Introduction to the Analysis

Designing computer support for the learning of mathematics is a major educational challenge today. Networked computers provide an attractive opportunity to explore collaborative-learning approaches to math education. The recent availability of dynamic-geometry software provides further prospects for innovation. This book reports on an extensive research effort involving teaching math teachers and their students in an online collaboration environment. Specifically, it documents the cognitive development of a particular team of three students learning about dynamic geometry in that virtual social setting. An extended case study shows how the team enacts software tools and adopts group practices within the educational research project, which was designed to extend and support their ability to collaborate, to engage in mathematical discourse, and to explore or construct dynamic-geometric figures. The book provides detailed empirical support, within a math-education context, for the theory and practice of group cognition.

Research Context

This volume builds on earlier publications about the Virtual Math Teams (VMT) Project, putting their arguments into practice, documenting their claims, fleshing out their theory, and fulfilling their promises. It culminates a cycle of books reporting on the project:

- *Group Cognition* (Stahl, 2006, MIT Press) introduced the VMT Project as a response to practical and foundational issues in computer-supported collaborative learning (CSCL) and computer-supported cooperative work (CSCW). It recommended adapting methods of interaction analysis to online text chat. It proposed that

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the investigation of small-group processes and practices could provide insight into online collaborative learning. It outlined a preliminary theory of group cognition as a framework appropriate for computer-mediated interaction.

- *Studying Virtual Math Teams* (Stahl, 2009, Springer) described the technology approach and affordances of the VMT software environment. The edited volume provided illustrative analyses of brief excerpts of student interaction in VMT by a number of international researchers. It suggested technology design features and methodological considerations. It expanded the philosophic and scientific bases of group-cognition theory.
- *Translating Euclid* (Stahl, 2013, Morgan & Claypool) reviewed the VMT Project: its multiuser technology, collaborative pedagogy, dynamic-geometry curriculum, design-based research approach, and educational goals. The multifaceted research project was situated within its historical, mathematical, and educational contexts. This recent project review discussed the integration of collaborative dynamic geometry into the VMT environment. It further elaborated the theory of group cognition as a basis for educational innovation.

The present book documents the findings of the VMT Project as a paradigmatic example of computer-supported collaborative learning (CSCL) exploration, incorporating a unique model of human-computer interaction (HCI) analysis. Directed by the author for the past twelve years, the VMT Project pioneered a method of analyzing interaction data, adapting ethnomethodologically inspired interaction analysis to the special conditions of computer-mediated collaboration and to the needs of design-based research in mathematics education. This fine-grained report on data from the VMT Project applies its methods longitudinally to the full eight hours of one student group's interaction. In this analysis, it details the team's cognitive development. It ties the development of their group cognition to the technological mediation, which takes place at multiple levels of the project:

- The students interact exclusively through the VMT online collaboration environment using text chat.
- The student team explores dynamic geometry in a computer simulation.
- The domain of dynamic geometry is defined by its software implementation.

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- The VMT curriculum is technologically scripted for use with minimal teacher intervention.
- All the data is collected electronically through comprehensive instrumentation of the collaboration environment.

The centrality of computer support to the project makes this book relevant to (1) CSCL, (2) HCI, (3) mathematics instruction, and (4) educational technology:

1. From a CSCL perspective, this book is paradigmatic in offering a detailed example of research based on the theory of group cognition. The cognitive development of the observed team of students is conceived as computer-supported collaborative learning, in which learning is primarily viewed at the small-group unit of analysis of collaboration and all the communication takes place through computer-mediated interaction. It provides a rich picture of learning on many levels, not just measuring a single learning outcome. It not only documents *that* learning took place by the student team but also details *how* the learning happened by observing the enactment of numerous group practices. It provides an examination of small-group cognitive development in terms of the adoption of group practices, including the enactment of tools. This approach is framed in the philosophy of group cognition, which has emerged from the VMT Project and is grounded in its findings. A rich picture of a prototypical instance of computer-supported collaborative learning emerges from this research.
2. From an HCI standpoint, the book's analysis is distinctive in that it documents an investigation in which computer-mediated interaction analysis played a central role in the design-based research process, providing feedback to the project at multiple points: advice to teachers between sessions, revisions for the next cycle, and formative evaluation of the overall project, including elaboration of the theoretical framework.
3. From a mathematics instruction view, the book offers several proposals. In terms of curriculum design, the set of topics illustrates a focus on a central theoretical concept of the domain: dependency relationships in dynamic geometry. The online presentation of the topics to small groups of students illustrates a form of guidance toward mathematical understandings through computer scripting or scaffolding, with minimal direct teacher intervention. The sequential accumulation of group practices provides a conceptualization

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of increasing mathematical understanding. Finally, the collaborative approach to work on challenging problems reveals the mutual contributions from student zones of proximal development, which are negotiated and adopted by the group as cognitive practices.

4. From an educational-technology approach, the book is unique in offering a longitudinal case study, which details cognitive development starting when the students first encounter online collaborative dynamic geometry. It identifies dozens of group practices by which the team of students learns to collaborate, to enact software tools, to understand geometric figures, and to discuss mathematical invariants and their dependencies. It thereby shows how an online collaboration environment can facilitate learning – specifically the critical development of geometric reasoning – by providing a supportive space for the emergence and adoption of group practices.

As the concluding volume reporting on the VMT Project, this book illustrates a successful implementation of group-cognition research and analysis. Since it was proposed in the 2006 volume, the theory of group cognition has been increasingly accepted within the research community as an alternative to the traditional educational-psychology approach to instructional technology, focused on measurable learning outcomes of individual minds. As a presentation of CSCL methodology, the book provides an alternative or complement to statistical coding approaches. Within HCI, it shows that an ethnomethodologically informed approach can generate implications for design systematically within a practical design-based research process. Within the mathematics-instruction literature, it offers several proposals concerning curriculum focus on underlying relationships, guidance toward mathematical principles, operationalizing deep understanding in terms of practices, and appreciating mechanisms of collaborative learning of mathematics. As an educational-technology intervention, it demonstrates the potential and details the challenges of using collaborative dynamic-geometry software to facilitate the development of mathematical cognition.

Presentation Structure

Constructing Dynamic Triangles Together: The Development of Mathematical Group Cognition rounds out the story of the VMT Project. It centers on an extended case study: the detailed longitudinal analysis of eight hours of interaction by a virtual math team of three middle-school girls working on an introductory sequence of dynamic-geometry challenges. It fulfills the promises and claims of previous publications on VMT by demonstrating the success of the methods they proposed and carrying out systematic analysis of one team's entire online collaborative-learning experience. Along the way, it provides lessons for online curricular design, for CSCL technology, and for HCI analysis. It also fills in the theory of group cognition with concrete results based on detailed data showing how collaborative learning of mathematics takes place through the enactment of specific group practices for collaboration, math discourse, and software tool usage.

Attempts to study collaborative learning are often confounded by ambiguity about what the learners already know. Even more generally, evidence of various factors effecting the learning is missing from the available data. For instance, there may be social influences or power relationships that are not captured in the data, or there may have been interactions, gestures, and speech that were off-camera or unintelligible. Even worse, self-reports and introspection about learning take place long after foundational instances of learning have been processed, transformed, and internalized. The learning analyzed in this book, in contrast, involves the students' initial encounters with a subject that is new for them: geometry, especially dynamic-geometry construction. Furthermore, their interactions about these encounters are captured live in full logs and replayer files, which reproduce the interactions just as they appeared to the students. We assume that the students had previous familiarity with the visual appearances of conventional basic shapes of everyday geometry, but we are interested in how the team develops beyond this knowledge. There are certainly other influences on the individual mental activity of the students, based on their past and on events not captured in the VMT system, but we are focused on the team's development at the group unit of analysis; everything that took place between the students and was shared by the team passed through the VMT technology and was logged. So the data analyzed here is about as complete as one could hope for and as required by our methodology. To the extent practical, the VMT data documents the beginnings of mathematical cognition in the domain of introductory dynamic geometry for the team.

The team's developmental trajectory during their VMT experience is guided by a carefully designed sequence of curricular units: the topics that the students worked on in their eight sessions. The following analysis considers the team's work on each topic in order. The topics are planned to introduce the students methodically to the fundamentals of dynamic geometry. In particular, the goal is to have the team develop an understanding of dependency relationships that establish invariances, such as the maintained equality of side lengths of an equilateral triangle. The curriculum builds systematically. It starts by letting the students play with the most basic steps of construction, while guiding the team to work collaboratively. It introduces the building of an equilateral triangle as a prototypical construction and then extends it for the construction of perpendicular bisectors and right triangles.

Because an understanding of problems and solutions in dynamic geometry is mediated by one's mastery of the software tools for manipulating and constructing dynamic-geometry objects, the most important tools are introduced before the topics that require them. As the team explores the use of the tools and engages in problem solving in response to the curricular topics, the team starts to adopt group practices. The analysis of the team interaction focuses on how the team enacts the tools, and it identifies various kinds of practices that the team adopts.

The adopted group practices are taken to be important constituents of the team's group cognition. The team learns by successively embracing specific practices. For instance, in its early sessions, the students learn to work together effectively by incorporating group collaboration practices. These practices are in part suggested by the curriculum. The team negotiates them and then begins to follow them. Similarly, they gradually integrate group mathematical practices – often involving using the software tools to drag and construct dynamic-geometry figures – into their joint work. These practices establish necessary foundations for computer-supported collaborative learning in this domain of mathematics.

By identifying the team's adoption of group practices, the analysis in this book provides a paradigmatic example of CSCL. The case study analyzes the computer technology as enacted by the team. It shows the mediation of the team's interaction by the integrated online pedagogy and domain-centered curriculum. It focuses on interaction at the group unit of analysis and illustrates the methodological approach of the theory of group cognition. Its longitudinal approach provides a rich example of how collaborative learning can take place, while suggesting design lessons for improving the next iteration of software, pedagogy, curriculum, analysis, and theory.

Overall, the detailed and extended longitudinal case study provides a rare view into how students learn in small groups. The many individual actions described are united into a narrative about the development of mathematical group cognition, framed in a theoretical and methodological perspective and leading to pedagogical and curricular lessons.

The presentation is divided into a number of chapters. The bulk of the volume conducts a fine-grained analysis of the student interaction and identifies the team's adoption of group practices into their interaction. These analysis chapters illustrate many aspects of sequential-interaction analysis, show how the students enact the use of the available technology, examine the student interpretation of curricular artifacts, and display the student engagement in specific group practices. Each analysis chapter concludes with an assessment of the team's cognitive development and a set of implications for redesign of project details. This core of the book is preceded by methodological considerations and followed by theoretical reflections.

The chapters are:

- *Researching Mathematical Cognition*. The initial methodological chapter emphasizes the importance of mathematical cognition in the modern world and the difficulty it presents for many students. It briefly considers issues of schooling and theories concerning the development of mathematical understanding. It then argues for a case study approach, incorporating sequential-interaction analysis. Building on Vygotsky's ideas, it suggests focusing on developmental processes at the group unit of analysis.
- *Analyzing Development of Group Cognition*. The VMT Project is described in the following chapter as design-based research, which incorporates cycles of refining technology, curriculum, and theory through iterative trials with classroom teachers and students. The goals of the project – providing the focus of analysis in this book – include development of collaboration skills, mathematical discourse, and usage of software tools. Dynamic geometry is briefly described, with its characteristics of dragging, constructing, and defining mathematical dependencies. The analytic methodology is then presented as sequential-interaction analysis, with a special emphasis on extended sequences of interaction involved in geometric problem solving. Such analysis can highlight the display by students of their collaborative mathematical development as they chat, manipulate graphical objects, explore problems, construct geometric figures, and articulate solutions. In this way, analysis not only indicates that certain learning transpired but also shows how it took place: through the adoption of group practices.
- *Session 1: The Team Develops Collaboration Practices*. This first analysis chapter shows how the three students developed into a collaborative

team, largely during their initial hour together online. At first, the students had no idea what to do in the VMT environment. However, they successively responded to suggestions within the environment – textual instructions, software displays, results of explorations. The chapter enumerates many specific group collaboration practices that they adopted in their first session, which served them well for the remainder of their work together.

- *Session 2: The Team Develops Dragging Practices*. Dragging points of geometric figures and observing the consequent changes is a central activity of dynamic geometry. Dragging can be used for a variety of purposes, such as aligning parts of a geometric figure, exploring a construction, or testing if dependencies hold during dragging. In their second session, the team developed a number of group practices related to dynamic-geometry dragging.
- *Session 3: The Team Develops Construction Practices*. Construction is a conventional focus in learning Euclidean geometry. In this session, the team engages in several traditional construction tasks. In the process, they adopt a series of group construction practices that are specific to dynamic geometry. The chapter also investigates difficulties the team had in constructing figures, how they overcame some of their problems, and how they missed opportunities that had been designed into the tasks. During this session, the team displayed significant progress in moving from a visual to a more formal mathematical approach to construction.
- *Session 4: The Team Develops Tool Usage Practices*. In its fourth session, the team honed its skills using the dynamic-geometry tools, including the procedure to define new custom tools. The team adopted additional group practices for using the tools.
- *Session 5: The Team Identifies Dependencies*. This chapter explores in even greater detail a particularly exciting developmental breakthrough by the team. Viewed superficially, the team seems to be floundering with a challenging problem involving inscribed triangles. They seem to have digressed even in their collaboration practices. However, at the end of the session, it appears that the student who often seems to be the weakest in mathematical understanding solves the problem. The particular geometry task is one that has been used often in the VMT Project and is rarely solved within an hour, even by mathematically experienced adults. A close analysis in this chapter shows how the team actively explored the problem and potential solution techniques through extensive investigation of dragging and

construction approaches. The eventual solution actually involved contributions from all three team members and displayed a clear understanding of the solution logic.

- *Session 6: The Team Constructs Dependencies*. The team was given another hour-long session to tackle a related dynamic-geometry problem. This time, the triangles were replaced by inscribed squares. The team had not worked with constructing squares before, but eventually arrived at an elegant solution for doing that. Once they constructed the outside square, the whole team immediately expressed knowledge of how to construct an inscribed square in it. This displayed their firm understanding of what they had accomplished in the previous session with the triangles. Their success also confirmed their impressive development of collaboration, dragging, construction, tool usage, and dependency practices.
- *Session 7: The Team Uses Transformation Tools*. For the students' next session, the teacher skipped ahead to an introduction to unrelated tools for rigid transformations (translation, reflection, rotation). Although the team had some success with this topic, they failed to gain much insight into the transformation paradigm of constructing dependencies. Here, analysis revealed the need for considerably more curricular scaffolding, especially supporting enactment of the new tools.
- *Session 8: The Team Develops Mathematical Discourse and Action Practices*. The team's final session involved the exploration of different quadrilaterals, to determine dependencies in their construction through dragging. The team investigated seven figures, with very different results. Some figures were too simple and others too difficult to understand through a few minutes of dragging. However, while working on the second quadrilateral, the team engaged in impressive dragging and in striking mathematical discourse about dependencies. This session displayed both the extent of the team's development along multiple dimensions and the fragility of this development. The analysis of the team's interaction suggests revisions to the curriculum for future research trials.
- *Contributions to a Theory of Mathematical Group Cognition*. In this theoretical chapter, the findings of the preceding analyses are reflected on as aspects of the theory of group cognition, specifically as applied to school mathematics. The sequences of group practices adopted by the team of students are conceptualized in the light of contemporary cognitive theory. For instance, the group

collaborative practices are seen as contributing to a sense of group agency, using insights from Latour and others. The mathematical discourse practices are contrasted to conclusions of Sfard. Group tool usage practices are considered in terms of Rabardel's concept of instrumental genesis. Dragging is related to embodied group cognition, construction to situated group cognition, and dependencies to designing.

- *Constructing Mathematical Group Cognition*. The concluding chapter has three parts. First, it considers the development of mathematical cognition as a dialectical process rather than a one-time acquisition. Then it recaps the book's implications for redesign of the VMT collaboration environment, especially the curriculum of dynamic-geometry tasks, focusing it even more tightly on dependencies. Finally, it reviews what has been learned from the VMT Project about the development of group practices and suggests prospects for future efforts continuing this research.