WHITE DWARF SEISMOLOGY : INVERSE PROBLEM OF g-MODE OSCILLATIONS

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Since light variability in white dwarfs was first discovered twenty years ago, eighteen DA white dwarfs, several pulsating DB white dwarfs, and hotter pre-white dwarfs have so far been found to be pulsating variables. The most conspicuous characteristics of pulsations in these stars are that they seem to consist of multiple g-modes of nonradial oscillations. Attention should be paid to multiplicity of modes. Stimulated by the success of helioseimology, a research field called 'asteroseismology', in which we may probe the internal structure of stars by means of observations of their oscillations, is going to develop. How well such a seismological approach succeeds is dependent on how many modes are observed in each of stars. Since the number of modes of an individual pulsating white dwarf is larger than those of other types of pulsating stars but for the Sun, the seismological study may be the most promising as to the white dwarfs. In fact, by applying the asymptotic relations among eigenfrequencies of high order g-modes with low degree, the degree l, and the radial order n, Kawaler(1987a,b,c) succeeded to get some constraints on the physical quantities of some of pulsating white dwarfs.

The eigenfrequencies of g-modes are mainly governed by the Brunt-Väisälä frequency distribution in the star. Therefore, we can, in principle, obtain some information about the Brunt-Väisälä frequency distribution by means of a seismological approach. The Brunt-Väisälä frequency distribution in a white dwarf is related with the entropy gradient, and depends on the cooling rate, the past nuclear reaction, mass loss, convection, diffusion of elements, degeneracy, and so on. None of these processes has so far been well understood. Therefore, seismology may provide a unique tool to investigate these elementary processes. The mathematical procedure to diagnose the Brunt-Väisälä frequency distribution in a white dwarf has been outlined by Shibahashi(1986). This method is a variant of an inversion method to probe the sound velocity distribution in the Sun from the p-mode oscillations of the Sun (Shibahashi 1987, Sekii and Shibahashi 1987). In this paper, we perform some numerical simulation to examine the validity of this method.

The wave equation for g-mode oscillations is, in some limiting cases, reduced to a form similar to the Schrödinger equation in quantum mechanics, which is written as

$$d^2v/dr^2 + N^2(r)r^{-2}[l(l+1)/\omega^2 - \Xi_l(r)]v = 0. \tag{1}$$

Here, v denotes an eigenfunction, ω means the eigenfrequency, l is the degree of the spherical harmonic function which describes the pattern of the mode on the stellar surface, and $\Xi_l(r)$ is the 'gravity wave potential' which consists of the inverse square of the Brunt-Väisälä frequency, $N(r)$, multiplied by $l(l+1)$ and an l-independent part;

$$\Xi_l(r) = l(l+1)/N^2(r) + \Theta(r). \tag{2}$$

The first term in the right hand side of equation (2) dominates over $\Theta(r)$ in the inner part of a white dwarf, while $\Theta(r)$ becomes large in the outer part. Based on the WKB method, the quantization rule leads to

$$(n+\varepsilon)\pi = \int_{r_1}^{r_2} [l(l+1)/\omega^2 - \Xi_l]^{1/2} N/r \, dr, \tag{3}$$

where n is the radial order of the mode and $r_i\,(i=1,2)$'s are the turning points at which $\Xi_l(r_i) = l(l+1)/\omega^2$. Strictly speaking, the quantization rule gives only a relation between discrete eigenvalues $l(l+1)/\omega^2$ and the corresponding integers l and n. But we extend this relation to nonintegers n by interpolation and treat equation (3) as if it were a continuous function of $l(l+1)/\omega^2$ and l. If we can identify each of observed modes, then we can regard equation (3) as an integral equation giving $N/r\ dr/d\Xi_l$, since, for a fixed value of l, the left hand side of equation (3) is a function of $l(l+1)/\omega^2$. The solution gives the distance between two turning points measured with the gravity wave velocity :

$$s(\Xi_l, l) \equiv \int_{r_1}^{r_2} N/r \, dr = 2 \int_{[l(l+1)/\omega^2]_{\min}}^{\Xi_l} \partial n/\partial[l(l+1)/\omega^2] \; [\Xi_l - l(l+1)/\omega^2]^{-1/2} d[l(l+1)/\omega^2]. \tag{4}$$

Here, the lower limit of the integral region corresponds to the minimum of the gravity wave potential, and it is obtained by extrapolating eigenvalues to $n=0$. Once we get solutions (4) for several values of l, we obtain

$$d[1/N(r)]/d\ln r_1 = (2l+1)/[2l(l+1)]\cdot(\partial s/\partial l)^{-1} \tag{5}$$

by differentiating solutions (4) with respect to l since the inner turning point of r_1 is approximately given by

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$$N^2(r_1) = \omega^2. \quad (6)$$

The right hand side of equation (5) is evaluated at a given $l(l+1)/\omega^2$, and then by using equation (6), we should regard equation (5) as an equation to give $d[1/N(r_1)]/d\ln r_1$ as a function of $N(r_1)$. By using a reasonable range of l, we eventually obtain the Brunt-Väisälä frequency distribution in the white dwarf.

To apply the present method to practical cases, we have to perform the mode identification prior to the inversion. It is a quite difficult work, because we cannot resolve the stellar disk image so that the available observational data do not include any direct information of spatial patterns of modes except for the solar case. So, the methods for mode identification is an important subject in asteroseismology. Probably, fitting the observed period spectrum to the asymptotic formula for periods, l, and n (e.g. Tassoul 1980) will be useful for the mode identification. But, at the moment, we suppose the modes are well identified, and we examine how well the Brunt-Väisälä frequency distribution in a white dwarf is inferred from the oscillation data in an ideal case in order to see the validity of the inversion method. To do so, we use a pre-white dwarf model calculated by Kawaler(1986), which is a 0.60 $M_\odot$ star with a luminosity of 100 $L_\odot$, and its theoretically calculated eigenfrequencies of 95 g-modes with $1 \le l \le 10$ and $1 \le n \le 10$. Figure 1 shows the eigenmodes in the $\{l, [l(l+1)/\omega^2]^{1/2}\}$-diagram, which corresponds to the so-called (k, ω)-diagram in helioseismolgy. The range of the degree l is wider than the really detectable modes, but we dare to use even high degree modes such as $l > 4$ because our present purposes are to develop the inversion method and to explore the possibility of the asteroseismological study of white dwarfs in an ideal case. Figure 2 shows the result of the inversion; the thin curve indicates the true Brunt-Väisälä frequency of the model as a function of $\int_0^r N/r\,dr$ (lower scale), and the thick curve indicates the Brunt-Väisälä frequency distribution obtained by using the present method. As seen in this figure, the solution reproduces well the real distribution. Since the inner turning points of g-modes with $l \le 10$ of the present model are at the deep interior, the outer part of the model is not inferred at all. In a case of cool DA white dwarfs, the outer part will be reproduced rather than the inner part.

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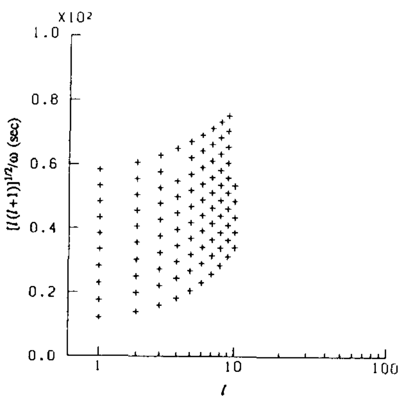

Fig.1. The eigenmodes used in the present paper. The ordinate and the abscissa indicate l and $[l(l+1)]^{1/2}/\omega$, respectively.

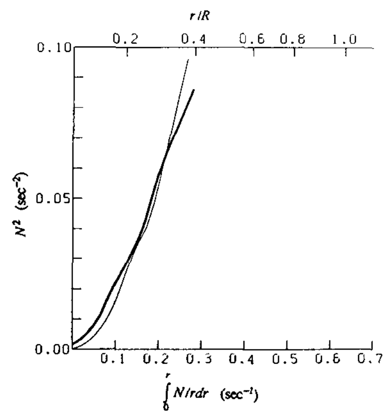

Fig.2. The Brunt-Väisälä frequency distribution in the model (thin curve) and the inverted result (thick curve). The upper scale of the abscissa indicates r/R as a reference.

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