RELATIVITY, UNCERTAINTY AND ELECTRODYNAMICS

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1. Introduction

It is known that the Unified Field Theories of Weyl [14] and of Einstein [4] give no indication of how Relativity and Quantum Theories should be connected into a comprehensive field theory of physics. Indeed, the only determined attempt to establish such a theory, due to Eddington [3] and [6], failed through lack of contact with the contemporary developments, especially in quantum electrodynamics and elementary particles. Its author tried to explain curvature of the space-time in terms of statistical fluctuations partly of a physical origin defined within a mechanical system, and partly of a geometrical origin of coordinates superimposed on the latter. It is clear however that both the fluctuations of Eddington refer to purely mathematical frames. The probabilistic nature of his theory takes no account of physical objects, such as particles or energy distributions. It is the author's belief that this is the cause of difficulties associated with the otherwise admirable work of Eddington.

In this paper, we shall refrain from attempting to apply statistical considerations to an a priori conceived geometry. In the absence of non-gravitational fields only, we assume that the physical world is described by a Riemann manifold V_4. As far as unified theory is concerned, we confine ourselves to the case when such non-gravitational fields are electromagnetic, that is when they are described by skew symmetric tensors $h^{\mu\nu}$ and $f_{\mu\nu}$, satisfying respectively the first and second sets of Maxwell's equations. Greek indices are assumed throughout to go from 1 to 4 and the usual summation convention is retained. The distinction between $h^{\mu\nu}$ and $f_{\mu\nu}$ arises from Mie's theory ([11], [8], [9] and [10]) and we shall find an occasion to refer to it again in the sequel.

The most characteristic feature of General Relativity, which distinguishes it from other theories of physics, is contained in the Principle of Geometrisation. The laws of nature are represented by measurable properties of the mathematical continuum which is the model of physical space-time. We interpret this here as implying that the geodesics of the manifold should be the paths of test particles in the presence of an electromagnetic field. For a charged particle, their differential equations are

$$\frac{d^2x^\alpha}{ds^2} + \left\{ {\alpha \atop \mu\nu} \right\} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = g^{\alpha\lambda} f_{\lambda\mu} \frac{dx^\mu}{ds},$$

where $g_{\mu\nu}$ is a symmetric tensor used in raising and lowering of tensor indices and in constructing Christoffel brackets $[\alpha, \mu\nu]$ and $\{{\alpha \atop \mu\nu}\}$. It has been shown (ref. 7) that the above equations arise from the extremal ("Fermat") variational principle

$$\delta \int ds = 0,$$

when the metric takes the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \phi_{\mu\nu} d_2 s^{\mu\nu},$$

and $\phi_{\mu\nu}$ is skew symmetric. "$d_2 s^{\mu\nu}$" is a skew, two-dimensional area element. It has been suggested that it represents a cross-section of a geodesic tube arising from subjecting the theory of motion of test particles to the Uncertainty Principle of Heisenberg. We seek in this paper to make this notion geometrically precise.

The area element can be constructed with the help of Lie differentiation. The geodesic tube itself is determined by introducing the concept of a dragged coordinate system [15]. Finally, we consider a generalisation of the electromagnetic action density of Mie [11] and of Infeld and Plebanski [5], required by the present theory.

It is this generalisation which offers a hope that our approach may be extended in a natural way to a unified field theory which would include gravitational, electromagnetic and continuous quantum fields. It is hard to see that a macroscopic field theory can be expected to do more. Purely quantum phenomena, exchange relations, probabilistic interpretation of wave functions, and so on, will take over somewhere in the passage to microphysics and will force the use of different mathematics.

2. Uncertainty principle and geodesic motion

Newton's First Law of Motion in general relativity is equivalent to the assertion that a free particle moves on a geodesic in a riemannian manifold representing the physical space-time. Similarly, it is natural to expect that in a unified field theory which describes, besides the gravitational field, also other physical fields, the geodesics should represent the paths of the particles in the presence of such fields. This requirement leads to non-riemannian geometrics which reduce to a riemannian V_4 only in the absence of non-gravitational fields. We shall investigate below the possibility of generating a suitable geometry of this kind by subjecting the V_4 to essentially quantal considerations.

It is self-evident that physical observations are necessary to determine whether a given particle describes a geodesic or not. Let us consider the conditions under which a relativistic observer (abbreviated to RO in the sequel) will carry out the relevant observations.

We must distinguish carefully between such phrases as "what really happens is $\cdots$" and "what an RO says happens is $\cdots$". What "really" happens is that the test particle, as long as it is reasonably small, will be subjected to Heisenberg's Uncertainty Principle because of the empirical methods involved in the process of observation.

The RO may have in mind a preconceived geodesic path L_{AB} joining two points A and B in an a priori imagined V_4. He wishes to test whether a particle moves on L_{AB} and in doing so makes successive observations of the particles position and momentum (that is, of the coordinates x^μ and the vector p^μ related to the vector dx^μ/ds tangent to the path). The observations are made at what constitutes in the RO's opinion, sufficiently numerous instants of time.

The errors Δx^μ and Δp^μ in the measurements of the position and the momentum, respectively, are related by

$$\Delta x^\mu \Delta p^\mu \geqq \frac{h}{2}.$$

It follows that all that the RO can actually observe is his particle moving inside a bundle of geodesics surrounding L_{AB} and described by Heisenberg's relation. We seek a quantitative determination of this bundle. To establish his conclusion that the test particle moves along L_{AB}, the RO is interested in what happens between A and B only. The measurement of x^μ at B is exact because RO ceases to be concerned with this particle beyond B(and allows the error in the momentum there to become infinite). The existence of a limiting velocity ($\lim \sqrt{g_{44}} < \infty$) puts an upper bound to the error with which p^μ can be determined close to A and this value can be taken as the initial momentum (or direction of motion at A). Hence RO can regard the terminal points A and B of L_{AB} as accurately fixed. The uncertainty bundle or tube of geodesics thus issues from A and ends at B.

It is still necessary to express mathematically the meaning of RO's conclusion that his particle does in fact move on L_{AB}. We show that this brings in the notions of a dragged coordinate system and of a Lie derivative ([15], Chapter I).

We replace the relation between the momentum (tangent) vector observed at some point P in the uncertainty tube (to within the error ΔP), and the momentum of the particle at "where it ought to be" on the geodesic L_{AB}, by an infinitesimal mapping of the tangent vector space at P onto a neighbouring region of the manifold. Now the RO's assertion that his particle describes L_{AB} if it is free, corresponds to a transformation of coordinates $S \to S'$ such that the vector in question has the same components at a point on L_{AB}, in S', as the original vector dx^μ/ds had at P, in S.

The above interpretation allows us to describe the situation in terms of Lie differentiation. We define the latter following Yano. Let X_n be an n-dimensional space of class C^m, which is the class of functions defining allowable coordinate transformation in the space

$$x'^\mu = x'^\mu(x^\nu), \qquad \mu, \nu = 1, 2, \cdots, n. \tag{1}$$

A set of position functions

$$G^\alpha(x), \qquad \alpha = 1, 2, \cdots, N, \; x \in X_n$$

is called a general geometric object G of class $p \leqq m$ if its components G^α transforms under (1) as

$$G'^\alpha = f^\alpha(G, x, x'),$$

where f^α are functions of G^α, x^μ, x'^μ and of the first p derivatives of x'^μ with respect to x^μ. Moreover, we require f^α to satisfy the usual group properties.

Let T be a mapping (possibly many valued) of a region R of X into $\bar{R}$ and let T^{-1} be its inverse. If $P, Q \in R$ and $\bar{P} \in \bar{R}$, we may have

$$T : P \to \bar{P}; \; T^{-1} : \bar{P} \to Q.$$

S is said to be dragged along by T, if $S \to S'$ so that the coordinates of Q in S' are the same as those of $\bar{P}$ in S

$$\bar{x}^\mu = x'^\mu. \tag{2}$$

Let the transformation (2) follow upon an infinitesimal mapping

$$\bar{x}^\mu = x^\mu + \vartheta^\mu d\eta. \tag{3}$$

Let also a new object $\bar{G}$ at x (i.e. at P) be defined by

$$\bar{G}'^\alpha(x) = G^\alpha(\bar{x}) = f^\alpha(\bar{G}(x), x, x').$$

The Lie differential of G^α with respect to ϑ^μ is then

$$\mathscr{L}_\vartheta G^\alpha d\eta = \bar{G}^\alpha(x) - G^\alpha(x). \tag{4}$$

The Lie derivative of G^α is defined as

$$\mathscr{L}_{\vartheta} G^{\alpha} = \vartheta^{\sigma} G^{\alpha}{}_{,\sigma} - \sum_{s=0}^{p} f_{\sigma}{}^{(\nu_s)\alpha}(G, x)\vartheta^{\sigma}{}_{,\nu_1,\nu_2,\cdots,\nu_s}$$

where comma denotes ordinary partial differentiation with respect to the coordinates, and

$$f_{\sigma}{}^{(\nu_s)\alpha}(G, x) = \frac{\partial f^{\alpha}}{\partial \bar{x}^{\sigma}{}_{,\nu_1,\cdots,\nu_s}}\Bigg|_{\bar{x}=x,\bar{x}^{\mu}{}_{,\nu}=\delta^{\mu}{}_{\nu},\bar{x}^{\mu}{}_{,\nu_1,\nu_2,\cdots}} = 0,$$

$$\vartheta^{\sigma}{}_{,\nu_0} = \vartheta^{\sigma}, \quad f_{\sigma}^{(\nu_0)\alpha} = \frac{\partial f^{\alpha}}{\partial \bar{x}^{\sigma}}\bigg|_{x}.$$

Lie derivative of G is a geometric object only if f^{α}'s are linear functions, tensors, spinors and affine connections being examples.

In a metric space a motion is defined as a mapping preserving distance ds between neighbouring points and an affine motion preserves in addition the property of parallelism. If $\mathscr{L}$ is the operator of Lie differentiation, and ∇ that of covariant differentiation, a motion is an affine motion if and only if

$$\mathscr{L}\nabla = \nabla\mathscr{L}.$$

Any motion in V_n is an affine motion.

Returning to our description of the motion of a test particle, we require that the mapping (3) should map geodesics into geodesics. We use the same affine parameter u for all curves of the congruence. Then, if p^{λ} is tangent to L_{AB} at some point $P(x^{\lambda})$ on it,

$$\frac{dp^{\lambda}}{du} + \Gamma^{\lambda}_{\mu\nu} p^{\mu} p^{\nu} = 0,$$

$\Gamma^{\lambda}_{\mu\nu}$ being the affine connection. A similar equation holds at $\bar{P}_{(\bar{x})}$. Define a new vector field $\bar{p}^{\lambda}$, and a new connection $\bar{\Gamma}^{\lambda}_{\mu\nu}$, by

$$p^{\lambda}(\bar{x}) = \bar{p}^{\lambda'}(x),$$

and

$$\Gamma^{\lambda}_{\mu\nu}(\bar{x}) = \Gamma^{\lambda'}_{\mu'\nu'}(x),$$

in a S' defined by (2). Then, we have, at P,

$$\frac{d\bar{p}^{\lambda'}}{du} + \bar{\Gamma}^{\lambda'}_{\mu'\nu'} \bar{p}^{\mu'} \bar{p}^{\nu'} = 0.$$

The dashes can be dropped on transforming back to S. Since (ref. 15) we have

$$\bar{p}^{\mu} = p^{\mu}$$

everywhere on L_{AB}, it follows that

$$({}_{\alpha}\Gamma^{\lambda}_{\mu\nu}) p^{\mu} p^{\nu} = 0.$$

Yano shows also, that

$$\mathscr{L}_\vartheta \Gamma^\lambda_{\mu\nu} = \nabla_\mu \nabla_\nu \vartheta^\lambda + R_{\sigma\mu\nu}{}^\lambda \vartheta^\sigma \tag{5}$$

where

$$R_{\sigma\mu\nu}{}^\lambda = 2\Gamma^\lambda_{[\mu|\nu|\sigma]} + 2\Gamma^\lambda_{[\sigma|\rho|}\Gamma^\sigma_{\mu]\nu}.$$

Furthermore, we have

$$p^\mu p^\nu \nabla_\mu \nabla_\nu \vartheta^\lambda = \nabla^2_u \vartheta^\lambda,$$

since L_{AB} is a geodesic.

It follows that ϑ^λ must satisfy the equation

$$\nabla^2_u \vartheta^\lambda + R_{\sigma\mu\nu}{}^\lambda \vartheta^\sigma p^\mu p^\nu = 0, \tag{6}$$

with $\vartheta^\lambda = 0$ at A and B.

The equation (6) is a constraint on the Lie mappings permitted in our description of geodesic motions. We may observe that when ϑ^λ is itself infinitesimal normal to L_{AB} at P, (6) becomes the equation of geodesic deviation. In our case, (6) defines the bundle of geodesics which we require in our unified field theory.

3. Area element and equations of motion

The primary aim of this paper is to define the skew symmetric area element mentioned in the Introduction and which was used in Ref. 7. Let, for this purpose, $\phi_{\mu\nu}$ be a skew symmetric tensor and ϑ^μ, w^μ define two infinitesimal Lie mappings with parameters ξ and η respectively. The cross section of the uncertainty tube of geodesics can be thought of as given by

$$(w^\mu \vartheta^\nu - w^\nu \vartheta^\mu) d\xi d\eta.$$

We assume therefore, that the metric of our space time is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \phi_{\mu\nu}(w^\mu \vartheta^\nu - w^\nu \vartheta^\mu) d\xi d\eta. \tag{7}$$

The equation (6), defining the bundle, must be amended since its derivation depended on the space time being a V_4. We now have

$$\mathscr{L}_\vartheta \Gamma^\lambda_{\mu\nu} p^\mu p^\nu = \mathscr{L}_\vartheta g^\lambda \tag{8}$$

where

$$g^\lambda = g^{\lambda\alpha} \frac{dx^\beta}{ds} f_{\alpha\beta} \tag{9}$$

and

$$f_{\alpha\beta} = \tfrac{1}{2}(\chi,_\beta n,_\alpha - \chi,_\alpha n,_\beta). \tag{10}$$

We have written, for the sake of brevity

$$n = \phi_{\mu\nu}(w^\mu \vartheta^\nu - w^\nu \vartheta^\mu), \tag{11}$$

and

$$\chi = \int_0^x \frac{d\xi}{ds}\frac{d\eta}{ds}\, ds \tag{12}$$

is a new scalar parameter. Definition (12) implies that ξ and η are to be regarded as functions of the arc length s of the Riemannian geodesic L_{AB}. The latter will not be, necessarily, an extremal curve in a geometry based on the metric (7). Since the Lie derivative of $\Gamma_{\mu\nu}$ in the new geometry is unchanged, the deviation equation which $V^\mu = \vartheta^\mu, w^\mu$, must satisfy, is

$$\nabla_u^2 V^\mu + B_\sigma^\lambda V^\sigma = 0 \tag{13}$$

where $B_\sigma^\lambda = R_{\sigma\mu\nu}{}^\lambda p^\mu p^\nu - \nabla_\sigma g^\lambda$, and $R_{\sigma\mu\nu}{}^\lambda$ is the Riemann-Christoffel tensor of the background V_4. When ds is given by (7), the equations of motion of a charged test particle can be obtained in the usual way from the variation

$$\delta \int_A^B ds = 0 \tag{14}$$

and take the form

$$\frac{d^2x^\alpha}{ds^2} + \left\{ {\alpha \atop \mu\nu} \right\} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = g^{\alpha\lambda} f_{\lambda\mu}\frac{dx^\mu}{ds}. \tag{15}$$

The curve given by (15) is, of course, a geodesic of the space-time and in this respect, our theory differs from that given in Ref. 12. Identifying $f_{\mu\nu}$ (given by equation 10) with an electromagnetic intensity tensor, the equations (15) are the Lorentz equations of motion of a charged particle moving in a gravitational field $g_{\mu\nu}$. The electromagnetic potential A_μ can be identified as either $\frac{1}{2}\chi,_\mu n$ or $-\frac{1}{2}\chi n,_\mu$, but it does not appear explicitly in the metric. In this way we avoid criticism which has been raised against Mie's theory, of assigning physical significance to a physically indeterminate object ([1] and [8]). We rely however, on Mie's work in our discussion of the action principle. The latter gives the most general way of arriving at an expression for the interaction between the gravitational and electromagnetic terms in the field equations. The variation ensures also that the resulting equations will have an invariant form.

4. Electromagnetic and U.F.T. action

We begin by considering electromagnetic action in general relativistic case but without reference to a unified field theory. As mentioned before, we retain some features of Mie's electrodynamics, namely, the formal independence of the material tensor $h^{\mu\nu}$, satisfying

(16) $$h^{\mu\nu}{}_{;\nu} = j^{\mu},$$

where $j^{\mu} = \rho_e \dot{x}^{\mu}$ is the current and a semicolon denotes covariant differentiation and of the intensity tensor $f_{\mu\nu}$ describes an electromagnetic field in vacuum. We make no assumption about $f_{\mu\nu}$ apart from its skew symmetry. This is in accordance with the ideas of Infeld and Plebanski [5] and of [8]. The Lagrangian density function L is then on arbitrary function of four quantities

$$L = L(h^{\mu\nu}, h^{\mu\nu}{}_{;\nu}, f_{\mu\nu}, g_{\mu\nu}).$$

Let

(17) $$\delta L = \tfrac{1}{2} p^{\mu\nu} \delta f_{\mu\nu} + \tfrac{1}{2} q_{\mu\nu} \delta h^{\mu\nu} + \phi_{\mu} \delta (h^{\mu\nu}{}_{;\nu}) - \tfrac{1}{2} \Gamma^{\mu\nu} \delta g_{\mu\nu}.$$

Following Born [1], we have, for integration over adjacent volumes of the manifold, mapped onto each other by (3)

$$\int_{V'} L' d\tau' = \int_{V} (L+\delta L) J d\tau = \int_{V} L d\tau,$$

where the Jacobian J is, to the first order in dη

$$J = 1 + \vartheta^{\mu}{}_{,\mu} d\eta.$$

It follows that

(18) $$(L+B)\delta^{\mu}_{\nu} = p^{\mu\alpha} f_{\nu\alpha} - h^{\mu\alpha} q_{\nu\alpha} + h^{\mu\alpha} [\phi_{(\nu,\alpha)} - \phi_{(\nu} \Gamma^{\lambda}_{\alpha)\lambda}] - T^{\mu}_{\nu}$$

where

$$B = \phi_{\sigma} h^{\sigma\rho}{}_{;\rho} + \tfrac{1}{2} \phi_{[\sigma,\rho]} h^{\sigma\rho} - \phi_{\sigma} \Gamma^{\lambda}_{\rho\lambda} h^{\sigma\rho}.$$

The equation (18) is considerably simplified if we assume (Ref. 8) that $f_{\mu\nu}$ and $h^{\mu\nu}$ appear in L, only in the combination

(19) $$U = f_{\mu\nu} h^{\mu\nu}.$$

Then

(20) $$L\delta^{\mu}_{\nu} = h^{\mu\alpha} [\phi_{(\nu,\alpha)} - \phi_{(\nu} \Gamma^{\lambda}_{\alpha)\lambda}] - T^{\mu}_{\nu} - B\delta^{\mu}_{\nu}$$

becomes the appropriate Lagrangian of Mie-Infeld-Plebanski theory. It should be noted that ϕ_{μ} is here a vector density.

In the theory of Mie's electrodynamics without potential (Ref. 8),

$$L = \tfrac{1}{4} f_{\mu\nu} h^{\mu\nu} - \phi_{\mu} j^{\mu} - \tfrac{1}{4} T,$$

where

$$\phi_{\mu} = \sqrt{-g} \frac{k^2}{\rho} \frac{\partial \Omega}{\partial \rho},$$

$$f_{\mu\nu} = 2\phi_{[\mu,\nu]} - \frac{1}{2g}(\phi_\nu g_{,\mu} - \phi_\mu g_{,\nu}),$$

$$T = -\sqrt{-g}\left(4\Omega + \rho\frac{\partial\Omega}{\partial\rho}\right),$$

Ω is an arbitrary function of U and of the density ρ of charged matter, given by

$$\rho^2 = k^2 j_\mu j^\mu, \qquad k = \text{const},$$

and g is the determinant of $g_{\mu\nu}$. In a Minkowski space g is constant and we obtain the usual expression for $f_{\mu\nu}$ as a curl of the potential ϕ_μ.

We must take account now of gravitation. This can be done in the same way as in [7]. We assume that

$$(21) \qquad \Omega = \Omega(R, U, \rho)$$

where R is the gravitational scalar

$$R = g^{\mu\nu}R_{\mu\nu},$$

and we have the identity

$$1 = \frac{1}{\rho_e^2} j_\mu j^\mu + \phi_{\mu\nu}\frac{d_2 s^{\mu\nu}}{ds^2},$$

$$d_2 s^{\mu\nu} = 2w^{[\mu}\vartheta^{\nu]} d\xi d\eta.$$

If we assume, following Mie's electrodynamics without potentials [8], that

$$\int\sqrt{-g}\frac{\partial\Omega}{\partial U} f_{\mu\nu}\delta h^{\mu\nu} d\tau = \int\sqrt{-g}\frac{\partial\Omega}{\partial U} h_{\mu\nu}\delta f^{\mu\nu} d\tau,$$

a lengthy, but standard calculation shows that the action principle

$$\delta\int\sqrt{-g}\,\Omega\, d\tau = 0,$$

leads to the field equations

$$(22) \qquad A_{\mu\nu} = 0,$$

and

$$(23) \qquad M_{\mu\nu} = 0.$$

Here

$$A_{\mu\nu} = \frac{\partial\Omega}{\partial R} R_{\mu\nu} - \tfrac{1}{2}\Omega g_{\mu\nu} - \tfrac{1}{2}\phi_{(\mu;\nu)} - 2g_{\alpha(\nu}h_{\mu)\beta}f^{\alpha\beta}$$
$$+ \tfrac{1}{2}g_{\mu\nu}[\phi_{[\rho,\sigma]}h^{\rho\sigma} + \phi_\sigma j^\sigma] + C_{\mu\nu}$$
$$+ \tfrac{1}{2}g^{\rho\sigma}[\rho\sigma, (\nu]\left(\frac{\partial\Omega}{\partial R}\right)_{,\mu}) - \tfrac{1}{2}g^{\rho\sigma}\left(\frac{\partial\Omega}{\partial R}\right)_{,\sigma}[\rho(\mu,\nu)],$$

$$C_{\mu\nu} = \left(\frac{\partial\Omega}{\partial R}\right)_{,\mu\nu} - \left(\frac{\partial\Omega}{\partial R}\right)_{,\rho\sigma} g^{\rho\sigma} g_{\mu\nu} - \left(\frac{\partial\Omega}{\partial R}\right)_{,\varepsilon} (\Gamma^{\varepsilon}_{\mu\nu} - g_{\mu\nu} g^{\rho\sigma} \Gamma^{\varepsilon}_{\rho\sigma} - g^{\varepsilon\rho} g_{\mu\nu,\rho}$$
$$+ g^{\rho\sigma}[\rho\sigma, (\mu]\delta^{\varepsilon}\nu)),$$

and

$$M_{\mu\nu} = -2 \frac{\partial\Omega}{\partial U} f_{\mu\nu} - \phi_{[\mu,\nu]}.$$

If we let $\partial\Omega/\partial U = \frac{1}{4}$, and identify $-\phi_{\mu}$ as the potential four vector, the equations (**23**) reduce to the second set of Maxwell's equations. Since, further, the velocity of light is dimensionless in our choice of units and ρ, U and R are all of dimensions of a density, we can require Ω to be a homogeneous function of these invariants. The simplest choice is

$$\Omega = R + U + \rho,$$

in which case

$$\rho = -2(R + 6U).$$

If independent means of measuring the electromagnetic energy content of the universe, such relations should be verifiable on a cosmic scale.

5 Conclusions

We have developed in the preceding sections of this paper the principles on which a unified field theory of gravitation and electromagnetism may be founded. Its initial axiom is the requirement that the geodesics of the space time should be the equations of motion of a charged particle. This results in associating with every pair of neighbouring points of the manifold a skew symmetric area element. The latter can be regarded as the carrier of radiation energy of the field. In this way, the distance measure between any two points is subjected to the explicit influence of electromagnetism. The area element is related to the uncertainty of physical measurements by means of which an RO determines empirically whether a test particle is free, that is whether it describes a geodesic path in the world. The resulting "uncertainty tube of geodesics" is determined with the help of Lie differentiation by equation **13**. The background manifold of the present theory, that is the world of uncharged gravitating particles is still the standard Riemannian V_4.

Summary

Starting with a Riemannian space V_4, we develop with the help of Lie differentiation the basic principles of a unified field theory of gravitation and electromagnetism. It is founded on the axiom that the geodesics

of space time should give the equation of motion of charged particles in the presence of electromagnetic fields. The metric is made to depend explicitly on the latter by involving Heisenberg's Uncertainty Principle.

References

[1] M. Born, *Ann. Inst. H. Poincaré*, 7 (1937), 155.
[2] P. A. M. Dirac, *Proc. Roy. Soc.* A257 (1960).
[3] A. S. Eddington, *Fundamental Theory*, (Cambridge, 1946).
[4] A. Einstein, *Meaning of Relativity*, *App. II*, (London, 1954).
[5] L. Infeld and J. Plebanski, *Proc. Roy. Soc.*, A224 (1954), 222.
[6] C. W. Kilmister and B. O. J. Tupper, *Eddington's Statistical Theory*, (Oxford, 1962).
[7] A. H. Klotz, *Nuovo Cim.*, 14 (1959), 135.
[8] A. H. Klotz, *Nuovo Cim.*, 23 (1962), 697.
[9] A. H. Klotz, *Thesis*, (1963, London).
[10] A. H. Klotz, *Journ. Maths.* $\pm$ *Mech.*, 14 (1965), 365.
[11] G. Mie, *Ann. d. Phys.* 37, 39, 40 (1912, 1913).
[12] G. Stephenson and C. W. Kilmister, *Nuovo Cim.*, 10 (1953), 230.
[13] J. L. Synge, *Theory of Relativity* (*General Theory*), (Amsterdam, 1960).
[14] H. Weyl, *Space Time Matter*, (London, 1922).
[15] K. Yano, *Theory of Lie Derivatives*, (Amsterdam, 1955).

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