

Surface magnetic fields across the HR Diagram

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Abstract. The past 20 years have seen remarkable advances in spectropolarimetric instrumentation that have allowed us, for the first time, to identify some magnetic stars in most major stages of stellar evolution. We are beginning to see the broad outline of how such fields change during stellar evolution, to confront theoretical hypotheses and models of magnetic field structure and evolution with detailed data, and to understand more of the ways in which the presence of a field in turn affects stellar structure and evolution.

Keywords. Magnetic fields, polarization, instrumentation: polarimeters, stars: atmospheres, stars: magnetic fields

1. Introduction

The objective of this meeting is to promote cross-fertilisation of ideas, methods and personal contacts between astronomers and astrophysicists with a common interest in polarisation and polarimetry of astrophysical objects. Participants include specialists from solar physics and many branches of stellar physics, as well as regional astronomers benefitting from the proximity of the meeting to learn about an often unfamiliar field.

Measurements of solar and stellar polarisation provide two major kinds of information. Linear polarimetry offers tools for probing circumstellar material such as disks and winds, and other kinds of departure from simple azimuthal symmetry around the line of sight. Circular polarimetry makes possible the study of magnetic fields in stars and other astrophysical environments. Both of these applications of polarimetry are discussed in detail in the various papers from the meeting. The task of this review is to provide a basic road map of stellar magnetism to help people from other fields of specialisation to follow the more detailed papers later in the volume.

2. Magnetic field measurements

How are measurements of polarisation connected with the detection and study of stellar magnetic fields? The basic connection goes back to the physics of the Zeeman effect. If we look at an isolated spectral line in a laboratory, it is found experimentally that applying a sufficiently strong magnetic field to the emitting gas causes the spectral line to split into multiple components. The atomic physics of this Zeeman splitting phenomenon is well understood. It arises because most atoms have a magnetic moment and thus have energy that depends on orientation in a magnetic field. Thus in a transition that releases a photon, each of the single upper and lower energy levels involved in the transition split into several closely spaced energy levels corresponding to different orientations of the ion in the magnetic field. Because of this splitting, a transition between a pair of energy

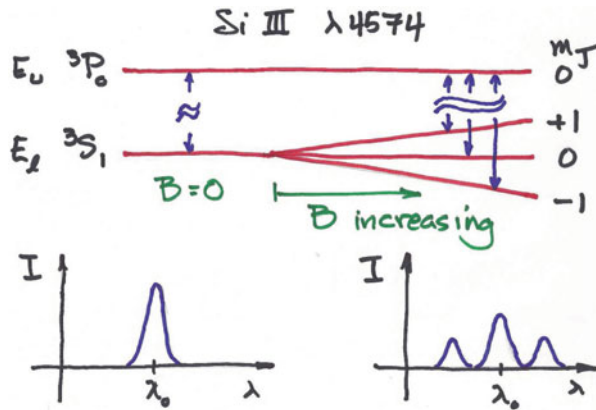


Figure 1. The Zeeman effect for the Si III line at 4574 Å. In the presence of a magnetic field, the lower of the two levels of this transition splits into three levels corresponding to the three possible orientations of the $J = 1$ state, while the upper $J = 0$ state is not split. This results in the single line splitting into three components, as sketched below.

levels becomes, in a magnetic field, a set of transitions between neighbouring levels and the single spectral line breaks into a group of spectral lines.

Because the energy added to each level by magnetic orientation increases linearly with magnetic field strength, the splitting of a single spectral line produces components whose separation is proportional to the magnetic field strength. The magnitude of the splitting is given by

$$\Delta\lambda(\text{\AA}) \sim 5 \cdot 10^{-13} B(\text{G})\lambda^2(\text{\AA}) \sim 0.013 \text{\AA}/\text{kG} \quad (2.1)$$

where $\Delta\lambda$ is the typical separation of one component from its undisplaced wavelength λ in a field of B Gauss. It is clear that a field of the order of 1 kG (10^{-1} Tesla), which is a field strength easily produced in a laboratory by an electromagnet, and is a typical field strength found in a star, produces only very small splitting of spectral lines in the visible spectrum.

The basic physics of spectral line splitting in the particular example of the Si III line at 4574 Å is illustrated in Figure 1. An example of actual stellar magnetic Zeeman splitting of spectral lines in the magnetic Ap (“peculiar A”) star HD 215441, which has an extremely large global field of about 34 kG, is shown in the upper part of Figure 2, where we see the Zeeman splitting of the absorption lines.

It is also found from laboratory experiments that if the magnetic field is orientated along the axis of the emission, the displaced components generally become *circularly polarised*, with one handedness dominating in components shortward of the unperturbed line wavelength and the opposite handedness dominating in the longward components. This means that if one measures the wavelength of the line in a circularly polarised spectrum, the mean line wavelength as measured in one sense of circular polarisation is different from the position measured in the other circular polarisation. These displaced components are called “sigma components”.

When we observe starlight through a pair of oppositely-handed circularly polarised filters, we often present the results in the form of a sum spectrum (the normal absorption-line or intensity spectrum in the upper part of Figure 2, labelled I) and a difference spectrum (the lower spectrum in the figure, labelled V/I , where V and I are components of the Stokes vector). In the presence of a strong line-of-sight field, when each of the sigma component is present mainly in only one of the two circularly polarised spectra,

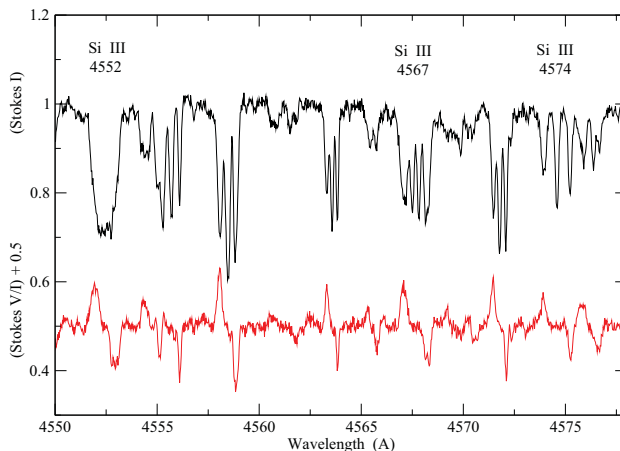


Figure 2. The Zeeman effect in the magnetic Ap star HD 215441, with a field of about 34 kG. All the spectral lines in this window are split by the Zeeman effect into multiple components, and in the circular polarisation difference spectrum V/I below, the net circular polarisation of the various Zeeman components produces upward or downward spikes.

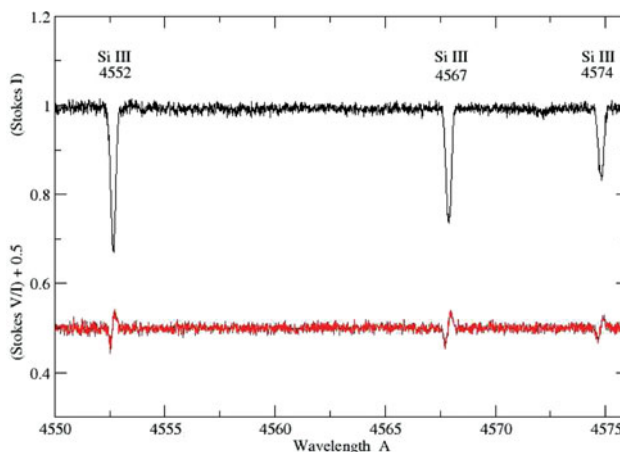


Figure 3. The intensity (Stokes I) and circular polarisation (Stokes V/I) spectra of the magnetic B star HD 96446. The field of about 5 kG is not strong enough to visibly split the absorption lines, which are slightly broadened by a few km s^{-1} of rotational velocity and pulsation, but the slightly different mean wavelengths of the lines as seen in right and left circular polarisation lead to clearly visible spikes in the V/I difference spectrum.

the result is a difference spectrum with downward spikes on one side of the line centres and upward spikes on the other.

Even when the field is so weak that its effect on the intensity spectrum is undetectable because the Zeeman splitting is small compared to other broadening mechanisms (for example thermal, instrumental, or rotational broadening) the fact that the mean wavelengths of the line in the two senses of circular polarisation are slightly different means that a detectable signal may still be present in the difference spectrum, as shown in Figure 3, where the absorption lines show no trace of Zeeman splitting but the V/I circular polarisation difference spectrum below clearly reveals the magnetic field.

It is found that observing the spectrum of a star in the two senses of circular polarisation and forming the V/I difference spectrum is in almost all cases the most sensitive

method of detecting a weak magnetic field on the surface of the star. This measurement can be made to very high precision because it is essentially equivalent to a differential radial velocity measurement with the two radial velocity spectra obtained simultaneously through almost identical optical paths. In practice we can measure polarisation differences in individual lines of the order of 10^{-3} or 0.1% of the continuum intensity, and if we make use of the obvious similarity of the circular polarisation signals of different lines to average the signal, it is possible to detect Zeeman polarisation of less than 0.001% of the continuum, revealing magnetic fields as small as a few G, only slightly larger than the surface field of the Earth.

3. High-precision magnetic field measurements

High precision searches for fields have been practical for many years for the brightest stars (e.g. Landstreet 1982, who reached an uncertainty of ± 7 G on Procyon). Recent advances in instrumentation have made it possible to obtain even higher precision on far fainter stars. This has mainly been achieved by installing polarisation analysers on extremely efficient cross-dispersed echelle spectrographs, so that the polarisation in individual lines could be measured with much higher signal than in the past, and the signal could be averaged over many, sometimes many thousands, of lines. The result is that fields as low as 1 G can be routinely detected in the most favourable stars (e.g. Aurière *et al.* 2009), and for the first time we can really survey much of the HR Diagram for magnetism.

Some of the instruments at the forefront of magnetic field searches and measurements are ESPaDOnS at the Canada-France-Hawaii Telescope, HARPSpol at ESO-La Silla, and the lower dispersion instrument FORS at ESO-Paranal.

The best field measurement uncertainty or standard error (usually written σ_B) that one can obtain depends strongly on the nature of the star observed. The brighter the star, of course, the more precisely the polarisation signal can be measured. Furthermore, as in radial velocity measurements, the amplitude of the signature to be detected in the difference spectrum is much larger for a star with extremely sharp lines (such as most red giants have) than in a star with broad lines (most A and B main sequence stars, for example). Finally, since averaging over many lines can reveal a very small signal, in stars with many lines (most cool main sequence and giant stars) much smaller fields can be detected than in stars with only a few usefully deep lines (such as rapidly rotating B stars). An example of how averaging allows us to detect a weak field in a star is shown in Figure 4, in which the almost undetectable circular polarisation signal in individual lines of the stellar spectrum emerges very clearly in the averaged spectral line and circular polarisation.

4. Magnetic fields over the HR Diagram

The improvements in instruments achieved in recent years have led to a number of large surveys, and to detection of magnetic fields in at least a few stars in almost every major stage of stellar evolution. Fields have been found to be generally present in low-mass pre-main sequence T Tau stars, and are detected in a few more massive pre-main sequence Herbig AeBe stars. Fields are found in most rapidly rotating low-mass main sequence stars, but above about $2M_{\odot}$, only about 10% of stars have fields, and the magnetic stars are usually slowly, not rapidly, rotating. Among evolved stars, very weak fields (typically a few G or less) are found in some red giants and AGB stars. Fields are also present

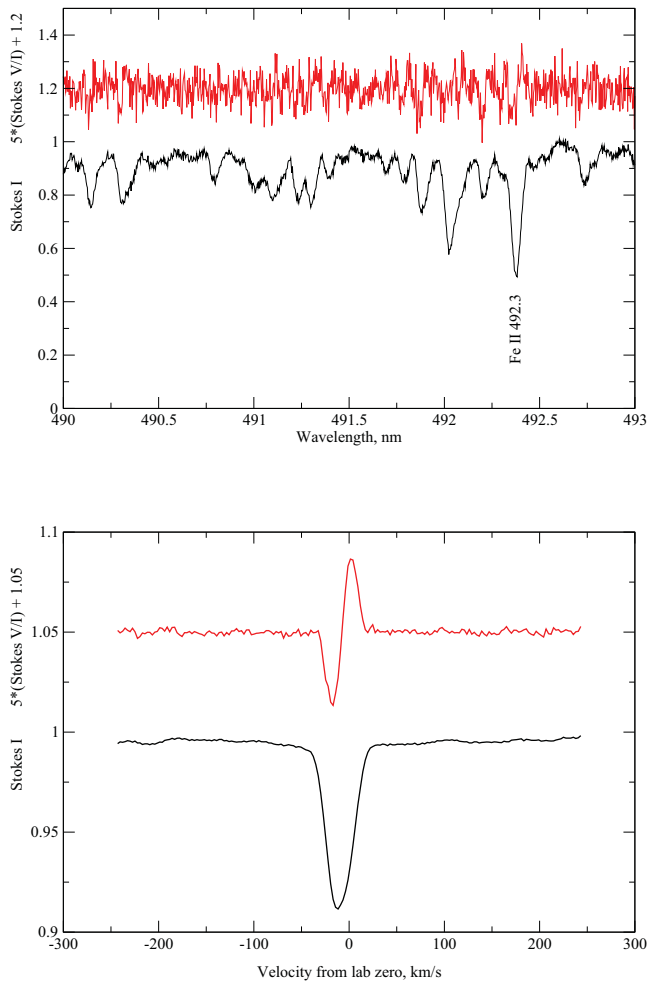


Figure 4. The upper figure shows a portion of the absorption spectrum of the magnetic B star HD 317857, together with the polarisation spectrum (above). Almost no trace of a magnetic signature is visible in the polarisation spectrum. However, when many lines are averaged together, as shown in the lower figure, the presence of a magnetic signature is clear, and the -920 G field is measurable with an uncertainty of less than 30 G.

at the end of evolution; about 10% of white dwarfs have large fields (typically a MG or more), and most neutron stars seem to form with fields of 10^8 to 10^{15} G.

When we look closely at the magnetic fields that are found across the HR Diagram, we identify two very different kinds of fields.

Dynamo or solar-type fields The fields of stars with atmospheric effective temperatures below about 6–7000 K generally appear to have dynamo fields. Such fields are reminiscent of that of the Sun. Dynamo fields have complex topology, often with rather small-scale structure. The structure typically changes on a short time-scale, often weeks or months. Such fields are often accompanied by solar-like activity – emission in the core of the Ca II K line, X-rays, flares, spots (like sunspots). The strength of the field, and the activity, are clearly related to the rotation period of the star, and are generally more intense in rapidly rotating stars than in slow rotators, and more intense in dwarfs than in

giants. We believe that such fields, like the solar field, are the product of dynamo action inside stars, driven by a combination of convection and (differential) rotation.

Fossil fields Most stars with effective temperatures above about 7000 K do not show any sign of magnetic fields. However, a few percent of such stars do have fields. These fields are very different in nature from dynamo fields. They are usually rather simple in global structure, and are mostly roughly dipolar. Such fields do not vary significantly over tens of years except for the changes in what is observed as the star rotates. The observed field strengths vary over about two orders of magnitude or more; in main sequence stars the field strength at the stellar surface can be between roughly 300 and 3×10^4 G; in white dwarfs the range is between perhaps 10^4 up to nearly 10^9 G. The field strength has no relation to the rotation period of the star, and in fact fields are found in some stars with extremely long rotation periods, up to many decades. We believe that such fields are fossils, i.e. they are not currently being generated by any active dynamo. Instead, they are the remnants of fields that were produced earlier in the star's history which have simply persisted (while also changing slowly with time) because of the very large electrical conductivity and the enormous size of stars.

Many aspects of these issues are reviewed by Donati & Landstreet (2009).

4.1. Pre-main sequence stars

Both low mass and intermediate mass stars pass through the T Tau phase during star formation, in which they have low effective temperature and deep convection in their outer layers, and are observed to be surrounded by a disk of material which is accreting onto the star, or the remnants of such material. These conditions produce strong magnetic fields, of a kG or so, in stars that rotate with periods of the order of a day. These are typical dynamo fields (e.g. Donati *et al.* 2011, Johns-Krull *et al.* 2011, and Hussain *et al.* 2012), and they strongly affect how material is accreted onto the star.

Most T Tau stars directly become low-mass main sequence stars, but some are massive enough ($M > 1.7M_{\odot}$ or so) that on the main sequence their effective temperatures are above the dynamo range. Before they become main sequence stars, such relatively massive stars briefly become Herbig Ae or Be stars, when they look similar to main sequence stars except for a circumstellar accretion disk. During the Herbig phase, we observe that most stars do not show evidence of magnetic fields, but a few percent of such stars appear to have rather typical fossil fields of a few hundred G or a kG. The transition between dynamo and fossil field must occur in at most a few million years because of the shortness of the star formation era for such stars (e.g. Catala *et al.* 2007, Wade *et al.* 2007, and Alecian *et al.* 2013).

4.2. Main sequence stars

As mentioned above, low-mass main sequence stars have dynamo fields whose strength is closely related to the rotation rate. These fields range from about 3 kG for rapid rotators down to the global field of the Sun, which is only about 1 G on average (although it reaches kG strength in sunspots). The variation of global field strength, the structure of the field, the degree of differential rotation, and the relationship of the field to stellar activity such as X-ray emission, and their relationships to fundamental stellar parameters such as mass, effective temperature, rotation rate, and age are all subjects of very active current study (see particularly Figure 3 of Donati & Landstreet 2009 for a clear summary of the situation as it was a few years ago; see also Morin *et al.* 2010).

Amongst main sequence stars with $M > 1.7M_{\odot}$ (intermediate and massive stars), we find fossil fields. As such fields have been recognised in sharp-line chemically peculiar A stars for almost 70 yr (Babcock 1947, 1958), much is already known about magnetism in

these stars. Fields range from roughly 300 to 3×10^4 G. They occur in roughly 10% of stars more massive than about $2.3M_{\odot}$ but become rapidly rarer with decreasing mass below this limit, and vanish at about $1.7M_{\odot}$. Fields are almost always found in stars that rotate much less rapidly than the average for main sequence A or B stars; rotation periods are typically a few days, but some magnetic A stars have rotation periods of many decades. Probably this is due to such stars shedding angular momentum during star formation by magnetic coupling (see Preston 1971, Landstreet 1992, Mathys 2012). It is now known that fields of such stars decline steadily through the main sequence stage, by a factor of several from their initial (ZAMS) values (Landstreet *et al.* 2008). Between masses of about $1.7M_{\odot}$ and about $8M_{\odot}$, main sequence stars with strong magnetic fields are generally “chemically peculiar” (Ap or Bp stars), usually with (often striking) underabundances (relative to solar abundances) of He, C and O, and overabundances of heavier atoms, especially such iron peak elements as Cr and Ti, and rare earths such as Eu, Nd, and Pr. At the high end of this range, some of the magnetic stars show striking overabundances even of He. It has recently been shown that the abundance anomalies tend to decrease with stellar age during the main sequence (Bailey *et al.* 2014).

Fossil fields are now being found in increasing numbers in more massive stars. These fields are often found in stars which do not appear chemically unusual, but which may be somewhat slower rotators than normal (Henrichs *et al.* 2000, Donati *et al.* 2002, Wade *et al.* 2012). The fields seem to be present in roughly the same fraction of massive stars as among the intermediate mass stars. In some of these massive magnetic stars, mass loss fills a trapped magnetosphere around the star, which can lead to a variety of phenomena such as periodically variable H α emission and “eclipses” in the light curves (Landstreet & Borra 1978, Townsend *et al.* 2005, and Petit *et al.* 2013).

4.3. Red giants and AGB stars

Very weak magnetic fields of a few G or less have now been reliably detected in a number of red giants, most of which up to now have masses of about $2M_{\odot}$ or more. It appears that fields are usually directly detected in stars that show other symptoms of “activity” such as strong X-ray emission, rapid rotation, flaring, etc. These fields are thought to be dynamo-type fields (Konstantinov-Antova 2013). Magnetic fields of higher field strength are found in a small number of quite slowly rotating red giants; these fields are thought to be the remnants of fields from earlier evolution in stars that have evolved from the magnetic Ap or Bp state on the main sequence (Aurière *et al.* 2012).

Extremely weak fields of the order of 1 G have been found in a number of massive red supergiants (Grunhut *et al.* 2010).

It should be noted that the fields of all these evolved stars are probably rather complex, and the average longitudinal field strength measured is almost certainly much smaller than the actual field strength locally on the stellar surface, because much of the signal from various points on the surface is cancelled by similar signal of opposite sign on other parts of the star.

4.4. Collapsed stars: white dwarfs and neutron stars

Magnetic fields were first detected in white dwarfs not through the usual Zeeman effect, but by broadband continuum circular (and then linear) polarisation by Kemp *et al.* (1970). The first few magnetic white dwarfs were found in this way. However, magnetic fields can also be detected in these stars via the usual Zeeman effect, both by circular polarisation detected (for example) in Balmer line wings, and by visible magnetic splitting of line cores. Zeeman splitting can range from barely enough to be visible in the line core of H α to such severe separation of magnetic sublevels that spectral lines are split into

almost unrecognisable patterns. Fields have now been identified in many hundreds of white dwarfs through Zeeman splitting detected in the course of the Sloan Digital Sky Survey (Külebi *et al.* 2009), and so a recent list of magnetic white dwarfs by Kawka *et al.* (2007) is now already quite out of date.

The fields found are now understood to range from around 10^4 up to nearly 10^9 G. Fields appear to be of the fossil type, and measurements either remain quite repeatable or change periodically with the rotation period of the underlying star. As in upper main sequence stars, the fields appear to be geometrically fairly simple, and have roughly dipolar structure (see for example Landstreet 1992, Schmidt 2001 or Landstreet *et al.* 2012).

Finally, of course, most or all neutron stars are formed as pulsars with magnetic fields in the range of 10^9 up to 10^{15} G.

5. Global evolution of magnetic fields

We now have observational evidence that (some) magnetic fields occur in most major stages of stellar evolution. The main stages for which evidence is still missing (or results so far have not revealed any fields) are those that are rapid rotators or fairly faint, such as the horizontal branch, hot subdwarfs, and nuclei of planetary nebulae.

In low mass stars, which during their evolution on the main sequence and the giant/AGB stages generally have low effective temperature (except for the horizontal branch phase, and early stages of the collapse to a white dwarf state), it appears that currently operating dynamo magnetic fields can occur through most of the evolution until the final collapse to the white dwarf state. It is not clear if the magnetic fields found in about 1/10th of white dwarfs descend in part from magnetic low mass stars, for example from those with the largest fields during the giant phases.

The evolution of magnetism in more massive stars is very interesting. As a massive star collapses from its natal gas cloud, it is believed that it passes through a period during the accretion stage when the outer layers (in fact, most of the star) are deeply convective. This is like the T Tau stage in lower mass stars, in which magnetic fields are almost always found, and it is thought that such more massive stars probably have dynamo magnetic fields during this phase. (In fact, fields have been observed in T Tau stars massive enough that they will become main sequence stars of intermediate mass, possibly with fossil fields; see Johns-Krull 2007 and Hussain 2012 for examples.)

After this T Tau-like stage, an intermediate mass star becomes a Herbig Ae or Be stars, of which only a small fraction have so far been found to have fields. In this stage, it appears that the field structure, if a field is present, is already the simpler geometry of a fossil field. This remains the situation on the main sequence, and we suspect (but do not know) that magnetic Herbig Ae and Be stars become magnetic Ap and Bp stars. When a magnetic intermediate mass main sequence star expands to become a red giant, it appears to retain some memory of the field it had on the main sequence. It is not known how long this remnant of main sequence magnetism persists, but probably eventually the field reverts to a normal dynamo, which may continue into the AGB stage. Finally, an intermediate mass star usually becomes a white dwarf, which (as on the main sequence) is only found to be detectably magnetic in about one star out of ten.

For massive stars the evolution overall is similar to that for intermediate mass stars, except that the usual outcome of fuel exhaustion is a supernova explosion followed by collapse of the stellar core to a (magnetised) neutron star.

This evolution is *very far from being understood*.

However, there is a growing body of theoretical work aiming to understand the internal structure and evolution of stellar magnetic fields, and the relationship of internal to surface magnetism.

In cool stars, the observed fields may mostly be rather directly produced by the action of a current dynamo whose action is determined by the depth of the outer convection zone and the distribution of angular momentum throughout the star. However, we see in the strong-field red giants that some memory of an earlier field seems to be preserved, so initial conditions may play a significant role, at least for a while.

In more massive stars, the phases with effective temperatures above about 7000 K lack the deep convection needed for outer envelope dynamos to operate. It seems very probable that the field we observe at present in such a star is due to active field generation earlier in the life of the star, modified by Ohmic decay, relaxation of the kinks and bends in the initial field structure, instabilities, and internal shearing flow due to the distribution and redistribution of internal angular momentum. This evolution is currently being studied with a combination of analytical and numerical simulations, for example by Braithwaite & Spruit (2004), Mestel & Moss (2010), and Duez & Mathis (2010).

There is evidence that some of the observed fossil-type fields may have been produced by binary mergers, particularly in white dwarfs (Tout *et al.* 2008).

A recent survey of theoretical issues has been presented by Braithwaite (2014).

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