OPTIMAL INSURANCE COVERAGE UNDER BONUS-MALUS CONTRACTS

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ABSTRACT

The paper analyses the questions: *Should – or should not – an individual buy insurance? And if so, what insurance coverage should he or she prefer?* Unlike classical studies of optimal insurance coverage, this paper analyses these questions from a bonus-malus point of view, that is, for insurance contracts with individual bonus-malus (experience rating or no-claim) adjustments. The paper outlines a set of new statements for bonus-malus contracts and compares them with corresponding classical statements for standard insurance contracts. The theoretical framework is an expected utility model, and both optimal coverage for a fixed premium function and Pareto optimal coverage are analyzed. The paper is an extension of another paper by the author, see Holtan (2001), where the necessary insight to – and concepts of – bonus-malus contracts are outlined.

KEYWORDS

Insurance contracts, bonus-malus, optimal insurance coverage, deductibles, utility theory, Pareto optimality.

1. INTRODUCTION

Should – or should not – an individual buy insurance? And if so, what insurance coverage should he or she prefer? These fundamental questions are of main practical interest within the field of insurance purchasing, and have been extensively studied under varying conditions in insurance economics. Classical references are e.g. Mossin (1968), Arrow (1974) and Raviv (1979). A common factor of all these studies is their straightforward focus on insurance contracts *without* individual bonus-malus (experience rating or no-claim) adjustments. However, both from a customer's point of view and from a theoretical point of view, insurance contracts *with* bonus-malus adjustments, like e.g. motor insurance contracts, are usually much more complex to consider with regard to optimizing the insurance coverage. The increased complexity is caused by the bonus hunger mechanism of the customers; that is,

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the tendency for insurance customers to carry small losses themselves in order to avoid an increase of future premium costs. The aim of this paper is to take this bonus hunger mechanism into account within the framework of optimal insurance coverage under bonus-malus contracts. The paper is an extension of another paper by the author, Holtan (2001), where the necessary insight to – and concepts of – bonus-malus contracts are outlined.

The paper is organized as follows: Sections 2 and 3 describe the general insurance contract and an expected utility approach to the problem. Sections 4-6 outline some new propositions of the field of optimal insurance coverage particularly for bonus-malus contracts. These propositions are compared to their correspondingly classical propositions for standard insurance contracts. Section 5 treats optimal coverage for a fixed premium function, while section 6 treats pareto optimal coverage. Section 7 gives a summary of the conclusions of the paper.

2. THE GENERAL INSURANCE CONTRACT

We recapitulate briefly the main features of a general bonus-malus insurance contract as outlined in Holtan (2001). Consider an insurance buyer representing a risk of loss X, where X is a stochastic variable with probability density function $f(x)$ and $x \geq 0$. The damage side of the contract is characterized by a contractual compensation $c(x)$ and a true compensation $c^*(x)$ if loss $X = x$ occurs, where

$$c^*(x) = \begin{cases} c(x) - z & \text{if } c(x) > z \\ 0 & \text{if } c(x) \leq z. \end{cases} \tag{1}$$

The true compensation function $c^*(x)$ is the actual compensation function because of its bonus hunger component z, while the contractual compensation function $c(x)$ is no more than the loss amount minus the contractual deductible. The fixed amount z is the excess point of the optimal choice of self-financing generated by the customer's bonus hunger strategy after the loss occurrence, or in other words, the present value of the loss of bonus, and is defined as

$$z = \int_0^{\infty} e^{-\lambda t} (p_1(s+t) - p_0(s+t)) dt, \tag{2}$$

where λ is the non-stochastic market rate of interest of self-financing, $p_1(s+t)$ is the premium paid at time t after a loss occurrence at time s if the loss is reported to the insurer, and $p_0(s+t)$ is the correspondingly premium if the loss is not reported. The premium processes $p_0(s+t)$ and $p_1(s+t)$ are assumed to be continuous non-stochastic for all $t > 0$. See Holtan (2001) for a more detailed and complete description of bonus-malus effects on an insurance contract.

An important statement which partly follows from (1) and (2) is that *independent of the contractual compensation function, the true compensation function has always an individual deductible*; see proposition 2 in Holtan (2001). As we discuss later, this statement explains much of the optimal coverage characteristics of bonus-malus contracts outlined in this paper.

3. An expected utility approach

The existence of a true compensation function obviously influences the individual in his or her choice of insurance coverage within bonus-malus insurance contracts. Recall hereby our introductory questions in section 1: Should – or should not – an individual buy insurance? And if so, what insurance coverage should he or she prefer? Or more precisely: What is the optimal insurance coverage for the individual? As pointed out earlier these questions have traditionally been treated within the framework of insurance economics; in general see e.g. Borch (1990), chapter 2.1, 2.9, 6.3 and 6.4, or a more updated overview in Aase (1993), chapter 8. A brief summary of this classical treatment is as follows:

Consider the insurance customer and the insurance contract described in section 2. Assume w to be the certain initial wealth of the customer. Assume the risk taking preference of the customer to be represented by expected utility $Eu(\cdot)$, that is, facing an uncertain choice the customer is assumed to maximize his expected utility of wealth. Or more precisely: The customer will prefer an uncertain wealth W_1 to another uncertain wealth W_2 if $Eu(W_1) \geq Eu(W_2)$. The preference period of the customer is assumed to be one-period, which is the usual contractual period in non-life insurance. Note that even if the loss of bonus is accumulated over many years, the customers act on the present value of the loss of bonus, and hence the one-period preference period is a consistent assumption in this context.

Classical optimal condition: For the moment consider the classical point of view where the insurance contract *has no* bonus-malus adjustments. Thus the necessary condition for the customer to purchase a coverage $c(\cdot)$ for a premium p is:

$$Eu(w - X + c(X) - p) \geq Eu(w - X). \tag{3}$$

In other words; the customer prefers to buy an insurance coverage $c(\cdot)$ if the expected utility of the coverage is greater than or equivalent to the expected utility of not buying insurance at all. Note that within this framework the random variable X represents the total risk exposure of the customer, which not only includes the uncertain loss amount, but also the uncertain probability of loss occurrence. The probability distribution of X, $f(x)$, is hereby a mixed distribution, containing the probability that no accident occurs at the mass point $x = 0$ and, conditional on one or more accidents, a continuous loss size distribution for $x > 0$.

There may of course exist more than one coverage which satisfies (3). Hence the optimal choice of insurance coverage is the one which maximizes the left hand side of (3) with respect to the function $c(\cdot)$ and the function p, where p in this context obviously must depend on $c(\cdot)$.

Bonus-malus optimal condition: Let us now consider the situation where the insurance contract contains bonus-malus adjustments. Thus (3) is not longer a valid purchasing condition for the customer. The corrected optimal condition is rather influenced by the generalized true compensation function which was defined by (2). More precisely, the necessary condition for the customer to purchase a contractual coverage for a premium p is simply:

$$Eu(w - X + c^*(X) - p) \geq Eu(w - X). \tag{4}$$

In (4) p follows the rules of a general bonus-malus system and is also a function of $c(\cdot)$. If (4) holds for at least one contractual coverage $c(x)$, then the bonus-malus optimal choice of coverage is simply the one which maximizes the left hand side of (4).

Within the framework of bonus-malus insurance contracts condition (4) will obviously influence a wide specter of classical propositions and statements within the theory of optimal insurance coverage. In sections 4-6 some of these classical propositions are presented and thereafter corrected by the effect of the true compensation function within a bonus-malus framework.

4. The indifferent premium

Classical proposition I: Assume the classical framework of a standard insurance contract with no bonus-malus adjustments. The maximum premium the customer will pay for the insurance coverage is the premium $p = p_{max}$ which generates a "=" instead of a "≥" in (3). The premium p_{max} is hence the premium where the customer is indifferent between buying and not buying the insurance coverage, and is therefore also called the *indifferent premium.* The existence of such a premium is actually one of the axioms of the von Neumann-Morgenstern utility theory.

The utility function $u(\cdot)$ is usually assumed to be concave and monotonically increasing, i.e. $u'(\cdot) > 0$ and $u''(\cdot) < 0$, which means that the customer is assumed to be risk averse. Hence, by trivial use of Jensen's inequality, we may find that

$$p_{max} > Ec(X), \tag{5}$$

which is one of the key propositions in insurance economics. A practical interpretation of (5) is that a risk averse customer is willing to participate in an unfair game ($p_{max} = Ec(X)$ is a fair game).

Bonus-malus proposition I: Indifferent premium

Within the framework of a bonus-malus contract the indifferent premium satisfies

$$p_{max} > Ec^*(X), \tag{6}$$

where $c^*(\cdot)$ is defined by (1). □

Proof: From Jensen's inequality it follows that $u(w - EX) > Eu(w - X)$ since $u''(\cdot) < 0$. Hence the equality sign in (4) will hold for some $p_{max} > Ec^*(X)$. □

The practical interpretation of (6) is in fact the same as for (5), that is, a risk averse customer is willing to participate in an unfair game, but the unfair premium limit (the indifferent premium) is different between (5) and (6).

5. Optimal coverage for a fixed premium function

The two introductory questions in section 1 concern the problem of rational insurance purchasing for a fixed set of bonus-malus contracts offered by the insurer. In other words, the terms of the insurance contract are assumed to be exogenously specified and imposed on the insurance customer. This approach reflects a realistic purchasing situation in an insurance mass market, where the customers just within certain limits have possibilities to influence the terms of the insurance contract. The next proposition give attention to a classical statement and to a correspondingly bonus-malus statement within such an exogenous point of view. The contractual compensation assumes to be on excess of loss form, which is probably the most common contractual compensation form in the world wide insurance market.

Classical proposition II:

Assume the classical framework of a standard insurance contract with no bonus-malus adjustments. Assume the contractual compensation to be $c(X) = max[X - d, 0]$, where $d \geq 0$ is the contractual excess point, and the premium to be $p(d) = (1 + \gamma)Ec(X) + k$, where $\gamma \geq 0$ is a safety loading factor and $k \geq 0$ is a flat cost fee. If $\gamma = 0$ (and $w > p(d) + d$) and k is not too high, *it is always optimal to buy maximal contractual coverage*, that is, $d = 0$ is the optimal choice of insurance coverage. If k is too high, the only alternative is not to buy insurance at all. □

This classical statement is e.g. outlined in Borch (1990), pp. 33-34. As we will find, this statement of maximal coverage is also valid under bonus-malus contracts. The point is, however, that the specification of maximal coverage is different under bonus-malus contracts.

Bonus-malus proposition II:

Within the framework of a bonus-malus contract assume the contractual compensation to be $c(X) = max[X-d, 0]$, where $d \geq 0$ is the contractual excess point, and the premium to be $p(d) = (1+\gamma)Ec^*(X)+k$, where $\gamma \geq 0$ is a safety loading factor and $k \geq 0$ is a flat cost fee. If $\gamma = 0$ (and $w > p(d) + z(d) + d$) and k is not too high, *it is always optimal to buy maximal true coverage*, that is, a value of d which gives $z'(d) = -1$ is the optimal choice of insurance coverage. If k is too high, the only alternative is not to buy insurance at all. □

Remark: It is not obvious that insurance companies explicitly calculate $Ec^*(X)$ in the premium expression $p(d) = (1+\gamma)Ec^*(X)+k$. However, implicitly they do because they use the actual reportet claims – which are affected by the bonus hunger of the customers – as data input to the risk premium estimation.

Proof: From (1) the true compensation is $c^*(X) = max[X-d-z(d), 0]$, where the bonus hunger excess point $z(d)$ obviously is a function of d since $p(d)$ depends on d.

The optimal coverage maximizes the left hand side of (4). Hence we have:

$$U(d) = Eu[w-X+c^*(X)-p(d)]$$
$$= \int_0^{d+z(d)} u[w-x-p(d)]f(x)dx + u[w-d-z(d)-p(d)] \int_{d+z(d)}^{\infty} f(x)dx. \tag{7}$$

The first order condition for a maximum is $U'(d) = 0$. Hence by straightforward calculus we find:

$$U'(d) = -p'(d) \int_0^{d+z(d)} u'[w-x-p(d)]f(x)dx$$
$$-(1+z'(d)+p'(d))u'[w-d-z(d)-p(d)] \int_{d+z(d)}^{\infty} f(x)dx. \tag{8}$$

We have:

$$p(d) = (1+\gamma)Ec^*(X)+k = (1+\gamma) \int_{d+z(d)}^{\infty} (x-d-z(d))f(x)dx + k, \tag{9}$$

which gives:

$$p'(d) = -(1+\gamma)(1+z'(d)) \int_{d+z(d)}^{\infty} f(x)dx, \tag{10}$$

and hereby:

$$1 + z'(d) + p'(d) = (1 + z'(d))\left[\int_0^{d+z(d)} f(x)dx - \gamma \int_{d+z(d)}^{\infty} f(x)dx\right]. \tag{11}$$

Hence, by substituting (10) and (11) into (8), followed by straightforward calculus, we find:

$$U'(d) = [1 + z'(d)] \int_{d+z(d)}^{\infty} f(x)dx \cdot \tag{12}$$

$$\left[(1+\gamma)\int_0^{d+z(d)} [u'(w-p(d)-x) - u'(w-p(d)-d-z(d))] f(x)dx + \gamma v'(w-p(d)- d-z(d)) \int_{d+z(d)}^{\infty} f(x)dx\right]$$

Since $u'(\cdot) > 0$ and $u''(\cdot) < 0$, we observe from (12) that if $\gamma = 0$ and $w > p(d) + z(d) + d$, we have

$$U'(d) = 0 \text{ if and only if } z'(d) = -1.$$

From (12) we also have generally

$$U'(d) > 0 \text{ if } z'(d) < -1$$
$$U'(d) < 0 \text{ if } z'(d) > -1,$$

which implies that $z'(d) = -1$ is a maximum point of $U(d)$, as shown illustratively in figure 1:

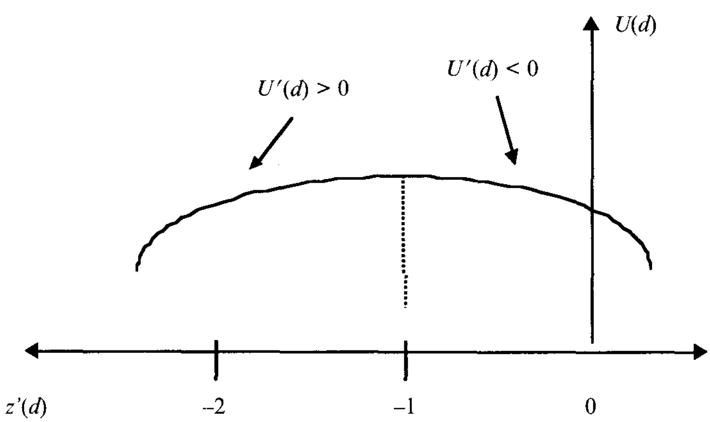

FIGURE 1

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Hence, if $\gamma = 0$ and $w > p(d) + z(d) + d$, then a value of d which gives $z'(d) = -1$ generates an optimal coverage solution (maximal expected utility) for the customer. □

If $D(d) = d + z(d)$ = the true deductible, then $D'(d) = 1 + z'(d)$, and hence $D'(d) = 0$ if $z'(d) = -1$. Since $D'(d) < 0$ when $z'(d) < -1$ and $D'(d) > 0$ when $z'(d) > -1$, then $z'(d) = -1$ represents a *minimum point* of $D(d)$. This minimum is greater than zero because $z(d) > 0$ for all $d \geq 0$.

From (10) we have correspondingly $p'(d) = 0$ if and only if $z'(d) = -1$. Since $p'(d) > 0$ when $z'(d) < -1$ and $p'(d) < 0$ when $z'(d) > -1$, then $z'(d) = -1$ represents a *maximum point* of $p(d)$.

Hence we conclude: $z'(d) = -1$ generates a maximum value of the premium $p(d)$ and a minimum value of the true deductible $d + z(d)$, which together gives maximal true coverage. In other words, *maximal true coverage gives maximal expected utility for the customer*, given that $\gamma = 0$ in the assumed premium function.

Note that there may exist more than one value of d satisfying $z'(d) = -1$; call them d_{max}. All other values different from d_{max} give lower expected utility from the customers point of view. Figure 2 gives an illustrative interpretation of this result by illustrating the existence of a tangent line $z'(d) = -1$ touching $z(d)$ in the maximum expected utility point d_{max}. For simplicity the figure assumes the existence of just one maximum point d_{max} satisfying $z'(d) = -1$ and a bonus-malus contract with *decreasingly* premium reduction generated by the deductible d.

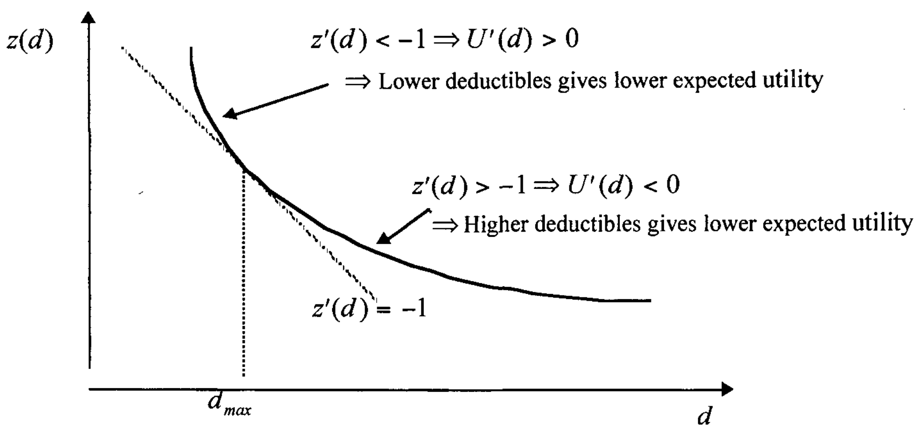

FIGURE 2

The bonus-malus rules of the contract, the market rate λ and the individual premium level at the purchasing time, decide the individual value(s) of d_{max} as well as the decreased expected utility for values of d different from d_{max}. This quite complex and individual dependent conclusion reflects to some extent the practical purchasing situation: Both the insurance company and the insurance customers find it difficult to recommend and choose an individual contractual deductible under bonus-malus contracts. And, if the premium reduction generated by the contractual deductible d has an upper limit (as most insurers have), there may not for some customers exist individual value(s) of d_{max} at all. Hence, these customers should probably not buy the insurance coverage either. These customers are typically customers with low bonus level, high premium level and hard malus rules in an economic market with low market rate λ. On the other hand, customers with high bonus level, low premium level and nice malus rules in an economic market with high market rate λ, should obviously buy the insurance coverage and choose d_{max} as the contractual deductible.

To summarize this section we conclude within our bonus-malus model: Given an excess of loss contractual compensation and a premium function without a safety loading factor, then there exists a specific choice of contractual coverage which gives maximum expected utility compared to other choices of coverage. This optimal contractual coverage is defined when the true insurance coverage is maximal, that is, when $z'(d) = -1$. This conclusion is in accordance with the correspondingly standard insurance contract without bonus-malus adjustments, where maximal (contractual) coverage is optimal for the customers.

Note that even if we in our model have defined the true deductible as a net present value based on an infinite-horizon consideration of the loss of bonus, the above conclusions will also hold for other considerations and assumptions of $z(d)$. The only condition is that $z(d)$ depends on d in some way.

6. Pareto optimal coverage

The conclusion in section 5 leads to a more general approach of deriving the optimal insurance coverage under bonus-malus contracts. A reversed key question is hereby: What is the optimality of a bonus-malus contract in an insurance market? And even more critical: Does there exist such an optimality at all? These problems involve *Pareto optimal* analysis techniques, where both the insurance customer and the insurer is analyzed from a risk-sharing point of view.

Hence consider a general insurance contract with bonus-malus adjustments. The necessary condition for the insurer to offer the true compensation $c^*(X) = max[c(X)-z(p), 0]$ for a premium p is obviously

$$Eu_0(w_0 - c^*(X) + p) \geq u_0(w_0), \tag{13}$$

where $u_0(\cdot)$ is the utility function of the insurer satisfying $u_0'(\cdot) > 0$ and $u_0''(\cdot) \leq 0$, w_0 is the initial wealth of the insurer and p follows the rules of a general bonus-malus system. In order for a bonus-malus contract to be acceptable to both the insurer and the customer, both (13) and (4) have to be satisfied. If such a contract exists at all, then the Pareto optimal contract is the one which maximizes the total risk-exchange utility for the insurer and the insured, that is, the contract which maximizes the left hand side of (4) and (13). This simple risk-exchange model is hereafter referred to as *the standard risk-exchange model*, which is e.g. part of Borch's classical 1960-theorem of Pareto Optimality. Within this framework Borch's theorem says in fact that a *sufficient* condition that our (bonus-malus) contract is Pareto Optimal is that there exist positive constants k_0 and k such that

$$k_0 u'_0(w_0+p-c^*(X))=ku'(w-p-X+c^*(X)),$$

which mathematically expresses a common linear maximizing of the left hand side of both (4) and (13). See Borch (1990), chapter 2.5 or Aase (1993), chapter 3, for a more detailed presentation.

We have:

Bonus-malus proposition III: A bonus-malus contract can not be Pareto Optimal within the standard risk-exchange model. □

Proof: A direct application of Borch's Theorem gives the first order condition for the Pareto optimal sharing rule between the insurer and the customer

$$u'_0(w_0+p-c^*(X))=\left[{}^{k}\!/_{k_0}\right]u'(w-p-X+c^*(X)), \tag{14}$$

where k and k_0 are arbitrary positive constants. Following Aase (1993), chapter 8, a differentiating of (14) with respect to X leads to

$$\frac{\partial}{\partial X}c^*(X)=\frac{R(w-p-X+c^*(X))}{R_0(w_0+p-c^*(X))+R(w-p-X+c^*(X))}, \tag{15}$$

where R and R_0 are the Arrow-Pratt measures of absolute risk aversions for the customer and the insurer. If both the customer and the insurer are risk averse, then directly from (15) we establish the general Pareto optimal criteria

$$0<\frac{\partial}{\partial X}c^*(X)<1 \text{ for all } X\geq 0. \tag{16}$$

On the other hand, under bonus-malus contracts we have $c^*(X) = max[c(X)-z(p), 0]$. Hence $\frac{\partial}{\partial X}c^*(X)=0$ for $c(X)\leq z(p)$, and hence quite generally the Pareto optimal criteria (16) will not hold for all $X\geq 0$. □

Under standard insurance contracts without bonus-malus adjustments the corresponding proposition is as follows; see Aase (1993), chapter 8, for a general proof which follows the same lines as the above proof:

Classical proposition III: The Pareto optimal sharing rule of a standard insurance contract without bonus-malus adjustments involves a positive amount of coinsurance within the standard risk-exchange model. A contractual compensation with a deductible can, however, not be Pareto optimal within the standard risk-exchange model. □

Proposition 2 in Holtan (2001) states that independent of the contractual compensation function, the true compensation function has always an individual deductible under bonus-malus contracts. Hence, given the standard risk-exchange model, it is intuitively correct that the Pareto optimal statement for standard contracts with a deductible is valid *in general* for bonus-malus contracts.

As concluded in Aase (1993), chapter 8, standard insurance contracts "with a deductible can only be Pareto optimal in models where one or more of the following are included; costs, moral hazard, asymmetric information, non-observability or alternative preferences (e.g. star-shaped utility)". Standard references within this context are: Arrow (1974), who included a fixed percentage (cost)loading to show optimality of deductibles, Raviv (1979), who found that a deductible is Pareto optimal if and only if the insurance costs depends on the insurance coverage, Rothschild & Stiglitz (1976), who included asymmetric information and found that low-risk individuals would choose high deductibles, and Holmstrøm (1979), who found that moral hazard gives rise to deductibles.

These expanded model assumptions are in general in accordance with the main intentions of a bonus-malus system in an insurance market:

1) *Adverse selection*: Measure and smooth out asymmetric information by individual a posteriori tariffication.
2) *Moral hazard*: Reduce the claim probability by economic punishment.
3) *Costs*: Reduce the administrative costs generated by claims handling.

Hence, since no one of these intentions was included in the model in this paper, we put forward the following conjecture:

Conjecture: A bonus-malus contract can only be Pareto optimal if the risk-exchange model includes one or more of the bonus-malus intentions 1-3. □

Proposition 3 and 4 in Holtan (2001) state that the compensation function of a bonus-malus contract without a contractual deductible is equivalent to the compensation function of a standard insurance contract with an individual deductible. Hence it should be easy to formally prove the existence of the conjecture for bonus-malus contracts without a contractual deductible.

On the other hand, if we do not restrict a bonus-malus contract in this way, then the size of the loss of bonus deductible depends on the individual choice of the contractual deductible, cf. the discussion in section 5. This dependency complicates the Pareto optimal analysis, and hence also the proof of the above conjecture.

As a concluding remark to the above discussion, we may point out that ordinary deductibles are usually used in the insurance market as the main instrument to reduce the claim probability (moral hazard) and to reduce the costs generated by claims handling. Therefore, the main intention of a bonus-malus system is to handle the problem of adverse selection generated by individual asymmetric information (even if Holtan (1994) outlines a model with high deductibles financed over a period of time as an adverse selection alternative to bonus-malus systems). Hence, as a general rule bonus-malus systems should only be used if individual loss experience is a significant risk parameter within the insurance market.

7. Summary

The paper outlines some new statements of optimal insurance coverage under bonus-malus contracts and compares them with corresponding classical statements under standard insurance contracts. The theoretical framework is an expected utility model, but neither adverse selection, moral hazard nor costs are part of the model. Under the assumption of an excess of loss contractual compensation and a premium function without a safety loading factor, it is outlined that maximal true coverage gives maximal expected utility for the customers. This result is in accordance with classical theory of standard contracts without bonus-malus adjustments. On the other hand and within the same expected utility model, it is outlined that bonus-malus contracts are not optimal to both the customers and the insurer at the same time, that is, Pareto optimal. The conjecture in section 6, which is not formally proved, states as a natural consequence that bonus-malus contracts can only be Pareto optimal if adverse selection, moral hazard and/or costs are included in the analysis model.

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