

# Toward improving the accuracy of Cepheid distances through parallax of pulsation

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**Abstract.** Improvement of the calibration of the Cepheid period–luminosity relation (Leavitt’s Law) is one of the main challenges to improve the accuracy of the Hubble constant,  $H_0$ . Many parallax-of-pulsation methods are promising but have not yet delivered sufficiently accurate distances: observational biases, such as the projection factor, still dominate. We propose a global parallax-of-pulsation method, combining all observables (photometry, spectroscopy, and interferometry), to (i) reduce statistical errors, (ii) use the redundancy among observables to validate our approach, and (iii) achieve 2% accuracy for individual Cepheid distances.

**Keywords.** Cepheids, stars: distances, stars: fundamental parameters, stars: individual ( $\ell$  Car)

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## 1. Introduction

Before Cepheids were definitely known to be pulsating stars, there were two theories to explain their brightnesses and spectral Doppler variations: pulsation and binarity. Lindemann (1918) noted that a way to test the pulsation theory was to verify the relation between the apparent brightness variation of the pulsator and its angular-diameter variations, computed from the integrated pulsation velocity. This is arguably the first time this remarkable relation was published, even before Baade (1926) and Wesselink (1946) did so.

The ‘parallax of pulsation,’ as this relation is now known, became a very successful way of determining distances to Cepheids, and hence calibrate Leavitt’s law, i.e., the period–luminosity relation. The most popular version, based on infrared surface brightness measurements (Fouqué & Gieren 1997), uses a combination of visible and infrared photometric magnitudes as a proxy for brightness and surface brightness. More recently, the use of optical interferometry has offered direct measurements of the angular-diameter variations (Lane *et al.* 2000) with potentially high statistical precision of distance determinations (Mérand *et al.* 2005).

However, empirical implementations of the parallax-of-pulsation method are limited in their accuracy by different biases, including:

- knowledge of the so-called ‘projection factor,’  $p$  (see, for example, Burki *et al.* 1982), which allows one to derive the true pulsation velocity from the radial velocity, derived from spectroscopy. This  $p$  factor depends not only on the line-formation process in the pulsating photosphere, but also on the actual method used to derive the radial velocity (for a demonstration of this effect, see Gray & Stevenson 2007);

- the two-filter Baade–Wesselink implementation contains an intrinsic degeneracy between distance and reddening;
- the possible presence of circumstellar envelopes (Kervella *et al.*, this volume) biases the infrared photometric and interferometric measurements.

## 2. Integrated Parallax of Pulsation

For the reasons outlined in the introduction, we think that it is imperative to move toward a more complete implementation of the parallax-of-pulsation distance determination, one that will use all observables at the same time: multiband photometry, high-resolution spectra, and interferometric angular diameters. Not only will it improve the precision of the derived distances (through the use of more data), but the combination of redundant data allows simultaneous derivation of ancillary parameters. We call this implementation the ‘integrated parallax of pulsation’ (IPOP).

For example, using more than two photometric bands, as well as spectra, offers a way to derive the reddening, since spectra contain information about the effective temperature and the magnitudes are determined from both the angular-diameter and temperature variations. To develop such a mechanism, the IPOP implementation must contain some physical modeling of the spectra and spectral-energy distributions (SEDs). For this reason, we use an implementation based on a grid of photospheric models (PHOENIX).

The method aims at obtaining a global fit to the observational data and relies on the following parameters:

- $P$ ,  $MJD_0$ : the ephemeris of the pulsation, from which the pulsation phase is computed;
- $v_{\text{puls}}(\phi)$ : phase-dependent pulsation velocity. This can be parametrized using Fourier series or period splines;
- $T_{\text{eff}}(\phi)$ : phase-dependent effective temperature. This can also be parametrized using Fourier series or period splines;
- $d$ : the distance to the system;
- $R_{\text{avg}}$ : average diameter of the star;
- $p$  factor: the projection factor to compute radial velocities from pulsation velocities;
- $E(B - V)$ :  $(B - V)$  color excess, used to parametrize the reddening.

From these parameters, it is possible to reconstruct any observables (radial velocities, spectra, magnitudes, colors, or interferometric angular diameters) using the grid of photospheric models. Our approach is to attempt a global (and simultaneous) fit to all available data using a complete set of parameters.

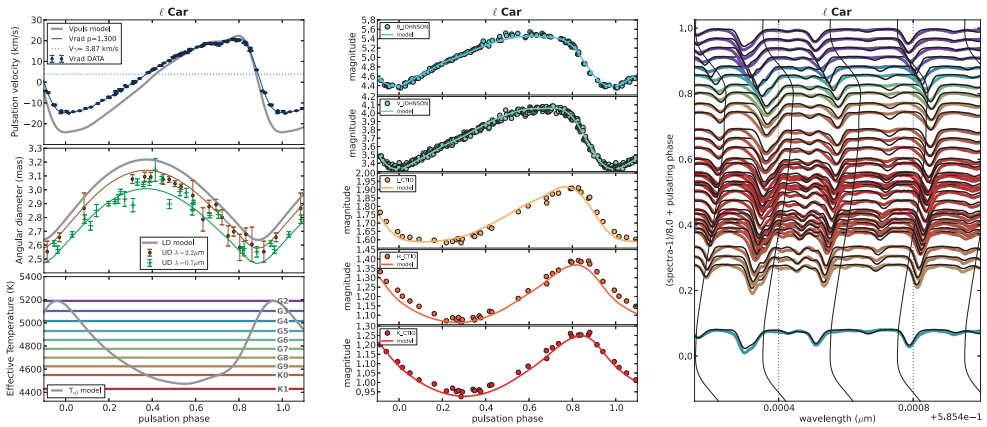
## 3. Implementation details

The photospheric models provide, for a given model, the intensity distribution as a function of wavelength and impact parameter,  $\mu$  ( $\mu = 0$  for the stellar limb,  $\mu = 1$  for the projected center). We currently use hydrostatic models and assume that pulsating Cepheids have a similar SED. This is, we think, a reasonable assumption, since Cepheid surface brightnesses are similar to those of non-pulsating stars (Kervella *et al.* 2004). Synthetic magnitudes are computed for the corresponding photometric system using transmission and zero-point descriptions from the ‘Filter Profile Service’ of the Spanish Virtual Observatory.† Synthetic spectra are produced by integrating the models  $I(\mu)$  after proper Doppler shifting ( $\mu v_{\text{puls}}$ ). The grid of photospheric models is also used to

† <http://svo.cab.inta-csic.es/theory/fps3/>

compute center-to-limb darkening corrections for interferometric uniform-disk diameter measurements, in the corresponding passband.

This kind of approach is commonly used to model eclipsing binaries, and we have already used the same approach to model such systems and validate the accuracy of our basic interpolation functions (Mérand *et al.* 2011). The minimization algorithm used is based on the Levenberg–Marquard approach to minimize a  $\chi^2$  computed for all data. As suggested at the conference in a comment following the presentation, the final optimized values and corresponding uncertainties can be derived using statistical resampling such as bootstrapping.



**Figure 1.** Implementation of IPOP for the Cepheid  $\ell$  Car. (left) From top to bottom: radial-velocity data (points), model (thin line), and pulsation velocity (thick line); angular-diameter variation model (thick line) and uniform-disk measurements in two different bands; effective-temperature model (thick line). (middle) Photometric data and best-fitting model for the  $B$ ,  $V$ ,  $J$ ,  $H$ , and  $K$  bands. The model combines the angular-diameter variations and effective-temperature models. (right) Small segment of the high-resolution spectra, vertically offset by their respective pulsation phases. The thin black lines are the models.

### 4. Preliminary Results

We have implemented a version of the IPOP and applied it to one star,  $\ell$  Car, which has a handful of different measurements available in the literature, including radial velocities, broad-band photometry, and an interferometric angular diameter. As can be seen in Fig. 1, the agreement between the model and the data set for  $\ell$  Car is satisfactory. We used data from Taylor *et al.* (1997) and Bersier (2002) for the radial velocities, Laney & Stobbe (1992) for infrared photometry, Berdnikov & Turner (2001) for visible photometry, Kervella *et al.* (2004) for infrared angular diameters, and Davis *et al.* (2009) visible angular diameters.

The distance estimate is on a par with literature values as well, which is to be expected since we used the same data sets. Our value is biased by our adopted value of the projection factor. The interesting result to consider is the uncertainty we obtain for the distance estimate. For example, for  $\ell$  Car (see Fig. 1), the uncertainty is 1.9% as derived from the Levenberg–Marquard algorithm, and approximately 3% as derived using a statistical resampling approach (bootstrapping).

## 5. Perspectives

Our preliminary results are encouraging: the good agreement between our physical modeling and the data yields a good precision for the estimated parameters (including the distance). This good agreement also instills confidence in our approach. We aim at applying this method to a large number of Cepheids with available observational data. At the same time, we would like to implement refinements, including high-resolution spectral synthesis (to avoid using the  $p$  factor) and subtle effects such as those caused by an infrared excess and rotation.

## References

- Baade, W. 1926, *Astron. Nachr.*, 228, 359  
Berdnikov, L. N. & Turner, D. G. 2001, *ApJS*, 137, 209  
Bersier, D. 2002, *ApJS*, 140, 465  
Burki, G., Mayor, M., & Benz, W. 1982, *A&A*, 109, 258  
Davis, J., Jacob, A. P., Robertson, J. G., *et al.* 2009, *MNRAS*, 394, 1620  
Fouqué, P. & Gieren, W. P. 1997, *A&A*, 320, 799  
Gray, D. F. & Stevenson, K. B. 2007, *PASP*, 119, 398  
Lane, B. F., Kuchner, M. J., Boden, A. F., *et al.* 2000, *Nature*, 407, 485  
Laney, C. D. & Stoble, R. S. 1992, *A&AS*, 93, 93  
Lindemann, F. A. 1918, *MNRAS*, 78, 639  
Kervella, P., Nardetto, N., Bersier, D., *et al.* 2004, *A&A*, 416, 941  
Kervella, P., Bersier, D., Mourard, D., *et al.* 2004, *A&A*, 428, 587  
Mérand, A., Kervella, P., Coudé du Foresto, V., *et al.* 2005, *A&A*, 438, L9  
Mérand, A., Kervella, P., Pribulla, T., *et al.* 2011, *A&A*, 532, A50  
Taylor, M. M., Booth, A. J., Booth, A. J., & Cottrell, P. L. 1997, *MNRAS*, 292, 662  
Wesselink, A. J. 1946, *Bull. Astron. Inst. Neth.*, 10, 88