A PRECISE DETERMINATION OF SOME CRITICAL TERMS IN THE SOLAR SYSTEM

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ABSTRACT

A new form to determine the contribution of some special small divisors in perturbation theory is presented in this paper. We can avoid to calculate all the $k\lambda+k'\lambda$ as it has to be done normally (λ and λ' designate the mean longitudes of the two regarded planets). For a chosen k and k' we calculate with a very high precision the contribution to the perturbation of the elements with the aid of the Hansen's coefficients.

INTRODUCTION

In determining the long periodic perturbations in the planetary motions we adapted a specific formula for the inverse distance and have chosen complex elements as has been described by R. Dvorak (1978). To calculate the long periods in the planetary motions we have taken only the mean values of the right members in the equations of Lagrange. We can also determine as another problem the single influence of critical terms on the motions. Although we can give solutions only to the first order of the masses it is a contribution of great interest, e.g. for the direct and indirect planetary pertubations on the Moon's motion. Another paper by J.Chapront and R.Dvorak (1978) is in preparation to explain the above mentioned subject in detail. Here we shall present only the method.

THE INVERSE DISTANCE AND THE HANSEN COEFFICIENTS

We used a set of complex elements

$$\begin{aligned} &a \text{ the semi major axes} \\ &\lambda \text{ the mean longitude} \\ &z = e.\exp j\tilde{\omega} \\ &\zeta = \sin(i/2).\exp j\Omega \end{aligned} \qquad (1)$$

R. L. Duncombe (ed.), Dynamics of the Solar System, 91–93.

(j stands for $(-1)^{1/2}$) and the development of $1/\Delta$ described by R. Dvorak (1977), where the short-periodic variables $(r/a)^n.\tau^m$ and $(a'/r')^{n'}\tau'^{m'}$ ($\tau = \exp j(v+\tilde{\omega})$) have been separated:

$$\Delta^{-s} = \left(\frac{a'}{r'}\right)^{-s} (2-\delta) \text{ Re} \sum_{n_i}^{N_1} A_{n_i} \left(\frac{r}{a}\right)^n \tau^m \left(\frac{a'}{r'}\right)^{n'} \tau'^{m'} \qquad (2)$$

The A_{n_i} (functions of the semi major axes a and a' and the inclinations i and i') are numbers which are given in tables calculated in advance for all couples of planets. The Hansen coeff icients are introduced to compute the values of $(r/a)^n \tau^m$ and $(a'/r')^{n'}\tau'^{m'}$ as it has been done first by V.Brumberg (1967):

$$\begin{aligned}\left(\frac{r}{a}\right)^n \tau^m &= \left(\frac{r}{a}\right)^n \exp jmv \,.\, \exp jm\tilde{\omega} = \\ &= \sum_k X_k^{n,m} \exp jk\lambda \,.\, \exp j(m-k)\tilde{\omega}\end{aligned} \qquad (3)$$

As we are interested only in one specific critical term of the two regarded planets, we employ the Hansen coefficient $X_k^{n,m}$ for the k fixed in advance. The same mechanism of calculation has to be applied for the $(a'/r')^{n'}\tau'^{m'}$ to be able to seperate above all a small divisor of the form $k\lambda + k'\lambda'$ in the inverse distance $1/\Delta$.

THE LAGRANGE EQUATIONS

The separation of the inverse distance $1/\Delta$ in the Lagrange equations for the variables a, λ, z and ζ gives the following final form for an element σ:

$$\frac{d\sigma}{dt} = \sum_{n=1}^{N_\sigma} \left(B_n^\sigma \{a_n, b_n, c_n, d_n\} + C_n^\sigma \frac{1}{r'^3} \right) \qquad (4)$$

where the parentheses stand for the multiplication

$$\{a_n, b_n, c_n, d_n\} = \frac{1}{\Delta^s} \left(\frac{r}{a}\right)^{a_n} \tau^{b_n} \left(\frac{a'}{r'}\right)^{c_n} \tau'^{d_n} \qquad (5)$$

This multiplication is basically one of two power series. But in our formulae this is a simple modification of the powers of (r/a) and τ as well as for (a'/r') and τ' in (2). It should be mentioned that we have to build a product of a complex number x and the real part of another complex number y (the development (2) for the inverse distance); we have to respect:

$$x\ \mathrm{Re}(y) = \frac{1}{2}(xy + x\bar{y}) \qquad (6)$$

($\bar{y}$ stands for the conjugate complex number).As a consequence we are led to a simple shifting of the powers in the calculation of the two quantities $(r/a)^n\ \tau^m$ and $(a'/r')^{n'}\tau'^{m'}$:

$$\{a_n, b_n, c_n, d_n\} = \left(\frac{r}{a}\right)^{n+a_n} \tau^{m+b_n} \left(\frac{a'}{r'}\right)^{n'+c_n} \tau'^{m'+d_n} + \left(\frac{r}{a}\right)^{n+a_n} \tau^{m-b_n} \left(\frac{a'}{r'}\right)^{n'+c_n} \tau'^{m'-d_n} \qquad (7)$$

RESULTS

We have already results of the couple Venus-Earth for some multiples of the longitudes and these values show a very good agreement with a first order theory (J.Chapront et al, 1975; N.Abu-El-Ata and J.Chapront, 1974). Because of the complexity of the programs we need computation times of some twenty minutes on a IBM 360/44, when we want a precision of 10^{-10} for a single element. We hope to be able to speed up the process in stocking many of the computed quantities (even the Hansen coefficients) which seems to be absolutely necessary for the determination of the long periods.

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