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MARTA FIORI CARONES, *Filling cages. Reverse mathematics and combinatorial principles*, University of Udine, Italy, 2020. Supervised by Alberto Marcone. MSC: 03B30, 03F35, 05C20, 05C55, 05C62, 06A07. Keywords: Reverse mathematics, graph theory, order theory.

### Abstract

In the thesis some combinatorial statements are analysed from the reverse mathematics point of view. Reverse mathematics is a research program, which dates back to the Seventies, interested in finding the exact strength, measured in terms of set-existence axioms, of theorems from ordinary non set-theoretic mathematics.

After a brief introduction to the subject, an on-line (incremental) algorithm to transitively reorient infinite pseudo-transitive oriented graphs is defined. This implies that a theorem of Ghouila–Hourī is provable in  $\text{RCA}_0$  and hence is computably true.

Interval graphs and interval orders are the common theme of the second part of the thesis. A chapter is devoted to analyse the relative strength of different characterisations of countable interval graphs and to study the interplay between countable interval graphs and countable interval orders. In this context the theme of unique orderability of interval graphs arises, which is studied in the following chapter. The last chapter about interval orders inspects the strength of some statements involving the dimension of countable interval orders.

The third part is devoted to the analysis of two theorems proved by Rival and Sands in 1980. The first principle states that each infinite graph contains an infinite subgraph such that each vertex of the graph is adjacent either to none, or to one or to infinitely many vertices of the subgraph. This statement, restricted to countable graphs, is proved to be equivalent to  $\text{ACA}_0$  and hence to be stronger than Ramsey's theorem for pairs, despite the similarity of the two principles. The second theorem proved by Rival and Sands states that each infinite partial order with finite width contains an infinite chain such that each point of the poset is comparable either to none or to infinitely many points of the chain. For each  $k \geq 3$ , the latter principle restricted to countable poset of width  $k$  is proved to be equivalent to ADS. Some complementary results are presented in the thesis.

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TOMASZ CIEŚLA, *Measurable combinatorics and orbit equivalence relations*, McGill University, Canada, 2020. Supervised by Marcin Sabok. MSC: 03E15. Keywords: Borel reducibility, equidecomposability, lifting probability measures.

### Abstract

A Borel action of a Polish group  $G$  on a Polish space  $X$  determines an equivalence relation  $E \subset X \times X$  whose classes are orbits of the action. Such a relation is called an *orbit equivalence relation*. Given two orbit equivalence relations,  $E \subset X \times X$  and  $F \subset Y \times Y$ , we say that  $E$  is *Borel reducible* to  $F$  if there exists a Borel function  $g: X \rightarrow Y$  such that  $\forall x, y \in X \ xEy \iff g(x)Fg(y)$ . Informally speaking, this means that  $F$  is at least as complicated as  $E$ . An orbit equivalence relation  $E$  is called *complete* (or *universal*) if all orbit equivalence relations are Borel reducible to  $E$ . In recent years many relations naturally arising in mathematics were proved to be complete, such as the isometry relation of Polish metric spaces (Gao, Kechris, and Clemens), the isometry relation of separable  $C^*$ -algebras (Sabok), and the homeomorphism relation of compact Polish spaces (Zielinski). The latter result was strengthened by Chang and Gao, who proved that the homeomorphism relation of continua (i.e., connected compact Polish spaces) is complete. They asked whether the homeomorphism

relation of locally connected continua is complete as well. The author of the thesis proved that the answer is affirmative by showing a stronger result: the homeomorphism relation of locally star-convex continua is complete.

The second part of the thesis is based on a joint work with Marcin Sabok. Let  $\Gamma \curvearrowright (X, \mu)$  be a free pmp action of an amenable group and let  $A, B \subset X$  be two sets of the same positive measure. It's easy to see that if  $A$  and  $B$  are equidecomposable (i.e., there exists a partition  $A_1 \sqcup A_2 \sqcup \dots \sqcup A_n$  of  $A$  and group elements  $\gamma_1, \dots, \gamma_n$  such that  $\gamma_1 A_1 \sqcup \gamma_2 A_2 \sqcup \dots \sqcup \gamma_n A_n$  is a partition of  $B$ ) then  $A$  and  $B$  satisfy a variant of the Hall condition in the Schreier graph of the action  $\Gamma \curvearrowright (X, \mu)$ . The main result asserts that for free pmp actions of  $\mathbb{Z}^d$  and sets  $A, B$  satisfying the following equidistribuity condition: there exists a constant  $c$  such that for any discrete cube  $F = \{0, 1, \dots, n\}^d \cdot x$  with  $x \in X$  it holds that  $\frac{||A \cap F| - |B \cap F||}{|F|} \leq \frac{c}{n}$ , the Hall condition is equivalent to existence of a measurable equidecomposition. This strengthens recent results of Grabowski, Máthé, and Pikhurko on measurable circle squaring.

The last part of the thesis is devoted to the problem of lifting invariant probability measures. Let  $X$  be a Polish space with a Borel probability measure  $\mu$  and  $T: X \rightarrow X$  be a Borel map preserving  $\mu$ . Let  $Y$  be a Polish space and  $S: Y \rightarrow Y$  be a continuous map. Let  $p: Y \rightarrow X$  be a Borel surjection such that  $p \circ S = T \circ p$ . Does  $\mu$  lift to an  $S$ -invariant Borel probability measure  $\nu$  on  $Y$ ? The main theorem provides a fairly general sufficient condition for that: if for  $\mu$ -almost all  $x \in X$  the set  $p^{-1}(x)$  is compact, then such a measure  $\nu$  exists. An analogous result holds true for left amenable semigroups acting on  $X$  by measure preserving maps and acting on  $Y$  by continuous maps so that the actions commute with  $p$ .

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