14

Quantum Eraser Experiments

14.1 Introduction

In recent years, numerous quantum optics experiments have demonstrated novel quantum effects based on quantum amplitude superposition. Some of these empirical observations raise disturbing questions about the nature of reality, particularly concerning quantum nonlocality. In this chapter we discuss some experiments that suggest (to us, at least) that the information void, that uncharted regime between signal preparation and outcome detection, is not well modeled by classical spacetime. Something strange seems to be lurking there.

For the purposes of exposition, the sequence of experiments we discuss in this chapter does not follow the historical sequence of those experiments, We discuss first the *delayed-choice quantum eraser* experiment of Kim et al. (Kim et al., 2000), referred to here as KIM.[1] This leads us to define a heuristic measure of *which-path* information that is central to the theme of this chapter. This leads to a discussion of Wheeler's thought experiments on the conundrum of *delayed choice* in physics, particularly evident in the phenomenon of galactic lensing (Wheeler, 1983), and its empirical implementation, the *delayed-choice interferometer* experiment of Jacques et al., referred to as JACQUES. Our final experiment in this sequence is the *double-slit quantum eraser* of Walborn et al. (Walborn et al., 2002), referred to as WALBORN. That experiment gives a significant challenge for quantized detector networks (QDN) to reproduce its empirical results.

Such experiments have led to suggestions that interference patterns formed by particles impacting on a screen may be influenced in some way by decisions made long after those particles had landed on that screen. Our objective in this chapter is to show by a detailed QDN analysis of those experiments that

[1] With profound apologies to co-authors, we will employ the convention that several experiments are referred to by the capitalized surname of the first named author.

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quantum principles do not support these suggestions. Nevertheless, something bizarre seems to be there.

There are two complementary aspects in these experiments that have deeply worried physicists. The first involves *spatial nonlocality* and is the point of Wheeler's concerns. Historically, it has been a great source of debate among physicists. It motivated Einstein to write to Born thus:[2]

> So clearly that you consider my attitude towards statistical quantum mechanics to be strange and archaic . . .
>
> . . . I cannot make a case for my attitude in physics which you would consider at all reasonable. I admit, of course, that there is a considerable amount of validity in the statistical approach which you were the first to recognise clearly as necessary given the framework of the existing formalism. I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky action at a distance.
>
> (*Born–Einstein Letters*, 1971, p. 158)

The second aspect involves *temporal nonlocality*: the temporal sequence of actions taken in the laboratory at different places seems to be irrelevant in certain experiments. We regard that as a vindication of the *stages* concept. The empirical evidence from experiments such as WALBORN, who specifically investigated the temporal aspects, is that the passage of "detector time" is synonymous with quantum information acquisition (QIA) occurring in a sequence of *stages* (Eakins and Jaroszkiewicz, 2005). Stages have rules that are different in some aspects from those of classical information acquisition, and this accounts for some of the apparent strangeness of quantum mechanics.

The rules governing QIA are those of quantum mechanics (QM) and indeed conform with all known physics. For example, QIA never violates the light-cone constraint of relativity that classical information cannot be acquired between space-like intervals. Underpinning this is the so-called *no-communication theorem* in QM, discussed in Section 16.7. This states that the actions taken by an observer on a substate of a total state of a system under observation (SUO) cannot be detected by another observer of another substate of that total state.

One way of understanding this is to note that quantum correlations appearing to violate the principle of Einstein locality always require observations to be completed *before* those correlations can be defined by observers, and this completion necessarily always takes place in a classically consistent matter. Currently,

[2] The specific English translation of what Einstein wrote here has been disputed: *mysterious* rather than *spooky* has been suggested by some authorities as closer to what Einstein intended to say.

there is no empirical evidence for, or theoretical necessity of, the contextually incomplete notion that information can flow backward in time, as suggested by Cramer's transactional interpretation of QM (Cramer, 1986), or for the existence of closed time-like curves (CTCs), as found in some relativistic spacetimes such as that of Gödel (Gödel, 1949).

On the other hand, quantum information *can* be shielded against the effects of decoherence and preserved in a state of stasis for arbitrarily long periods of laboratory time. QDN uses the concept of null test to encode this phenomenon, as demonstrated in our discussion of particle decay experiments in Chapter 15. In the delayed-choice experiments discussed in the present chapter, suitably shielded observations involving separate detectors can be taken in apparently random order relative to laboratory time without affecting correlations.

14.2 Delayed-Choice Quantum Eraser

We turn first to the *delayed-choice quantum eraser*. The architecture of an experiment proposed in Kim et al. (2000), and referred to here as KIM, is shown in Figure 14.1. In that stage diagram, we leave out dotted lines indicating stages and use subscripts for this purpose. In the following description, we use the term *photon* for convenience only.

A source S produces an initial total state 1_0 that is a superposition of two coherent photon pairs. By stage Σ_1 these are split by suitable apparatus into four components 1_1, 2_1, 3_1, and 4_1 as shown. Components 1_1 and 3_1 are from different initial pairs and are passed onto a detecting screen DS, consisting of detectors $5_2, 6_2, \ldots, K_2$, where it is supposed that they interfere, just as in the original double-slit experiment. Our analysis will show that the situation is more subtle than that.

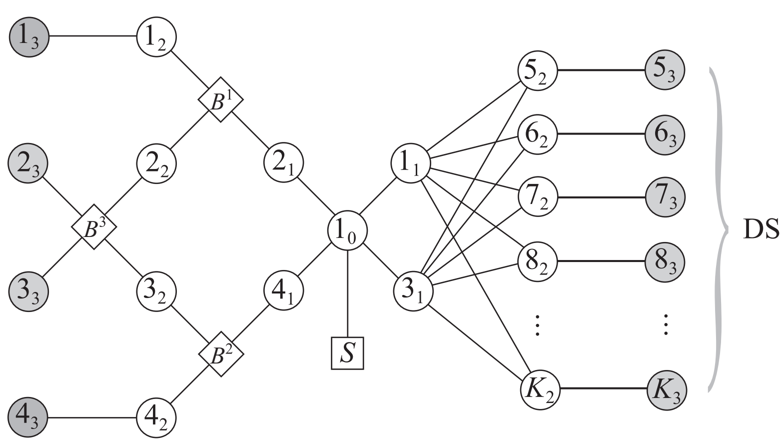

Figure 14.1. KIM, a proposed delayed-choice quantum eraser experiment (Kim et al., 2000).

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Meanwhile, components 2_1 and 4_1 are each passed through beam splitters B^1, B^2 as shown. The transmitted components from these beam splitters are sent on to detectors 1_3 and 4_3 as shown, while the reflected components 2_2 and 3_2 are passed through beam splitter B^3. The outcome channels of this beam splitter are monitored by detectors 2_3 and 3_3.

Analysis of this arrangement suggests that choices made by the experimentalist at B^3 can influence the signal patterns seen on DS, even though the signals in that screen had been captured earlier at stage Σ_2. Kim et al.'s argument is that correlations between signals in 1_3 and DS or between signals in 4_3 and DS give *which-path* information, whereas correlations between signals in either 2_3 and DS, and 3_3 and DS do not. Therefore, there should be no interference terms on DS when there are signals in 1_3 or 4_4, whereas interference terms are expected when 1_3 or 3_3 register a signal.

We proceed with our stage analysis as follows. From Figure 14.1 we read off the following parameters for our CA program MAIN: $N = 3$, $r_0 = 1$, $r_1 = 4$, $r_2 = r_3 = K, d_0 = d_1 = d_2 = d_3 = 1$, where K is chosen to be large enough so as to show what happens on the screen DS. In MAIN a value $K = 6$ was sufficient to show all the essential features of the experiment.

As with the double-slit experiment discussed in Chapter 10, we represent our initial preparation switch at stage Σ_0 by the total state $|\Psi_0) = |s_0\rangle \otimes \widehat{\mathbb{A}}_0^1 \mathbf{0}_0$.

Stage Σ_0 to Stage Σ_1

The jump to stage Σ_1 is defined by the production of two correlated photon pairs. Photon polarization is ignored here, but the formalism can readily deal with any situation where this is not the case. Therefore, MAIN assumes a one-dimensional internal Hilbert space in this experiment with normalized basis $|s_n\rangle$ for the internal degrees of freedom at any given stage Σ_n.

The semi-unitary transformation from $\Sigma_0 \to \Sigma_1$ is given by

$$\mathbb{U}_{1,0} \left\{ |s_0\rangle \otimes \widehat{\mathbb{A}}_0^1 \mathbf{0}_0 \right\} = |s_1\rangle \otimes \left\{ \alpha \widehat{\mathbb{A}}_1^1 \widehat{\mathbb{A}}_1^2 + \beta \widehat{\mathbb{A}}_1^3 \widehat{\mathbb{A}}_1^4 \right\} \mathbf{0}_1, \tag{14.1}$$

where $|\alpha|^2 + |\beta|^2 = 1$. As we stated above we assume that photon spin is not relevant to the issues being explored here.

Stage Σ_1 to Stage Σ_2

For this jump we have

$$\mathbb{U}_{2,1} \left\{ |s_1\rangle \otimes \widehat{\mathbb{A}}_1^1 \widehat{\mathbb{A}}_1^2 \mathbf{0}_1 \right\} = |s_2\rangle \otimes \sum_{i=5}^{K} C^i \widehat{\mathbb{A}}_2^i \{ t^1 \widehat{\mathbb{A}}_2^1 + i r^1 \widehat{\mathbb{A}}_2^2 \} \mathbf{0}_2,$$

$$\mathbb{U}_{2,1} \left\{ |s_1\rangle \otimes \widehat{\mathbb{A}}_1^3 \widehat{\mathbb{A}}_1^4 \mathbf{0}_1 \right\} = |s_2\rangle \otimes \sum_{i=5}^{K} D^i \widehat{\mathbb{A}}_2^i \{ t^2 \widehat{\mathbb{A}}_2^4 + i r^2 \widehat{\mathbb{A}}_2^3 \} \mathbf{0}_2. \tag{14.2}$$

Here, one component beam from each pair is focused onto the detecting screen DS, while the other component is channeled onto either beam-splitter B^1 or B^2,

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as shown. The $\{C^i\}$ coefficients represent the amplitudes for landing on the screen DS from 1_1, while the $\{D^i\}$ are from 3_1. The coefficients t^i, r^i are characteristic transmission and reflection parameters associated with beam splitter B^i, $i = 1, 2, 3$, and satisfy the rule $(t^i)^2 + (r^i)^2 = 1$. It is useful not to set these parameters to the conventional value $1/\sqrt{2}$ but to keep them open and available to be changed. It is in these parameters that we encode the observer's freedom of choice in this particular experiment.

Stage Σ_2 to Stage Σ_3

The final transition from stage Σ_2 to Σ_3 involves four terms, each of which involves null tests in one way or another. Most significantly for this discussion, it is supposed that the DS screen detectors have registered their signals irreversibly by stage Σ_2. Therefore, this information is carried to stage Σ_3 by null tests, shown by the horizontal lines on the right-hand side of Figure 14.1.

We write, for $i = 5, 6, \ldots, K$:

$$\begin{aligned}
\mathbb{U}_{3,2}\left\{|s_2\rangle \otimes \widehat{\mathbb{A}}_2^i \widehat{\mathbb{A}}_2^1 \mathbf{0}_2\right\} &= |s_3\rangle \otimes \widehat{\mathbb{A}}_3^i \widehat{\mathbb{A}}_3^1 \mathbf{0}_3, \\
\mathbb{U}_{3,2}\left\{|s_2\rangle \otimes \widehat{\mathbb{A}}_2^i \widehat{\mathbb{A}}_2^2 \mathbf{0}_2\right\} &= |s_3\rangle \otimes \widehat{\mathbb{A}}_3^i \{t^3 \widehat{\mathbb{A}}_3^3 + i r^3 \widehat{\mathbb{A}}_3^2\} \mathbf{0}_3 \\
\mathbb{U}_{3,2}\left\{|s_2\rangle \otimes \widehat{\mathbb{A}}_2^i \widehat{\mathbb{A}}_2^3 \mathbf{0}_2\right\} &= |s_3\rangle \otimes \widehat{\mathbb{A}}_3^i \{t^3 \widehat{\mathbb{A}}_3^2 + i r^3 \widehat{\mathbb{A}}_3^3\} \mathbf{0}_3 \\
\mathbb{U}_{3,2}\left\{|s_2\rangle \otimes \widehat{\mathbb{A}}_2^i \widehat{\mathbb{A}}_2^4 \mathbf{0}_2\right\} &= |s_3\rangle \otimes \widehat{\mathbb{A}}_3^i \widehat{\mathbb{A}}_3^4 \mathbf{0}_3.
\end{aligned} \tag{14.3}$$

This is all the information needed for our CA program MAIN to answer all possible maximal questions. MAIN gives four sets of nonzero maximal question answers, equivalent to two-site correlations:

$$\begin{aligned}
&\Pr(\widehat{\mathbb{A}}_3^1 \widehat{\mathbb{A}}_3^i \mathbf{0}_3 | \Psi_0) = |t^1 C^i \alpha|^2, \\
&\Pr(\widehat{\mathbb{A}}_3^2 \widehat{\mathbb{A}}_3^i \mathbf{0}_3 | \Psi_0) = |r^1 r^3 C^i \alpha - i r^2 t^3 D^i \beta|^2, \\
&\Pr(\widehat{\mathbb{A}}_3^3 \widehat{\mathbb{A}}_3^i \mathbf{0}_3 | \Psi_0) = |r^1 t^3 C^i \alpha + i r^2 r^3 D^i \beta|^2, \\
&\Pr(\widehat{\mathbb{A}}_3^4 \widehat{\mathbb{A}}_3^i \mathbf{0}_3 | \Psi_0) = |t^2 D^i \beta|^2, \qquad i = 5, 6, \ldots, K.
\end{aligned} \tag{14.4}$$

These give the total sum

$$\sum_{i=1}^{4} \sum_{j=5}^{K} \Pr(\widehat{\mathbb{A}}_3^i \widehat{\mathbb{A}}_3^j \mathbf{0}_3 | \Psi_0) = |\alpha|^2 \sum_{j=5}^{K} |C^i|^2 + |\beta|^2 \sum_{j=5}^{K} |D^i|^2, \tag{14.5}$$

which is consistent with probability conservation if we take the semi-unitarity conditions $\sum_{j=5}^{K} |C^i|^2 = \sum_{j=5}^{K} |D^i|^2 = 1$ into account. Note also that semi-unitarity requires $\sum_{j=5}^{K} C^{i*} D^i = 0$, where C^{i*} is the complex conjugate of C^i.

There are several observations to be made about these results.

1. The only genuine *interference* found in our analysis occurs in the two-signal correlation probabilities $\Pr(\widehat{\mathbb{A}}_3^2 \widehat{\mathbb{A}}_3^i \mathbf{0}_3 | \Psi_0)$ and $\Pr(\widehat{\mathbb{A}}_3^3 \widehat{\mathbb{A}}_3^i \mathbf{0}_3 | \Psi_0)$.
2. The parameters t^i, r^i for beam splitter B^i represent places in the apparatus where the experimentalist could make changes, either before or after signals

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have been registered on the screen DS during any given run of the experiment. In other words, choices can be made at B^1, B^2, and B^3 that affect various incidence rates. The question is, does any change made by the experimentalist at any beam splitter affect anything that has been measured *before* that change was made? In particular, can any change in B^3 affect what has already happened on the screen?

By inspection of (14.3), we see that no change in t^3 or r^3, subject to $(t^3)^2 + (r^3)^2 = 1$, has any effect whatsoever on $\Pr(\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$ or $\Pr(\widehat{\mathbb{A}}_3^4\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$. These coincidence rates actually involve signal detection completed during *earlier* stages. The conclusion therefore is that any suggestion that delayed choice can erase information irreversibly acquired in the past is incorrect and misleading.

3. By inspection, it is true that changes in t^3 and r^3 affect $\Pr(\widehat{\mathbb{A}}_3^2\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$ and $\Pr(\widehat{\mathbb{A}}_3^3\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$. However no acausality is involved, because a coincidence rate is undefined until signals from both detectors involved have been counted. $\Pr(\widehat{\mathbb{A}}_3^2\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$ and $\Pr(\widehat{\mathbb{A}}_3^3\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$ cannot be measured until *after* the choice of t^3 and r^3 has been made.

   Suggestions that events in stage Σ_3 could influence events in earlier stages do not take into account the crucial role of *post-selection*[3] in such experiments. The proper way to understand what is happening is to view the role of the four detectors $1_3, 2_3, 3_3$, and 4_3 as post-selection apparatus for the processing of data already accumulated on the screen DS.

4. Program MAIN allows for partial questions to be answered. If we look at the individual total counting rates at each of the four detectors i_3, $i = 1, 2, 3, 4$, we find

$$\begin{aligned}
\Pr(\widehat{\mathbb{A}}_3^1\mathbf{0}_3|\Psi_0) &\equiv \sum_{i=5}^{K} \Pr(\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0) = |t^1\alpha|^2, \\
\Pr(\widehat{\mathbb{A}}_3^2\mathbf{0}_3|\Psi_0) &\equiv \sum_{i=5}^{K} \Pr(\widehat{\mathbb{A}}_3^2\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0) = |r^1 r^3\alpha|^2 + |t^3 r^2\beta|^2, \\
\Pr(\widehat{\mathbb{A}}_3^3\mathbf{0}_3|\Psi_0) &\equiv \sum_{i=5}^{K} \Pr(\widehat{\mathbb{A}}_3^3\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0) = |r^1 t^3\alpha|^2 + |r^2 r^3\beta|^2, \\
\Pr(\widehat{\mathbb{A}}_3^4\mathbf{0}_3|\Psi_0) &\equiv \sum_{i=5}^{K} \Pr(\widehat{\mathbb{A}}_3^4\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0) = |t_2\beta|^2, \qquad (14.6)
\end{aligned}$$

using the semi-unitarity of the $\{C^i\}$ and $\{D^i\}$ coefficients. Again, changes made at B^3 would have no effect on $\Pr(\widehat{\mathbb{A}}_3^1\mathbf{0}_3|\Psi_0)$ or $\Pr(\widehat{\mathbb{A}}_3^4\mathbf{0}_3|\Psi_0)$ but would

[3] In ordinary usage, the term *post-selection* means selection occurring *after* some given process. In probability theory, given an event A, then the probability of some event B conditional on that given is a conditional probability, denoted $P(B|A)$. Either way, the concern is with what happens *after* something has been done or occurred.

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affect $\Pr(\widehat{\mathbb{A}}_3^2 \mathbf{0}_3|\Psi_0)$ and $\Pr(\widehat{\mathbb{A}}_3^3 \mathbf{0}_3|\Psi_0)$. These probabilities sum to unity as expected.

5. If we look at the individual relative probabilities for a given detector i_3, $i = 5, 6, \ldots, K$, on the screen DS, we find

$$\Pr(\widehat{\mathbb{A}}_3^i \mathbf{0}_3|\Psi_0) \equiv \Pr \sum_{j=1}^{4} \Pr(\widehat{\mathbb{A}}_3^j \widehat{\mathbb{A}}_3^i \mathbf{0}_3|\Psi_0) = |\alpha C^i|^2 + |\beta D^i|^2, \tag{14.7}$$

which shows no interference on DS. No change at any of the beam splitters affects the single detector rates observed on the screen DS. This is essentially the same phenomenon, discussed in Section 13.4, found in the two-photon interferometer experiment of Horne, Shimony, and Zeilinger (Horne et al., 1989).

We should ask: *Given that signals on the detector screen DS came from the double slits, why is there no interference on that screen? Surely there should be such interference.*

The answer is deep. This experiment is not just a double-slit experiment. The mere fact that a photon has gone off toward the beam splitters B^1 or B^2 *and* the observer could in principle find out which beam splitter it was is enough to provide which-path information. It is that contextual information that destroys any possibility of an interference pattern on DS. In fact, the beam splitters are not needed for this. It is enough to have 2_1 and 4_1 as identifiable potential sites for signal detection to destroy any chance of interference on DS.

14.3 Which-Path Measure

The double-slit and delayed-choice eraser experiments discussed up to this point in this book belong to an important class of experiment that, to use colloquial terminology, provide partial or complete information about which path a photon had taken in its journey from initial to final stages. We shall discuss in the next section another example, Wheeler's delayed choice experiment.

Each of these experiments carries with it contextual attributes arising from the experimental setup that determine the extent to which photon paths can be determined from the data or not. For example, the double-slit experiment with both slits open gives zero information about which slit a particular detected photon came from. On the other hand, the same setup with one of the slits blocked up gives total information as to where any of the detected photons originated.

It is of interest therefore to find some measure or parameter Φ that is characteristic of any given experimental setup and that gives us an indication as to how much which-path information we could extract from the experiment. There is no precise theory for this known to us at present, so in the absence of any deeper analysis, our choice is to define Φ heuristically as the total probability of determining with certainty full path information from a single detected "indicator" photon, that is, from a single signal in a specific set of detectors. We define

$$\Phi_{DS} \equiv Prob(\text{full path information}|\text{any single indicator detector fires}). \quad (14.8)$$

For example, in the case of the double-slit experiment discussed in Chapter 10, the detecting screen contains the indicator detectors. Then with both slits open, we find $\Phi_{DS} = 0$. When one of the slits is open, then $\Phi_{DS} = 1$. In the case of the delayed choice quantum eraser discussed above, the indicator detectors are 1_3 and 4_3, so we define

$$\Phi_{DC} \equiv \Pr(\widehat{\mathbb{A}}_3^1 \mathbf{0}_3|\Psi_0) + \Pr(\widehat{\mathbb{A}}_3^4 \mathbf{0}_3|\Psi_0) = |t^1\alpha|^2 + |t_2\beta|^2.$$

In the conventional symmetric situation, $|\alpha| = |\beta| = t^1 = t^2 1/\sqrt{2}$, giving $\Phi_{DC} = 1/2$, as we should expect. When $t^1 = t^2 = 0$, transmission to 1_3 or 4_3 cannot occur, so there is normally interference for sure, giving $\Phi_{DC} = 0$, meaning no path information can be established.

There is a pathology in this last case, because $t^1 = t^2 = 0$ gives $\Pr(\widehat{\mathbb{A}}_3^2 \mathbf{0}_3|\Psi_0) = |r^3\alpha|^2 + |t^3\beta|^2$ and $\Pr(\widehat{\mathbb{A}}_3^3 \mathbf{0}_3|\Psi_0) = |t^3\alpha|^2 + |r^3\beta|^2$. If now, in addition to setting $t^1 = t^2 = 0$, the experimentalist had set $r^3 = 0$, then a single photon would be detected at 2_3 or 3_3 and it would clear which path had been taken by the photons. But overall, this is equivalent to having no beam splitters, so this scenario is of no value here.

14.4 Wheeler's Double-Slit Delayed Choice Experiment

The physicist John Wheeler gave a theoretical discussion of a quantum interference experiment that has stimulated a great deal of puzzlement and controversy concerning the nature of reality (Wheeler, 1983). It is our considered judgment that his concern reflects the inadequacy of our classical views about reality.

There are two forms of his experiment that are commonly discussed: one is a modified Mach–Zehnder interferometer (MZI) and the other is a modified double-slit (DS) experiment. We shall refer to these as WHEELER-1 and WHEELER-2, respectively.

WHEELER-1

Figure 13.1 is relevant to WHEELER-1. Specifically, WHEELER-1 is concerned with stage-Σ_1 nodes 1_1 and 2_1 and their relationship to stage-Σ_2 nodes 1_2 and 2_2. Suppose the laboratory time between stage Σ_1 and stage Σ_2 is significantly long enough for anything done to one of the beams, at say module ϕ, to be long *after* the action of beam splitter B^1 in any reasonable sense of the word. Then there appears to be a clash of particle and wave concepts.

To be specific, consider a pulse of light 1_0 coming from a distant galaxy billions of years ago at stage Σ_0 being split by gravitational lensing (equivalent to B^1)

into components 1_1 and 2_1. These two components follow very different paths across the Universe until an observer here on Earth observes stage-Σ_3 signals at detectors 1_3 or 2_3. Now two kinds of process could affect outcome probabilities at those detectors: (1) there could be some galactic cloud affecting, say, the optical path from 1_1 to 1_2, equivalent to module ϕ, and (2) the observer could choose to do something significant at B^2 so that either an interference pattern is built up or no such pattern is built up.

The conventional picture here is of a particle known as a photon traveling through space. Intuitively, we would like to think of such objects. But it seems incredible that the presence of some disruptive process ϕ on the potential path $1_1 \to 1_2$ could affect a photon traveling from 2_1 to 2_2. Yet that is just what the photon paradigm requires us to contemplate, particularly if the observer decides to arrange the apparatus at B^2 to detect an interference pattern at the detectors 1_3 and 2_3, or not.

Of this scenario, Wheeler wrote:

> We get up in the morning and spend the day in meditation whether to observe by "which route" or to observe interference between "both routes." When night comes and the telescope is at last usable we leave the half-silvered mirror out or put it in, according to our choice. The monochromatizing filter placed over the telescope makes the counting rate low. We may have to wait an hour for the first photon. When it triggers a counter, we discover "by which route" it came with the one arrangement; or by the other, what the relative phase is of the waves associated with the passage of the photon from source to receptor "by both routes" – perhaps 50,000 light years apart as they pass the lensing galaxy. But the photon has already *passed* that galaxy billions of years before we made our decision. This is the sense in which, in a loose way of speaking, what the photon *shall have done* after it has *already* done it. In actuality it is wrong to talk about the "route" of the photon. For a proper way of speaking we recall once more that it makes no sense to talk of the phenomenon until it has been brought to a close by an irreversible act of amplification. "No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon." (Wheeler, 1983)

The problem as we see it lies not with the quantum physics but with mental imagery associated with the particle-wave concept. There is, for instance, a classically conditioned vacuous assertion glaringly obvious in Wheeler's statement above: that "the photon has already passed that galaxy billions of years before we made our decision." Proof? There can be none, but of course, we are all strongly conditioned to believe this counterfactual, vacuous statement and many like it. According to quantum physics principles, as we see them in this book, we have no direct empirical entitlement to make that assertion. In quantum physics, we have to resist the constant temptation to think classically in those places where such thinking is unwarranted.

WHEELER-2

Wheeler's delayed choice experiment can be discussed as a double-slit experiment with a modified screen. Some of the detectors on the screen can receive quantum signals from both slits, while the others can receive a signal from only one of the slits. The interest in this arrangement comes from the possibility that the observer can decide in principle which detectors receive which signal(s) long *after* light has left the two slits.

An idealized version of WHEELER-2 is shown in Figure 14.2. The details are much the same as the double-slit experiment studied in Chapter 10, but with the difference that now there are three groups of detectors on the screen. Detectors 1_2 to R_2 can receive a quantum amplitude from 1_1 only, $(R+1)_2$ to $(R+S)_2$ can receive quantum amplitudes from 1_1 and from 2_1, and $(R+S+1)_2$ to K_2 can receive an amplitude from 2_1 only. Here, $K = R + S + T$.

Following Wheeler, we imagine that the experimentalist can shuffle the values of R, S, and T during any given run, *after* they are sure that light has left the two slits 1_1 and 2_2 and *before* any impact on the final detecting screen at stage Σ_2. Any actual experiment would require analysis of the final data, post-selecting signals corresponding to equivalent values of R, S, and T.

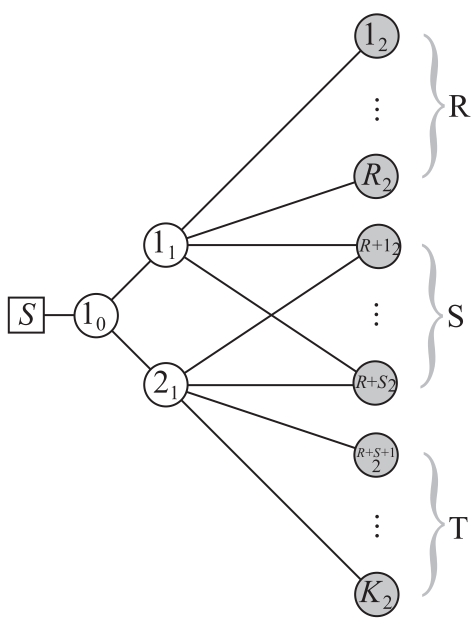

Figure 14.2. Idealization of Wheeler's delayed-choice experiment, WHEELER-2.

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For this discussion, we do not need CA assistance. Photon polarization is not an issue in this particular instance either, so the initial labstate $|\Psi_0)$ is given by $|\Psi_0) \equiv |s_0\rangle \otimes \widehat{\mathbb{A}}_0^1 \mathbf{0}_0$. The stage dynamics goes as follows.

Stage Σ_0 to Stage Σ_1

The evolution of the preparation switch is given by

$$U_{1,0}\left\{|s_0\rangle \otimes \widehat{\mathbb{A}}_0^1 \mathbf{0}_0\right\} = |s_1\rangle \otimes \left\{\alpha \widehat{\mathbb{A}}_1^1 + \beta \widehat{\mathbb{A}}_1^2\right\} \mathbf{0}_1, \tag{14.9}$$

where $|\alpha|^2 + |\beta|^2 = 1$.

Stage Σ_1 to Stage Σ_2

The evolution of the two separate components at stage Σ_1 is given by

$$U_{2,1}\left\{|s_1\rangle \otimes \widehat{\mathbb{A}}_1^1 \mathbf{0}_1\right\} = |s_2\rangle \otimes \left\{\underbrace{\sum_{i=1}^{R} R^i \widehat{\mathbb{A}}_2^i \mathbf{0}_2}_{\text{Region } R} + \underbrace{\sum_{i=R+1}^{R+S} S^{i,1} \widehat{\mathbb{A}}_2^i \mathbf{0}_2}_{\text{Region S}}\right\},$$

$$U_{2,1}\left\{|s_1\rangle \otimes \widehat{\mathbb{A}}_1^2 \mathbf{0}_1\right\} = |s_2\rangle \otimes \left\{\underbrace{\sum_{i=R+1}^{R+S} S^{i,2} \widehat{\mathbb{A}}_2^i \mathbf{0}_2}_{\text{Region } S} + \underbrace{\sum_{i=R+S+1}^{K} T^i \widehat{\mathbb{A}}_2^i \mathbf{0}_2}_{\text{Region T}}\right\}. \tag{14.10}$$

Here the coefficients $\{R^i\}, \{S^{i,1}, S^{i,2}\}$, and $\{T^i\}$ satisfy the semi-unitary conditions

$$\sum_{i=1}^{R} |R^i|^2 + \sum_{i=R+1}^{R+S} |S^{i,1}|^2 = \sum_{i=R+S+1}^{K} |T^i|^2 + \sum_{i=R+1}^{R+S} |S^{i,2}|^2 = 1, \quad \sum_{i=R+1}^{R+S} S^{i,1*} S^{i,2} = 0. \tag{14.11}$$

The outcome probabilities are found to be

$$\begin{aligned} \Pr(\mathbb{A}_2^i \mathbf{0}_2 | \Psi_0) &= |\alpha R^i|^2, && 1 \leqslant i \leqslant R \\ &= |\alpha S^{i,1} + \beta S^{i,2}|^2, && R < i \leqslant R + S \\ &= |\beta T^i|^2 && R + S < i \leqslant K \equiv R + S + T. \end{aligned} \tag{14.12}$$

These probabilities sum to unity as required.

From this we find the which-path parameter Φ_{W2} for $W2$ to be the sum of the probabilities over regions R and T, that is,

$$\Phi_{W2} = |\alpha|^2 \sum_{i=1}^{R} |R^i|^2 + |\beta|^2 \sum_{i=R+S+1}^{K} |T^i|^2. \tag{14.13}$$

This reduces to unity when $S = 0$ as expected and to zero when both R and T are zero.

14.5 The Delayed Choice Interferometer

A delayed choice experiment that confirms the predictions of QM was done by Jacques et al. with a Mach–Zehnder interferometer (Jacques et al., 2007), the relevant stage diagram being the same as Figure 13.1. In this experiment, referred to here as JACQUES, the final beam-splitter B^2 could be removed while the light was on its way from the first beam-splitter B^1.

Although this experiment is of type WHEELER-1, it is also equivalent to the above WHEELER-2 scenario with $K = 2$, because a Mach–Zehnder experiment represented by stage diagram Figure 13.1 is equivalent to a double-slit experiment with just two detectors in the final stage-detecting screen. In JACQUES, the configuration with the second beam splitter B^2 removed corresponds to taking $R = T = 1$, $S = 0$ in WHEELER-2, while that with B^2 in operation corresponds to $R = T = 0$, $S = 2$.

14.6 The Double-Slit Quantum Eraser

The above discussed experiments do not involved photon spin specifically. The experiment we discuss next requires a careful analysis of spin.

Prior to the delayed-choice quantum eraser experiment of Jacques et al. (Jacques et al., 2007), the double-slit quantum eraser experiment of Walborn et al. (Walborn et al., 2002) had demonstrated the empirical validity of the stage concept in quantum mechanics. The Walborn et al. experiment consists of three subexperiments, referred to here as WALBORN-1, WALBORN-2, and WALBORN-3.

WALBORN-1: No Which-Way Information

The first subexperiment, WALBORN-1, is shown in Figure 14.3. Source S prepares a spinless photon pair 1_0, which is then split, with photon 2_1 passing onto a double-slit $(2_2,\ 3_2)$, and then onto a screen with final stage Σ_3 detectors $2_3, 3_4, \ldots, K$. Meanwhile, 1_1 is passed onto a detector 1_3. Coincidence measurements are taken involving screen impacts and 1_3 detection, with no polarization involved.

The initial total state $|\Psi_0)$ is given by $|\Psi_0) \equiv |s_0\rangle \otimes \widehat{\mathbb{A}}_0^1 \mathbf{0}_0$.

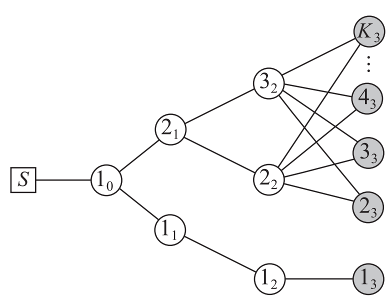

Figure 14.3. WALBORN-1: zero which-path information.

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Stage $\boldsymbol{\Sigma_0}$ to Stage $\boldsymbol{\Sigma_1}$

Evolution from $\Sigma_0 \to \Sigma_1$ is given by

$$U_{1,0}\left\{|s_0\rangle \otimes \widehat{\mathbb{A}}_0^1\mathbf{0}_0\right\} = |s_1\rangle \otimes \widehat{\mathbb{A}}_1^1\widehat{\mathbb{A}}_1^2\mathbf{0}_1. \tag{14.14}$$

Stage $\boldsymbol{\Sigma_1}$ to Stage $\boldsymbol{\Sigma_2}$

Evolution from $\Sigma_1 \to \Sigma_2$ is given by

$$U_{2,1}\left\{|s_1\rangle \otimes \widehat{\mathbb{A}}_1^1\widehat{\mathbb{A}}_1^2\mathbf{0}_1\right\} = |s_2\rangle \otimes \left\{\alpha\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2 + \beta\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\right\}\mathbf{0}_2, \tag{14.15}$$

where $|\alpha|^2 + |\beta|^2 = 1$.

Stage $\boldsymbol{\Sigma_2}$ to Stage $\boldsymbol{\Sigma_3}$

The final stage transition $\Sigma_2 \to \Sigma_3$ is given by

$$\begin{aligned} U_{3,2}\left\{|s_2\rangle \otimes \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2\mathbf{0}_2\right\} &= |s_3\rangle \otimes \sum_{i=2}^{K} C^i\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^i\mathbf{0}_3, \\ U_{3,2}\left\{|s_2\rangle \otimes \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\mathbf{0}_2\right\} &= |s_3\rangle \otimes \sum_{i=2}^{K} D^i\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^i\mathbf{0}_3, \end{aligned} \tag{14.16}$$

where the screen consists of $K-1$ detectors and the $\{C^i\}, \{D^i\}$ coefficients satisfy the usual semi-unitarity conditions.

The coincidence rates $\Pr(\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$ are given by

$$\Pr(\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0) = |\alpha C^i + \beta D^i|^2, \quad i = 2, 3, \ldots, K, \tag{14.17}$$

and these are the same as the single site detection rates $\Pr(\widehat{\mathbb{A}}_3^i\mathbf{0}_3|\Psi_0)$. These results demonstrate double-slit interference because the detection of the 1_3 signal provides no which-way information, so that this version of the experiment is equivalent to a standard double-slit experiment.

WALBORN-2: Creation of Which-Path Information

WALBORN-1 is now reconsidered with some modifications, shown schematically in Figure 14.4. Walborn et al. placed two quarter-wavelength polarizers P^2 and P^3 in front of 2_3 and 3_3 as shown (Walborn et al., 2002). Each polarizer alters the beam it acts on in a way that distinguishes it from the other beam. The consequence is that each signal observed on the detecting screen contains information about the path taken. Hence no interference should be observed on the screen.

To understand the action of the P^2 and P^3 modules, we need to consider three sets of orthonormalized photon polarization bases, and the fact that we have a two-spin photonic internal space.

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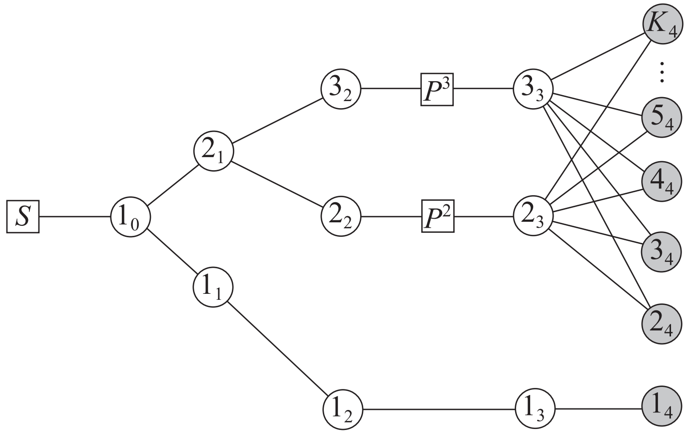

Figure 14.4. WALBORN-2: creation of which-path information.

Spin Bases

The first orthonormal photon spin basis $B^{HV} \equiv \{|H\rangle, |V\rangle\}$ consists of a horizontal polarization state $|H\rangle$ and a vertical polarization state $|V\rangle$; the second photon spin basis $B^{LR} \equiv \{|R\rangle, |L\rangle\}$ consists of a right circularly polarized state $|R\rangle$ and a left circularly polarized state $|L\rangle$; and the third photon spin basis is $B^{WP} \equiv \{|P\rangle, |N\rangle\}$, a conceptual "which-path" basis defined by Walborn et al. (Walborn et al., 2002). For clarity, our $|P\rangle$ and $|N\rangle$ states are the same as $|+\rangle$ and $|-\rangle$ states, respectively, used in Walborn et al. (2002).

These bases are related as follows:

$$\begin{aligned} |H\rangle &= \frac{1}{\sqrt{2}}\{|P\rangle + |N\rangle\}, \qquad |R\rangle = \frac{1-i}{2}\{|P\rangle + i|N\rangle\}, \\ |V\rangle &= \frac{1}{\sqrt{2}}\{|P\rangle - |N\rangle\}, \qquad |L\rangle = \frac{1-i}{2}\{i|P\rangle + |N\rangle\}. \end{aligned} \tag{14.18}$$

Photonic Space Basis

The photonic internal spin space involves symmetrized spin states of two photons denoted p and s by Walborn et al., where the p beam is passed into module X while the s beam is passed onto the double slit. Therefore, we have to consider tensor products of the form $|H^p\rangle|H^s\rangle$, $|H^p\rangle|V^s\rangle$, and so on, where we drop the usual tensor product symbol $\otimes$. For clarity, we keep a tensor product symbol between "inner" (SUO) states and "outer" (detector) labstates. In our CA program MAIN, we use the following four states, $\{s[i,n] : i = 1,2,3,4\}$, to serve as an orthonormal basis for the internal photonic degrees of freedom, defined at stage Σ_n by

$$\begin{aligned} |H^p_n\rangle|H^s_n\rangle &\to s[1,n], \quad |H^p_n\rangle|V^s_n\rangle \to s[2,n], \\ |V^p_n\rangle|H^s_n\rangle &\to s[3,n], \quad |V^p_n\rangle|V^s_n\rangle \to s[4,n]. \end{aligned} \tag{14.19}$$

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Double-Slit Polarizers

The two modules P^2 and P^3 have the following active[4] actions on their respective input beams:

$$\text{beam } 2_2 \begin{cases} P^2|H_2^s\rangle \underset{P^2}{\rightarrow} \quad |L_3^s\rangle = \frac{1}{\sqrt{2}}\{|H_3^s\rangle + i|V_3^s\rangle\}, \\ P^2|V_2^s\rangle \underset{P^2}{\rightarrow} \; i|R_3^s\rangle = \frac{1}{\sqrt{2}}\{i|H_3^s\rangle + |V_3^s\rangle\}, \end{cases} \tag{14.20}$$

$$\text{beam } 3_2 \begin{cases} P^3|H_2^s\rangle \underset{P^3}{\rightarrow} \quad |R_3^s\rangle = \frac{1}{\sqrt{2}}\{|H_3^s\rangle - i|V_3^s\rangle\}, \\ P^3|V_2^s\rangle \underset{P^3}{\rightarrow} -i|L_3^s\rangle = \frac{1}{\sqrt{2}}\{-i|H_3^s\rangle + |V_3^s\rangle\}. \end{cases} \tag{14.21}$$

The stage dynamics is as follows.

Stage Σ_0 to Stage Σ_1

$$U_{1,0}|\Psi_0) = \frac{1}{\sqrt{2}}\left\{\underbrace{|H_1^p\rangle|V_1^s\rangle}_{s[2,1]} + \underbrace{|V_1^p\rangle|H_1^s\rangle}_{s[3,1]}\right\} \otimes \underbrace{\widehat{\mathbb{A}}_1^1\widehat{\mathbb{A}}_1^2\mathbf{0}_1}_{a[2^0+2^1,1]\equiv a[3,1]}. \tag{14.22}$$

We show in (14.22) how the various terms are transcribed into the notation used in program MAIN.

Stage Σ_1 to Stage Σ_2

$$U_{2,1}\left\{|H_1^p\rangle|V_1^s\rangle \otimes \widehat{\mathbb{A}}_1^1\widehat{\mathbb{A}}_1^2\mathbf{0}_1\right\} = \frac{1}{\sqrt{2}}|H_2^p\rangle|V_2^s\rangle \otimes \left\{\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2\mathbf{0}_2 + \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\mathbf{0}_2\right\},$$

$$U_{2,1}\left\{|V_1^p\rangle|H_1^s\rangle \otimes \widehat{\mathbb{A}}_1^1\widehat{\mathbb{A}}_1^2\mathbf{0}_1\right\} = \frac{1}{\sqrt{2}}|V_2^p\rangle|H_2^s\rangle \otimes \left\{\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2\mathbf{0}_2 + \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\mathbf{0}_2\right\}. \tag{14.23}$$

Stage Σ_2 to Stage Σ_3

Here the modules P^2 and P^3 take effect:

$$U_{3,2}\left\{|H_2^p\rangle|V_2^s\rangle \otimes \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2\mathbf{0}_2\right\} = \frac{1}{\sqrt{2}}\{i|H_3^p\rangle|H_3^s\rangle + |H_3^p\rangle|V_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^2\mathbf{0}_3,$$

$$U_{3,2}\left\{|V_2^p\rangle|H_2^s\rangle \otimes \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2\mathbf{0}_2\right\} = \frac{1}{\sqrt{2}}\{|V_3^p\rangle|H_3^s\rangle + i|V_3^p\rangle \otimes |V_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^2\mathbf{0}_3,$$

$$U_{3,2}\left\{|H_2^p\rangle|V_2^s\rangle \otimes \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\mathbf{0}_2\right\} = \frac{1}{\sqrt{2}}\{-i|H_3^p\rangle|H_3^s\rangle + |H_3^p\rangle \otimes |V_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^3\mathbf{0}_3,$$

$$U_{3,2}\left\{|V_2^p\rangle|H_2^s\rangle \otimes \widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\mathbf{0}_2\right\} = \frac{1}{\sqrt{2}}\{|V_3^p\rangle|H_3^s\rangle - i|V_3^p\rangle \otimes |V_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^3\mathbf{0}_3. \tag{14.24}$$

The point here, as stressed by Walborn et al., is that all four photon polarization states on the right-hand side of (14.24) are mutually orthogonal. Therefore,

[4] Here "*active*" means that the changes are physically observable.

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no interference is to be expected in any subsequent pattern of signals. To confirm that QDN gives such a conclusion, we need to evolve the total state to the final stage.

Stage Σ_3 to Stage Σ_4

No polarizations are affected on passage through the double slit, so we have

$$U_{4,3}\left\{|s_3^i\rangle \otimes \widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^2\mathbf{0}_3\right\} = \sum_{j=2}^{K} C^j|s_4^i\rangle \otimes \widehat{\mathbb{A}}_4^1\widehat{\mathbb{A}}_4^j\mathbf{0}_4,$$

$$U_{4,3}\left\{|s_3^i\rangle \otimes \widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^3\mathbf{0}_3\right\} = \sum_{j=2}^{K} D^j|s_4^i\rangle \otimes \widehat{\mathbb{A}}_4^1\widehat{\mathbb{A}}_4^j\mathbf{0}_4, \quad (14.25)$$

for $i = 1, 2, 3, 4$. Here the $\{C^j\}$, $\{D^j\}$ coefficients satisfy the usual semi-unitarity relations.

The above information was encoded into program MAIN, with the following results for the nonzero maximal question answers:

$$\Pr\left(\widehat{\mathbb{A}}_4^1\widehat{\mathbb{A}}_4^j\mathbf{0}_4|\Psi_0\right) = \frac{1}{2}|C^j|^2 + \frac{1}{2}|D^j|^2, \qquad j = 2, 3, \ldots, K. \quad (14.26)$$

This confirms that QDN reproduces the empirical results of Walborn et al. (Walborn et al., 2002). Essentially, placing P^2 and P^3 in front of their respective slits destroys the lack of which-way information observed in the unpolarized double-slit experiment WALBORN-1 discussed above.

MAIN also confirms that $\Pr\left(\widehat{\mathbb{A}}_4^j\mathbf{0}_4|\Psi_0\right) = \Pr\left(\widehat{\mathbb{A}}_4^1\widehat{\mathbb{A}}_4^j\mathbf{0}_4|\Psi_0\right)$, $j = 2, 3, \ldots, K$, that is, that the single signal pattern on the screen shows no interference.

WALBORN-3: Erasure of Which-Path Information

The variants WALBORN-1 and WALBORN-2 confirm standard QM expectations: interference occurs in WALBORN-1 because no which-path information is available, while the modules P^2 and P^3 in WALBORN-2 provide such information by labeling the two beams passing through the double slit, and so there is no interference. The essence of WALBORN-3 is that the module labeled X in Figure 14.5 counteracts the effects of P^2 and P^3 so that interference in correlations is now observed. What is incomprehensible from a classical perspective is that the action of X is nonlocal relative to P^2 and P^3. This is the point made by Wheeler: P^1 and P^2 could be operating on one side of the Universe and X on the other, but the interference destroyed in WALBORN-2 is restored by the action of module X.

The QDN analysis of the total state evolution for WALBORN-3 is identical to that for WALBORN-2 up to stage Σ_2. However, at stage Σ_3, the most suitable internal polarization basis to use is B^{WP} rather than B^{HV}, because X essentially filters out the two elements $|P\rangle$ and $|N\rangle$ of that basis.

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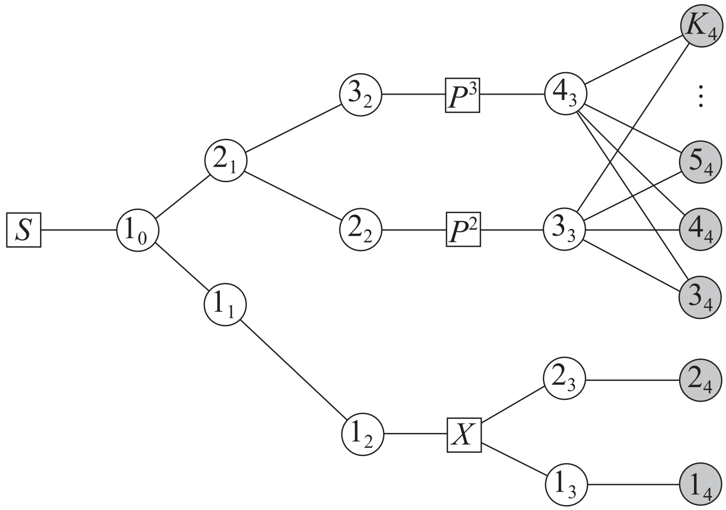

Figure 14.5. WALBORN-3: erasure of which-path information.

The combined actions of P^2, P^3, and X are now given by

$$\text{beam } 2_2 \begin{cases} P^2|H_2^s\rangle \underset{P^2}{\rightarrow} \quad |L_3^s\rangle = \frac{(1-i)}{2}\{i|P_3^s\rangle + |N_3^s\rangle\}, \\ P^2|V_2^s\rangle \underset{P^2}{\rightarrow} \; i|R_3^s\rangle = \frac{(i+1)}{2}\{|P_3^s\rangle + i|N_3^s\rangle\}, \end{cases} \tag{14.27}$$

$$\text{beam } 3_2 \begin{cases} P^3|H_2^s\rangle \underset{P^3}{\rightarrow} \quad |R_3^s\rangle = \quad \frac{(1-i)}{2}\{|P_3^s\rangle + i|N_3^s\rangle\}, \\ P^3|V_2^s\rangle \underset{P^3}{\rightarrow} -i|L_3^s\rangle = -\frac{(i+1)}{2}\{i|P_3^s\rangle + |N_3^s\rangle\}. \end{cases} \tag{14.28}$$

Module X has the following effects on the beam from 1_2:

$$\text{beam } 1_2 \begin{cases} X|H_2^p\rangle \underset{X}{\rightarrow} \quad \frac{1}{\sqrt{2}}\{|P_3^p\rangle + |N_3^p\rangle\} \\ X|V_2^p\rangle \underset{X}{\rightarrow} \quad \frac{1}{\sqrt{2}}\{|P_3^p\rangle - |N_3^p\rangle\}. \end{cases} \tag{14.29}$$

Although this looks like a passive basis change, module X has the property of splitting the beam into two separately observable components and this is critical to the experiment. Module X is really an active transformation, with stage Σ_3 output beams denoted 1_3 and 2_3.

Stage Σ_2 to Stage Σ_3

Here the pattern of information flow is intricate, requiring great care in the programming. The difficulty of correctly encoding what the experimentalists had done turned out to generate the most significant test of the QDN formalism: a single sign error would easily invalidate the whole calculation. This underlines what we said at the end of Chapter 12, that there is an obvious need for a more sophisticated process of transcribing stage diagrams into CA code. We find the following transition rules:

$$U_{3,2}\left\{|H_2^p\rangle|V_2^s\rangle\otimes\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2\mathbf{0}_2\right\}=\tfrac{(1+i)}{2\sqrt{2}}\left[\begin{array}{c}\{|P_3^p\rangle|P_3^s\rangle+i|P_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^3\mathbf{0}_3+\\ \{|N_3^p\rangle|P_3^p\rangle+i|N_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^2\widehat{\mathbb{A}}_3^3\mathbf{0}_3\end{array}\right],\tag{14.30}$$

$$U_{3,2}\left\{|H_2^p\rangle|V_2^s\rangle\otimes\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\mathbf{0}_2\right\}=-\tfrac{(1+i)}{2\sqrt{2}}\left[\begin{array}{c}\{i|P_3^p\rangle|P_3^s\rangle+|P_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^4\mathbf{0}_3+\\ \{i|N_3^p\rangle|P_3^p\rangle+|N_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^2\widehat{\mathbb{A}}_3^4\mathbf{0}_3\end{array}\right],\tag{14.31}$$

$$U_{3,2}\left\{|V_2^p\rangle|H_2^s\rangle\otimes\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^2\mathbf{0}_2\right\}=\tfrac{(1-i)}{2\sqrt{2}}\left[\begin{array}{c}\{i|P_3^p\rangle|P_3^s\rangle+|P_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^3\mathbf{0}_3-\\ \{i|N_3^p\rangle|P_3^p\rangle+|N_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^2\widehat{\mathbb{A}}_3^3\mathbf{0}_3\end{array}\right],\tag{14.32}$$

$$U_{3,2}\left\{|V_2^p\rangle|H_2^s\rangle\otimes\widehat{\mathbb{A}}_2^1\widehat{\mathbb{A}}_2^3\mathbf{0}_2\right\}=\tfrac{(1-i)}{2\sqrt{2}}\left[\begin{array}{c}\{|P_3^p\rangle|P_3^s\rangle+i|P_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^1\widehat{\mathbb{A}}_3^4\mathbf{0}_3-\\ \{|N_3^p\rangle|P_3^p\rangle+i|N_3^p\rangle|N_3^s\rangle\}\,\widehat{\mathbb{A}}_3^2\widehat{\mathbb{A}}_3^4\mathbf{0}_3\end{array}\right].\tag{14.33}$$

Stage Σ_3 to Stage Σ_4

The evolution is given by (14.25), taking into account that detector 2_4 is not associated now with the double-slit detecting screen but is part of the X module output channel detectors.

The above information when fed into program MAIN gives the results:

$$\begin{aligned}\Pr\left(\widehat{\mathbb{A}}_4^1\widehat{\mathbb{A}}_4^j\mathbf{0}_4\right)&=\frac{1}{4}|C^j|^2+\frac{1}{4}|D^j|^2+\frac{i}{4}\left(C^jD^{j*}-C^{j*}D^j\right),\\ \Pr\left(\widehat{\mathbb{A}}_4^2\widehat{\mathbb{A}}_4^j\mathbf{0}_4\right)&=\frac{1}{4}|C^j|^2+\frac{1}{4}|D^j|^2-\frac{i}{4}\left(C^jD^{j*}-C^{j*}D^j\right),\qquad j=3,4,\ldots,K,\end{aligned}\tag{14.34}$$

demonstrating the interference observed by Walborn et al. Moreover, the two alternative output channels, 1_4 and 2_4, show the out-of-phase interference referred to by Walborn et al. as *fringe* and *antifringe*, respectively.

QDN does indeed simulate the empirical results of Walborn et al. The following comments are relevant.

The No-Signaling Theorem Is Vindicated

The action of module X is entirely local, being applied only to beam 1_2. The question is, could anything done by X be observed at the possibly remote screen consisting of detectors $3_4, 5_4, \ldots, K_4$ alone? The answer is emphatically *no*. Specifically, we find the single detector outcome probabilities to be given as

$$\begin{aligned}\Pr\left(\widehat{\mathbb{A}}_4^j\mathbf{0}_4\right)&\equiv\Pr\left(\widehat{\mathbb{A}}_4^1\widehat{\mathbb{A}}_4^j\mathbf{0}_4\right)+\Pr\left(\widehat{\mathbb{A}}_4^2\widehat{\mathbb{A}}_4^j\mathbf{0}_4\right)\\ &=\frac{1}{4}|C^j|^2+\frac{1}{4}|D^j|^2,\ j=3,4,\ldots,K,\end{aligned}\tag{14.35}$$

that is, showing no interference terms.

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The *Stage* Concept Is a Valid Model of Empirical Time
Most significantly, Walborn et al. repeated their experiment with the screen and p photon detection order reversed with significant time differences and found no change in the results (Walborn et al., 2002). This is strong evidence for the validity of the stages concept in such quantum process.

14.7 Concluding Remarks

Our analysis supports the notion that QM never actually needs to involve any acausality in order to account for empirical data. We should be worried if it did, for then our entire view of what probability and information represent would need drastic revision.

Detailed QDN analysis reveals the basic fact that interference phenomena involve a lack of information about quantum *states*, and not specifically about particles per se. Conceptual problems arise when our classical conditioning is relied on too much. We would like to believe in photons as particles and we would like to believe that time runs continuously. Both concepts have their uses, but QM seems to require a generalization of both. In the case of the former, experiments tell us that we have to deal with interference of amplitudes, not particles. In the case of the latter, we cannot expect quantum processes to evolve strictly according to an integrable timetable, such as coordinate time, or even the physical time in a laboratory. What is important is whether or not quantum information has been extracted. If it has been placed "on hold," as can be seen in our analysis of the delayed-choice eraser and the double-slit eraser, then it can remain in a stage that could in principle persist until the end of the Universe.