsky calibration, Table 6; the calibration formula is from (Paper III, Section 4.1).

• Age estimation for the families, Table 7; includes the adopted Yarkovsky calibration, the STD due to the fit, the STD due to the calibration and the combined STD.

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Table 3. Fit region: family number and name, choice, cause of the family truncation on the left, minimum value of proper a, minimum value of the diameter selected for the inner side; cause for truncation on the right, maximum value of proper a, minimum value of the diameter selected for the outer side. FB stands for the Family Box.

number/ name	cause	min pr. a	min D IN	cause	max pr. a	min D OUT
221 Eos	7/3	2.96	6.25	FB?	3.14	3.33
1040 Klumpkea	11/5	3.08	4.00	5J-2S-2	3.18	4.00
1303 Luthera	FB	3.2	10.00	2/1	3.235	10.00
302 Clarissa	FB	2.385	1.18	FB	2.42	1.54
650 Amalasuntha	FB?	2.29	2.22			
752 Sulamitis	2/1M	2.42	3.33			

Table 4. Family albedos: number and name of the family, albedo of the parent body with standard deviation and code of reference (W for WISE, and I for IRAS), mean and standard deviation of the WISE albedo, by using only data with $S/N > 3$.

number/ name	albedo value	largest STD	ref	albedo mean	STD
221 Eos	0.165	0.034	W	0.157	0.050
1040 Klumpkea	0.239	0.048	W	0.204	0.100
1303 Luthera	0.052	0.009	W	0.053	0.010
302 Clarissa	0.052	0.010	I	0.054	0.020
650 Amalasuntha	0.027	0.002	W	0.060	0.020
752 Sulamitis	0.033	0.005	W	0.055	0.010

Table 5. Slopes of the V-shape for the families: family number/name, number of members of dynamical family, side, slope (S), inverse slope ($1/S$), standard deviation of the inverse slope, ratio OUT/IN of $1/S$, and standard deviation of the ratio.

number/ name	no. members	side	S	$1/S$	STD $1/S$	ratio	STD ratio
221 Eos	14376	IN	-2.063	-0.485	0.022		
		OUT	1.955	0.512	0.044	1.05	0.10
1040 Klumpkea	1950	IN	-4.429	-0.226	0.025		
		OUT	4.494	0.223	0.048	0.99	0.24
1303 Luthera	251	IN	-6.465	-0.155	0.014		
		OUT	6.633	0.151	0.019	0.97	0.15
302 Clarissa	222	IN	-27.170	-0.037	0.007		
		OUT	33.6409	0.030	0.005	0.81	0.16
650 Amalasuntha	1598	IN	-1.992	-0.502	0.052		
752 Sulamitis	188	IN	-4.633	-0.216	0.023		

All the notations and symbols in the tables are the same as in Paper III [Tables 1-12]; the families are partitioned in three groups: fragmentations, young (that is age < 100 My), and one sided; the group of cratering is represented among the 6 new ages only by the family 302, which is also young, and by family 752, which is also one-sided.

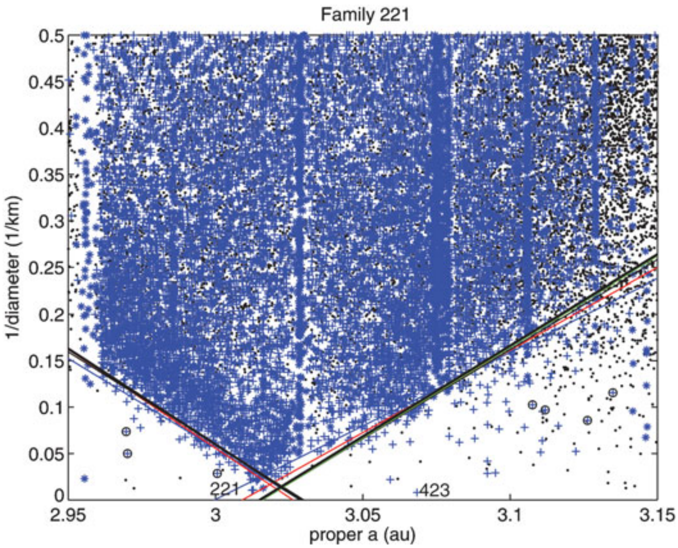

Figure 3. V-shape for the family of (221) Eos. A bulge below the fit line around proper $a = 3.068$ au may be linked with the large C-type asteroid (423) Diotima.

Comments

• Family of (221) Eos: this appears to be a very complex dynamical family, with number density changing wildly as a function of proper a. Nevertheless, a well-defined single V-shape is visible in Figure 3, and indeed a good fit can be obtained for both the IN and the OUT slope, with ratio of the OUT and IN inverse slopes 1.05 ± 0.10. This suggests that most of the members are from a single collisional event. Note that some of the scattered points in the bottom right corner of the plot are from family 507, thus merging 507 into 221 would imply a model with one additional collision for the Eos family.

• Family of (1040) Klumpkea: the OUT slope is less well determined than the IN slope, this should be due to the 2/1J mean motion resonance. Still the two slopes are well consistent, implying a single collision origin.

• Family of (1303) Luthera: has 251 members in the new classification, which is barely enough for the reliable use of the V-shape method. Still the two slopes are well consistent.

• Family of (302) Clarissa: although it has still only 222 members, a good fit is possible on both sides, see Figure 4, because the range in proper a is small. Note the conspicuous central gap, due to the YORP effect, see (Paper III, Section 5.2). This family is of cratering type, having the volume of the fragments 7.5% of the total including (302).

• Family of (752) Sulamitis: it has still only 188 members, but it has a one-sided V-shape (OUT side wiped out by the 3/1J resonance), thus the IN side slope has enough data points for a good fit, see Figure 5. It is of cratering type, with fragments accounting for 12% of the total volume.

• Dynamical family of (135) Hertha: it is well known (Cellino *et al.* 2002) that this is a complex family, with at least two collisional families, because of the dis-homogeneous taxonomy (Paper I, Figure 11) corresponding to a “double jet” shape in the proper (a, e) projection (paper I, Figure 10). Thus we define a *dark subfamily* 650 by a combination of proper elements and WISE albedo data, we get a one-sided V-shape and we fit a quite accurate IN slope (the OUT side wiped out by the 3/1J resonance), see Figure 8.

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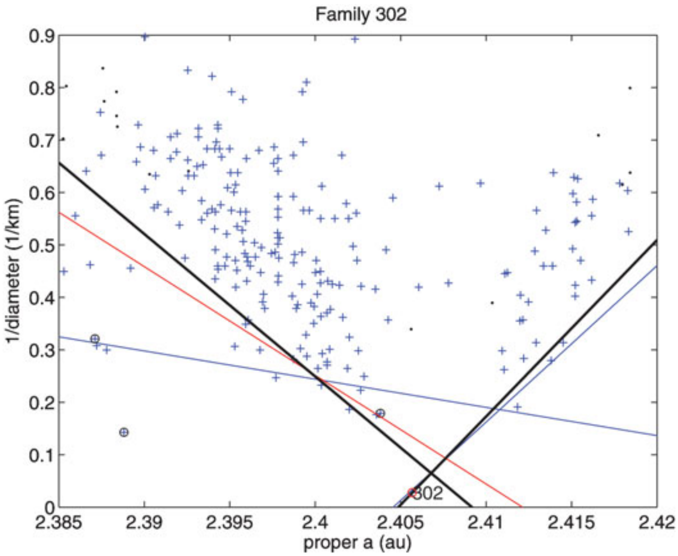

Figure 4. V-shape for the family of (302) Clarissa.

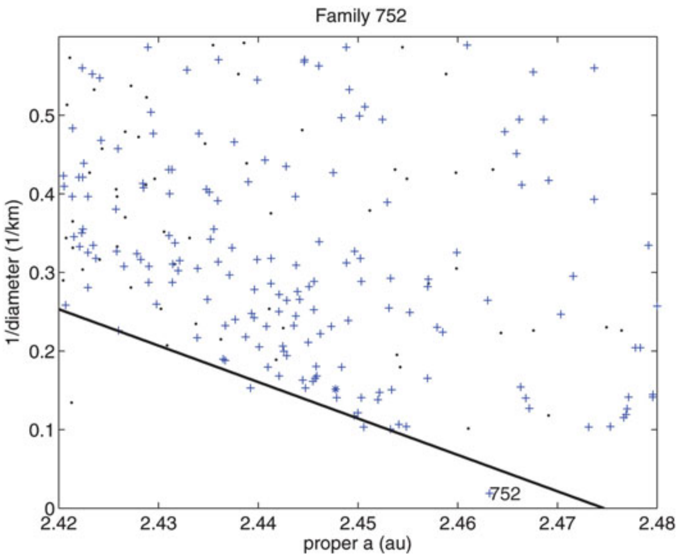

Figure 5. One sided V-shape for the family of (752) Sulamitis.

Complex families

A few additional explanations are needed for the two complex families 221 and 135.

The family of (221) Eos is indeed complex, both dynamically (because of the effect of several resonances, 2-body, 3-body and secular) and collisionally. However, a possible simple explanation can be found for the distribution in proper elements space of a large majority of the family members. If the family has originated near (221) (proper $a = 3.012$ au), then the OUT side with positive da/dt due to Yarkovsky must have met the 9/4J resonance at $a = 3.028$ au, which is strong enough to result in ejection of a good fraction of the resonance-crossing asteroids from the main belt (Morbidelli *et al.* 1995). This explains

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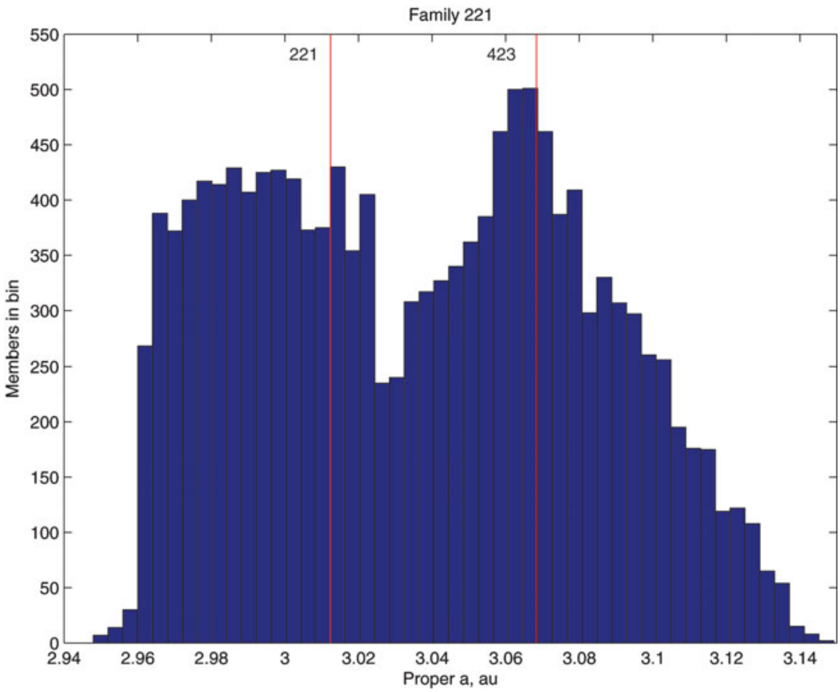

Figure 6. The distribution of the family 221 members, as a function of proper a, is highly dis-homogeneous and should be interpreted.

the sharp drop in the number density across the resonance, see Figure 6, without the need for any multiple collision model. With our method we get a two-sided V-shape with consistent IN and OUT slopes, and an age consistent with Vokrouhlický *et al.* (2006).

However, there is again an increase in the number density after the 9/4, with a peak in the region near the proper $a = 3.068$ of (423) Diotima, which is an interloper in the family both because of its size, see Figure 3, and its IRAS albedo 0.05. Actually, (423) is more than 200 km in diameter, almost twice as large as (221). By carefully looking at Figure 3 in the strip $3.05 < a < 3.074$ it is possible to see a bulge of data points above the fit line, which appears as a local anomaly of the fit residuals. In both Figure 6 and 7 there is a bimodality, with one of the peaks corresponding to the proper a and the albedo of (423) Diotima, respectively. Unfortunately, both the size of the data set and the S/N of the measurements is much lower if we try to use the albedo data, and the two histograms appear quite different, in particular in the number ratio between the hypothetical Eos (bright) and Diotima (dark) families.

Thus it is possible, although we cannot prove it, that the family 221 as found by our HCM contains an entire family of interlopers, all C-complex asteroids from a cratering event on (423) Diotima. Indeed, in Masiero *et al.* (2013)[Figure 7] there appears to be an indication of a family 423 in the output of an HCM procedure limited to the dark asteroids of zone 4, but the authors have not included a Diotima family in their list of families. We are not claiming to have built a complete model of the collisional and dynamical history of family 221, which would be a formidable task: we are just adopting a model on the basis of Occam's razor, that is the simplest one which does not contradict the data: most of the 221 family members originate from a single collision. Only with more data we will be able to answer in a reliable way the question about the existence of a subfamily.

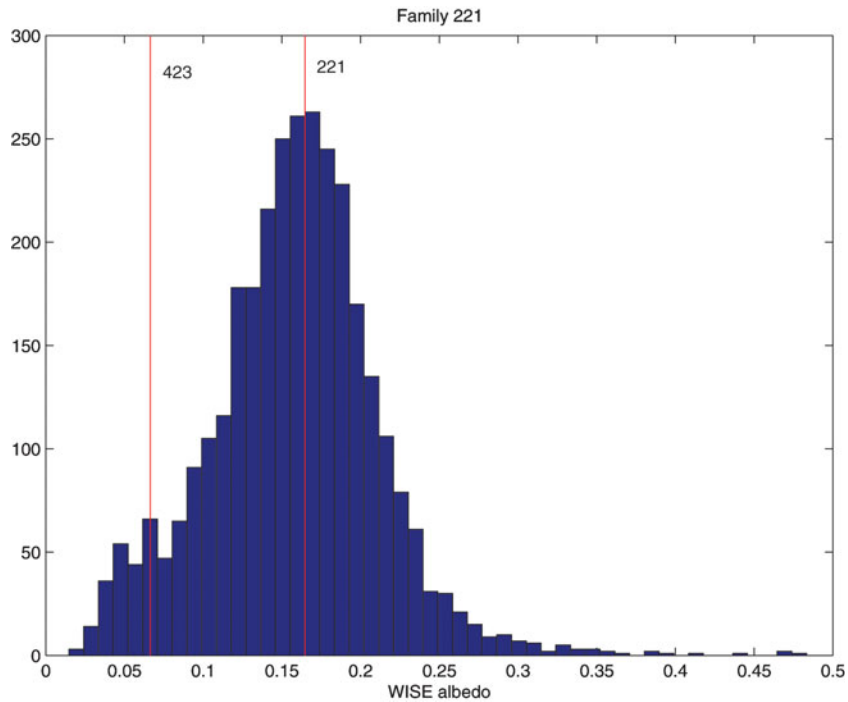

Figure 7. The distribution of the family 221 members, as a function of WISE albedo (only data with $S/N > 3$), is bimodal but in a way different from Figure 6.

For the family of (135) Hertha, we propose to split the family by combining proper elements and WISE albedoes p_v (limited to the ones with $S/N > 3$) as follows:

$$Dark = (a < 2.39 \ AND \ e < 0.162) \ OR \ (a > 2.39 \ AND \ e < 0.162 \ AND \ p_v < 0.09)$$
$$Bright = (e > 0.162) \ OR \ (a > 2.39 \ AND \ e < 0.162 \ AND \ p_v > 0.16)\,;$$

the choice of the corner point $(a, e) = (2.39, 0.162)$ and the values $0.09, 0.16$ for the albedo are justified from the Figures 10 and 11 of Paper I. This results in a partial success, as we can fit an IN slope and obtain an age (Table 7) for the Dark subfamily of 761 ± 242 My. However, we have found an unexpected (at least for us) result. From Figure 8 it is clear that (142) Polana is too large, given its proper a, to belong to the Dark subfamily; being in the 2/1 Mars resonance, the long term stability of its proper e is dubious. Thus the Dark subfamily, according to the convention on family naming, has namesake (650) Amalasuntha, the lowest numbered (also largest) member of the subfamily. Note that an Amalasuntha family had already been proposed by Zappala *et al.* (1990).

At this point it is useful to compare our results with the ones of Walsh *et al.* (2013): they propose an Eulalia family which is about the same as our family 650, apart from the fact that their namesake (495) has the possibility of having been a member of the family, but precisely because it is on a strongly chaotic orbit it is not possible to prove this claim. Then they propose a "new Polana" family, which beside (142) Polana could include the interlopers, discarded from the slope fit because they are too large, which can be seen in Figure 8. This family we have not identified with our HCM, which does not imply it cannot exist, but just that we cannot show that it is statistically significant. In conclusion, if we avoid being confused by the conventional family naming, but

Table 6. Data for the Yarkovsky calibration: family number and name, proper semimajor axis a and eccentricity e for the inner and the outer side, 1-A, density ρ at 1 km, taxonomic type, a flag with values m (measured), a (assumed), g (guessed), and the relative standard deviation of the calibration.

number/ name	proper a IN	proper e IN	proper a OUT	proper e OUT	1-A	ρ (1 km)	tax. type	flag m	rel. STD
221 Eos	2.96	0.075	3.14	0.06	0.95	2.275	S	m	0.20
1040 Klumpkea	3.08	0.19	3.17	0.20	0.93	2.275	S	g	0.30
1303 Luthera	3.20	0.12	3.23	0.12	0.98	1.41	C	g	0.30
302 Clarissa	2.385	0.108	2.415	0.106	0.98	1.41	C	a	0.25
650 Amalasuntha	2.29	0.14			0.98	1.41	C	g	0.30
752 Sulamitis	2.42	0.087			0.98	1.41	C	g	0.30

Table 7. Age estimation for the families: family number and name, side, da/dt calibration, age estimation, uncertainty of the age due to the fit, uncertainty of the age due to the calibration, and total uncertainty of the age estimation.

number/ name	side IN/OUT	da/dt $10^{-4} au/My$	Age My	STD(fit) My	STD(cal) My	STD(age) My
221 Eos	IN	-3.43	1412	65	282	290
	OUT	3.33	1537	131	307	334
1040 Klumpkea	IN	-3.40	664	72	199	212
	OUT	3.37	661	142	198	244
1303 Luthera	IN	-5.55	279	26	84	88
	OUT	5.52	273	34	82	89
302 Clarissa	IN	-6.41	57	11	14	18
	OUT	6.37	47	3	12	12
650 Amalasuntha	IN	-6.59	761	79	228	242
752 Sulamitis	IN	-6.34	341	37	102	109

look at the content of the families, that is most of the members, there is not really a disagreement.

The V-shape of the Bright subfamily does not allow a satisfactory fit. We believe this is due to the fact that the Bright subfamily, as defined above, is still a complex family, containing more than one collisional family. This can be confirmed by our visualizer, by plotting the dynamic family 135 in the proper (a, e) plane: with the enlarged data set, the family appears to have three lobes, consistently with the proposal by (Dykhuis *et al.* 2015) of two "Hertha" families, which could appear different in SDSS colors.

Calibration and ages

We can conclude by determining the Yarkovsky calibration for each of the 6 families for which the slope has been computed (see Table 5). Then the nominal estimated age is just (inverse slope)/(calibration), and the STD can be propagated taking into account both the STD of the slope fit and the STD of the calibration.

In this paper we are continuing to use the calibration procedure of Papers I and III because we wish to have additional ages computed with a method consistent with

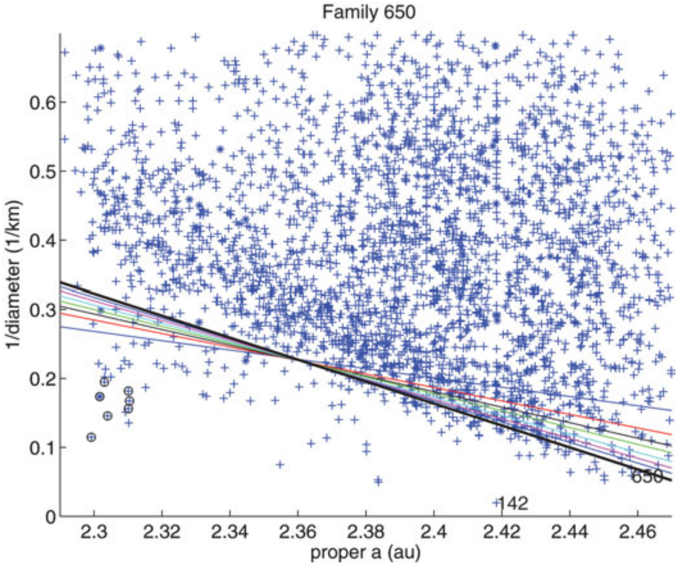

Figure 8. V-shape for the family of (650) Amalasuntha. Note the outliers of the fit (marked by $\oplus$) on the low a portion of the plot; they are discussed in the text.

the one used for the 37 ages already published in Paper III. In this way we can legitimately summarize the results in a single plot such as Figure 9. If the goal was to obtain the most accurate possible age for an individual family, we would recommend to compute a family-specific Yarkovsky calibration, possibly by obtaining with dedicated observation campaign either values of secular da/dt, or densities, or thermal information, for family members (or possibly for asteroids with the same composition as the family).

The most obvious conclusion we can draw from all the ages we have been able to compute (see Figure 9) is that our methods (both for family classification and for age estimation) are most efficient for ages between 100 My and 1.5 Gy, while for ages above 2 Gy only especially large families have been found to be suitable. This can be interpreted as a consequence of the Yarkovsky effect, which over Gy removes from the regions containing the families most small members. In this way the number density may be reduced below the level at which it can be shown to be statistically significant; this could explain the elusive Flora family (Knežević, *this volume*). In other cases, the family may be detected by HCM, but not have enough members for the V-shape fit. For the young families (with ages between 10 and 100 My) the number of families with enough members (at least 150 $\sim$ 200) to allow for the usage of our V-shape method is expected to increase regularly with the increase of the size of the asteroid accurate orbit catalogs, and of the corresponding catalogs of proper elements.

All the figures, and the data files used in the computations of ages for both the ages of Paper III and those of this paper can be found in the AstDyS web site at `http://hamilton.dm.unipi.it/astdys2/fam_ages/`

4. Conclusions and state of the art

In this paper we have presented our new classification, upgraded by using a proper elements catalog with $> 500,000$ asteroids, numbered and multiopposition. Thus it is the largest classification in terms of the amount of information used in input, taking into

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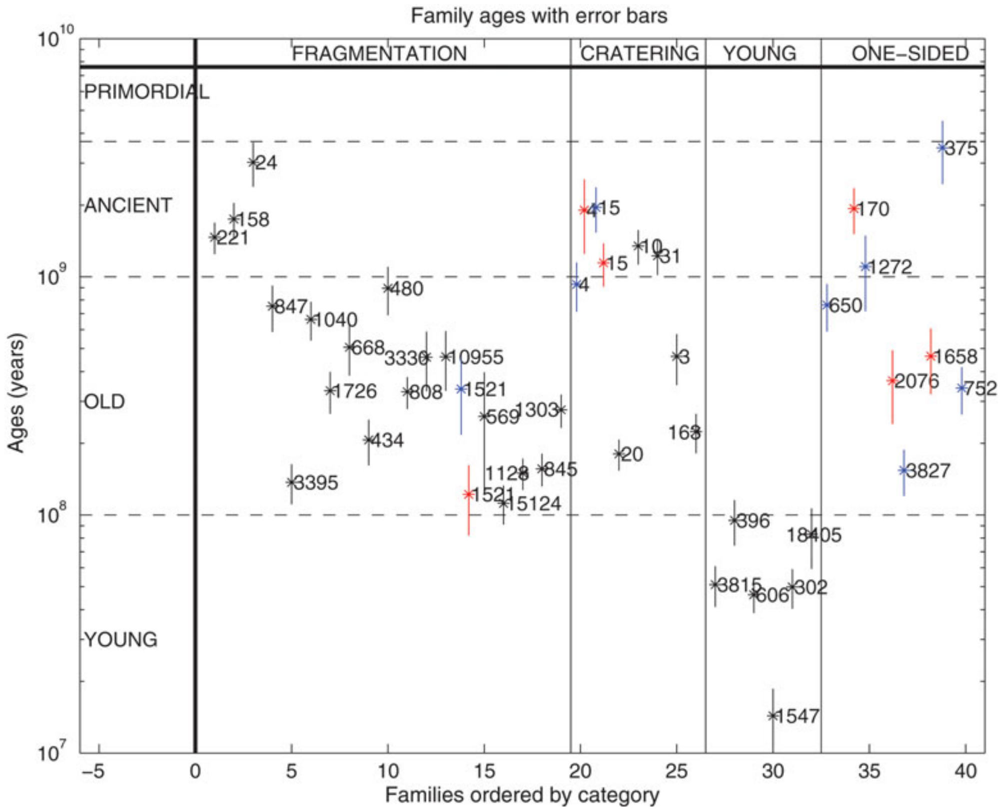

Figure 9. Global view of the family ages computed so far. For families appearing twice, the value and error bar on the left belong to the IN side, the one on the right to the OUT side. Families with only one value and error bar are either one-sided (on the right column of the figure) or reported with the combined value from both sides.

account that proper elements are by far the largest contributors of information, as shown in (Paper I, Table 1).

The careful utilization of this enlarged information input has allowed to simplify the classification, by decreasing the number of families: indeed, 7 small/tiny families (with < 100 members) and 2 medium families (173 and 19466) have been merged with larger ones. 1 tiny family (< 30 members) has been removed because its lack of growth suggests that it should be a statistical fluke. Thus we now have a total of 115 families (vs. 128 in Paper I).

To the contrary, the increase in family members (for a total $> 120,000$) is an important improvement: the goal is to obtain large families, such that details of the family shape are statistically significant. We now have 25 families with $> 1,000$ members: out of these, for 21 we now have some age estimate. We have another 19 with > 300 members: out of these, for 12 now we have some age estimate.

The new ages we have been able to compute have been 6; some of them required just to exploit the increased number or members resulting from the larger input data set, for others we had to make a more detailed analysis than the one used in Paper III. We consider plausible the models we found to explain at least part of the properties of two particularly complex cases, the dynamical families 135 and 221, which are also the largest dynamical families, with 15,442 and 14,376 members, respectively. Large dynamical families with complex structures are still presenting many open problems, which cannot be solved all at once. By solving some of these problems, we have at least understood the complex relationship between dynamical and collisional families.

We are certainly not claiming that our methods are unique, but we also cannot agree that other methods solve all the problems. E.g., knowing that complex families with multiple collisions exist, there is no reason for a priori assuming a single age: thus every age determination must be preceded by an assessment of the possibility of multiple ages. As for the calibration uncertainty, it is obvious, even from Figure 9, that it is the main source of uncertainty for most ages (in a logarithmic time scale, the calibration uncertainty being relative gives a roughly constant error bar); however, all the methods to compute ages by the Yarkovsky effect have a hidden calibration, the numerical integrations too. Our published inverse slopes allow anyone interested in more accurate ages to recompute the calibration and revise the estimated ages (with uncertainty).

We would like to conclude with a recommendation for the future discussion regarding asteroid family classifications, on the family name problem. There is a much larger fraction of *interlopers* (classified in the family, but really belonging to the background) among the largest members than among the small ones. Hence, in the fragmentation families it does occur that the largest member is an interloper: examples are families 93 (now 1272) and 283 (now 1521); in 93 even the second largest and second by number, (255), is an interloper. In cratering families it happens that the few largest after the parent body are interlopers: example in 4, with (63), (556) as interlopers.

Thus those working on asteroid families should understand that the conventional namesake is the less stable property for many families. Discussions (and publications) containing only family names are fruitless, let us speak of stable properties like the bulk of the membership. Then we will find, as in the Nysa-Polana-Hertha-Amalasuntha-Eulalia soap opera, that there is not that much disagreement as it appears from some discussions.

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