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A COMPREHENSIVE STUDY OF OPTION PRICING WITH TRANSACTION COSTS

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Option pricing has become a key problem studied in academia as well as in the finance industry ever since the publication of the seminal papers by Black and Scholes (1973) and Merton (1973). The Black–Scholes model laid a solid foundation for the rapid development of various option pricing theories in the next half a century. However, the Black–Scholes model imposes some unrealistic assumptions to achieve analytical tractability, among which the assumption of no transaction costs when trading stocks contradicts the fact there would always be costs associated with transactions of stocks in real markets. Although significant development has been made in studying the effects of transaction costs on option pricing in recent years, there are still gaps to fill in the literature.

In general, there are two different approaches to tackle the problem of pricing options with transaction costs: hedging strategies and utility indifference pricing. Each of these two approaches has its own merits for pricing options under incomplete markets. The former method is easy to implement, but does not take investors' preferences into consideration and the latter is very computationally intensive. This thesis aims to provide a comprehensive study of option pricing with transaction costs under both the hedging strategy and the utility maximisation theory, where the effects of transaction costs and stochastic volatility on option prices are analysed, with the emphasis on American option prices and their optimal exercise boundaries.

The thesis is composed of seven chapters, with Chapter 1 being the introduction and Chapter 2 providing a review of preliminary knowledge necessary for the work in later chapters. Chapter 3 presents a pricing model for European options with transaction costs under Heston-type stochastic volatility. This approach is formulated using the

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hedging strategy with some approximation to simplify the calculation of the expected transaction costs in hedging. This new approach is different from the existing literature (Mariani and SenGupta [3]) in two different aspects: Heston volatility is used and the option price does not depend on another option. The solution of the nonlinear partial differential equation is obtained by a finite-difference scheme, proving a fair price range.

Then we focus on the hedging strategy and utility indifference method for pricing options with transaction costs under constant volatility in Chapters 4–6. Our ultimate goal is to study the American option pricing problem with transaction costs via a utility indifference approach. Due to the nonlinearity resulting from the early exercise right of an American option, pricing American options via a utility indifference approach raises two key issues: the optimal exercise boundary which needs to be solved as part of the solution and the heavy computational demands of the nonlinear problem. To solve such a complicated option pricing problem, we start by dealing with two fundamental problems as the base of our ultimate goal. First, we prove that the utility indifference approach is equivalent to the hedging strategy for the American option pricing problem in a complete market in Chapter 4. This problem is not well addressed due to the nonlinearity of the problem resulting from the optimal exercise boundary. A numerical study is conducted to deal with such an important problem. Then, in Chapter 5, we derive a new pricing approach for European options with transaction costs, where the ideas of hedging strategy and utility indifference approach are combined to achieve a balance of efficiency and accuracy. Our utility indifference approach reduces the dimension of the portfolio problem without options, thus, achieves better efficiency than the standard utility approach. In Chapter 6, we price American options with transaction costs via these two approaches. Since the impact of transaction costs on the American option price, particularly on the most important feature of American options, is much less investigated, we provide a supplementary study in this area by analysing the effect of transaction costs on the optimal exercise price of an American option in addition to the option price itself through a utility-based approach. With a computationally efficient numerical scheme, we are able to demonstrate clearly how the optimal exercise price should be calculated and consequently how the option prices for the buyer and writer as well as the early exercise decision are affected by the inclusion of transaction cost.

The comparison presented in the thesis proves that compared to hedging strategies, utility indifference methods produce more realistic option prices in the presence of transaction costs. The utility indifference approach proposed in this study could provide a useful, computationally efficient way for pricing options with transaction costs.

Some of this research has been published in [1, 2, 4, 5].

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