SESSION 2

THEORY OF ACTIVE REGION STRUCTURE

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COMPUTATIONAL MODELING OF SOLAR MAGNETIC FIELDS

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ABSTRACT Modeling techniques for solar magnetic fields are discussed. As an extension of the source surface model for large scale solar magnetic fields, a method based on the Green's function approach to the Laplace equation is proposed. In the force-free field modeling, a proper definition of the boundary condition is an important but still unresolved problem. A condition based on the continuity of the Maxwell stress is newly proposed.

INTRODUCTION

The magnetic field on the solar surface can be measured by using magnetographs. The magnetic field in prominences is measured by means of the Hanle effect (Leroy 1988). The field in the corona may be derived from radio observations (Dulk and McLean 1978). The latter two measurements are, however, not performed so frequently as compared to almost routine-based mapping of photospheric magnetic fields by means of the magnetographs. Therefore an extrapolation of magnetic fields into the chromosphere and the corona based on the measured photospheric magnetic fields provides a useful means to infer the field above the photosphere.

The equation for the magnetic field is derived from the equation of motion in magnetohydrodynamics. For static equilibria we obtain

$$-\nabla p + \frac{1}{4\pi}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \rho \boldsymbol{g} = 0 \ . \tag{1}$$

Here ρ is the density, p is the pressure, $\boldsymbol{B}$ is the magnetic field, and $\boldsymbol{g}$ is the gravitational acceleration. In the solar corona, the pressure and the gravity forces can generally be neglected compared to the magnetic force. Therefore, we obtain the equation,

$$(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = 0 \ . \tag{2}$$

The magnetic field determined from this equation is called the force-free field. The equation can also be written as

$$\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B} \ , \tag{3}$$

where α should be constant along the field line,

$$\boldsymbol{B} \cdot \nabla \alpha = 0 \ , \tag{4}$$

in order for $\boldsymbol{B}$ to be divergence-free. A particular case in which α is constant in space is called the constant-α force-free field, or linear force-free field. The equation can be reduced to the Helmholtz equation. In contrast, the case in which α varies in space cannot generally be reduced to a linear equation, and hence called non-linear force-free fields.

The case of $\alpha = 0$, namely,

$$\nabla \times \boldsymbol{B} = 0 \tag{5}$$

is the current-free field, or the potential field. The magnetic field is described in terms of a scalar potential ψ,

$$\boldsymbol{B} = -\nabla \psi \ , \tag{6}$$

and the potential should satisfy the Laplace equation,

$$\Delta \psi = 0 \ . \tag{7}$$

POTENTIAL MAGNETIC FIELDS

The methods to solve the equation for the potential magnetic field are fully developed, and are summarized in Sakurai (1989). There are two categories of methods. One is the Green's function method and the other is the harmonic expansion.

In the Green's function methods, the observed magnetic field values are replaced with suitable distribution of magnetic sources, and the potential is represented as a superposition of the sources. Namely,

$$\psi(\boldsymbol{r}) = \int G(\boldsymbol{r}, \boldsymbol{r}') B_{\mathrm{b}}(\boldsymbol{r}') d\boldsymbol{S}', \tag{8}$$

where B_{b} is the boundary value of the magnetic field and $d\boldsymbol{S}'$ is the surface element at $\boldsymbol{r}'$. The function G is called the Green's function. It is assumed that the sources only exist in the observed area, and therefore the method assumes that the region of interest is isolated magnetically.

In the harmonic expansion methods, the potential is expanded in terms of Fourier series (planar case) or spherical harmonics (spherical case). The expansion coefficients are determined by using the observed magnetic fields. In Fourier expansions, the observed region is periodically duplicated in the solar surface. In order to avoid interference from these periodic extension of the observed data, one should supplement an area of vanishing magnetic field around the observed region.

In the case of spherical harmonics expansion, usually the data are accumulated over one rotation of the sun as synoptic data. In such a global modeling, the effects of the solar wind is taken into account by using the so-called source surface approximation.

A MODIFICATION TO THE SOURCE SURFACE MODEL

As the data have to be accumulated over one solar rotation, the conventional source surface model only deals with magnetic structures whose life time is as long as one month. On the other hand, there are some cases in which phenomena of shorter time scales have to be modeled. Poletto and Kopp (1988) tried to reproduce the magnetic structure after a flare by considering the flare model proposed by Kopp and Pneuman (1976). Namely, the late phase of a flare might be characterized by a magnetic configuration with partially open magnetic field lines. The open field lines gradually turn into a closed magnetic arcade via magnetic field line reconnection. The expected helmet streamer type magnetic configuration might be modeled in the framework of the source surface model. A major difference is that the location of the source surface is low, $r = 1.1 - 1.3r_{\odot}$, whereas $r \simeq 2.5r_{\odot}$ in the conventional source surface model. Further, in order to accommodate a higher spatial resolution in the computation, Poletto and Kopp (1988) adopted an approach in which only data within a longitude span of 60 degrees were retained and then periodically extended to the whole solar surface.

Here we propose an alternative approach based on the Green's function method. The situation we consider is as follows. Instead of making a global magnetic field model by accumulating data over one solar rotation, we only work on a single magnetogram. The magnetic field distribution on the invisible solar hemisphere is assumed to be the mirror reflection of that of the visible hemisphere. Sakurai(1982) derived the Green's function in such a case, when the source surface is at infinity. The solution is

$$\begin{aligned} G(\boldsymbol{r},\boldsymbol{r}') = &\frac{\boldsymbol{n}'\cdot\boldsymbol{l}}{2\pi}\left(\frac{1}{R}+\frac{1}{R_*}\right) \\ &+\frac{\boldsymbol{\mu}\cdot\boldsymbol{R}}{2\pi}\left[\frac{1}{R(R+\boldsymbol{l}\cdot\boldsymbol{R})}-\frac{1}{R_*(R_*+\boldsymbol{l}\cdot\boldsymbol{R}_*)}\right] \\ &+\frac{1}{4\pi a}\log\frac{R_*+\boldsymbol{l}\cdot\boldsymbol{R}_*}{R+\boldsymbol{l}\cdot\boldsymbol{R}}\,, \end{aligned} \tag{9}$$

$$\boldsymbol{R}=\boldsymbol{r}-\boldsymbol{r}',\quad \boldsymbol{R}_*=\boldsymbol{r}-\boldsymbol{r}'_*,\quad \boldsymbol{\mu}=\boldsymbol{l}\times(\boldsymbol{n}'\times\boldsymbol{l}).$$

Here a is the radius of the sun, $\boldsymbol{l}$ is the line-of-sight direction, and $\boldsymbol{r}'_*$ is the mirror-symmetric point of $\boldsymbol{r}$ located in the invisible hemisphere.

On the other hand we can show that the potential

$$\begin{aligned} F = &-\frac{m}{4\pi a}\log\left|(\tilde{R}-\boldsymbol{l}\cdot\tilde{\boldsymbol{R}})(\tilde{R}_*+\boldsymbol{l}\cdot\tilde{\boldsymbol{R}}_*)\right| \\ &+\frac{m'}{4\pi a}\log\frac{\tilde{R}_\dagger+\boldsymbol{l}\cdot\tilde{\boldsymbol{R}}_\dagger}{\tilde{R}_{\dagger*}+\boldsymbol{l}\cdot\tilde{\boldsymbol{R}}_{\dagger*}}\,, \end{aligned} \tag{10}$$

$$m'=-\frac{a}{b}m,\quad \tilde{\boldsymbol{R}}=\boldsymbol{r}-\tilde{\boldsymbol{r}},\quad \tilde{\boldsymbol{R}}_*=\boldsymbol{r}-\tilde{\boldsymbol{r}}_*,\quad \tilde{\boldsymbol{R}}_{\dagger*}=\boldsymbol{r}-\tilde{\boldsymbol{r}}_{\dagger*},$$

gives a vanishing line-of-sight component of the field on the sphere $r = a$. Here $*$ denotes the symmetric reflection and $\dagger$ denotes the mirror inversion. Therefore we may seek for certain values of the monopole strength m and its location b, such that the sum $G + F$ gives a resultant magnetic field which is nearly radial

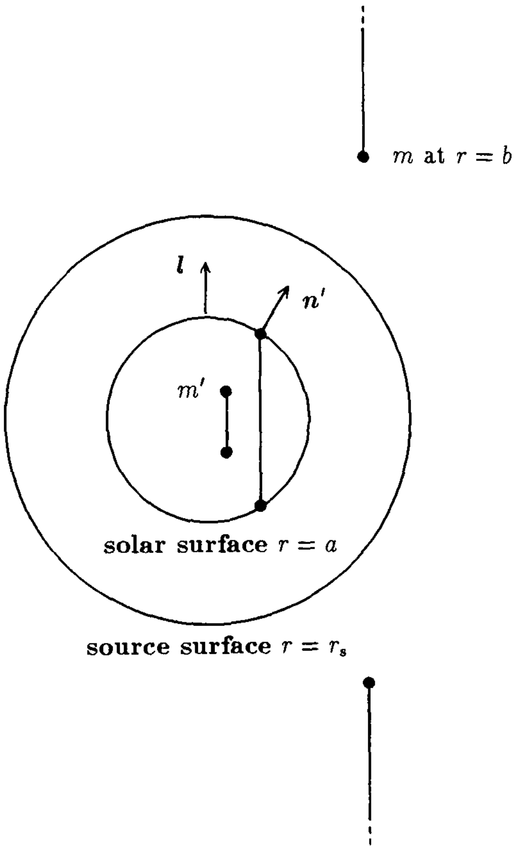

Fig.1. Source distribution for the Green's function described in the text.

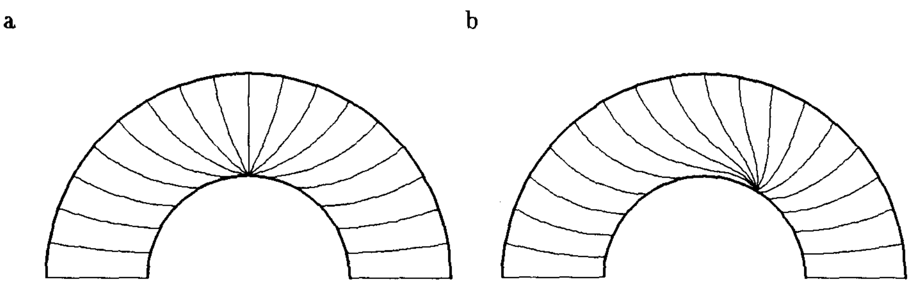

Fig.2. (a) Field lines of the Green's function when the source is at the center of the solar disk. (b) Same as (a), but when the source is 30° away from the disk center.

on the surface $r = r_s$. The function $G + F$ thus obtained is regarded as the Green's function with the source surface effect being taken into account.

Figure 1 explains the distribution of magnetic sources corresponding to $G + F$. Figure 2 shows the field lines of this Green's function for $r_s = 2a$. In figure 2a the source is at the disk center, while in figure 2b the source is 30 degrees away from the disk center. In both cases one can see that the field has no line-of-sight component on the solar surface (except at the location of the source itself), and the field is nearly radial on the source surface.

Similar calculations were performed for various source surface radii and source locations. It was found that the location of the outer monopole is generally around $r = 0.75r_s^2/a$. Therefore we assume that $b = (3/4)\ r_s^2/a$, and the most suitable strength of the monopoles were calculated. As the source surface radius decreases, the deviation of the field vectors from the radial direction increases. For $r_s < 4/3a$, the outer monopole comes inside the source surface and our scheme breaks down.

FORCE-FREE MAGNETIC FIELDS: BOUNDARY CONDITIONS

The equation for the force-free magnetic fields is generally non-linear, and no standard methods of solution exist. Several methods proposed so far were summarized in Sakurai (1989).

Some of the methods proposed utilize the observed magnetic field vectors as the boundary condition. Here the problem arises in that the methods of solution assume that the force-free condition applies to the boundary as well, whereas in actuality the layer where the magnetic field vectors are measured (i.e. the photosphere) may not be force-free.

Let us introduce two heights $z = z_1$ and $z = z_2$ in the solar atmosphere. Above z_2 we assume that the field is force-free. The magnetic field measurements refer to the layer z_1, where the force-free condition may be violated. The layer $z_1 < z < z_2$ is a transition layer from non-force-free to force-free states. The force-free magnetic field may be determined by specifying the z-components of the magnetic field (B_n) and the electric current density (j_n). If the thickness of the transition layer ($z_2 - z_1$) is small compared with the size of observational pixels, $B_n(z_1) \simeq B_n(z_2)$. This relation is derived from $\operatorname{div} \boldsymbol{B} = 0$ and from the assumption that the three components of $\boldsymbol{B}$ are of the same order in the transition layer.

On the other hand one cannot deduce $j_n(z_1) = j_n(z_2)$ from $\operatorname{div} \boldsymbol{j} = 0$. The reason is that, to connect the force-free layer $z > z_2$ and the non-force-free layer $z < z_1$, one must have large horizontal currents in the transition layer. In the limit of infinitely thin transition layer ($z_1 \to z_2$), the layer would have a surface current.

In order to look for the relation between $j_n(z_1)$ and $j_n(z_2)$, we integrate equation (1) over the layer $z_1 < z < z_2$ and obtain

$$0 = -\nabla_h P + \left[\frac{1}{c}\boldsymbol{J}_h \times \boldsymbol{B}\right]_h, \quad P = \int_{z_1}^{z_2} p dz, \quad \boldsymbol{J}_h = \int_{z_1}^{z_2} j_h dz . \qquad (11)$$

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The subscript h denotes the horizontal component. This equation describes the continuity of the horizontal mechanical stress. By taking the curl of this equation, we arrive at

$$\boldsymbol{n} \cdot \mathrm{curl}\ (B_{\mathrm{n}}\boldsymbol{B}_{\mathrm{h}})|_{z=z_1} = \boldsymbol{n} \cdot \mathrm{curl}\ (B_{\mathrm{n}}\boldsymbol{B}_{\mathrm{h}})|_{z=z_2}\ . \tag{12}$$

The vector $\boldsymbol{n}$ is the unit vector in the z-direction, and the subscript n denotes the normal component. This equation states that, even if the gravity and the pressure forces are present, the rotational component of the horizontal Maxwell stress should be continuous across the transition layer. Further, we may subtract the contribution from the potential field $\boldsymbol{B}_{\mathrm{hp}}$, which is regarded constant in the transition layer $z_1 < z < z_2$. Therefore we obtain

$$\boldsymbol{n} \cdot \mathrm{curl}\ [B_{\mathrm{n}}(\boldsymbol{B}_{\mathrm{h}} - \boldsymbol{B}_{\mathrm{hp}})]|_{z=z_1} = \boldsymbol{n} \cdot \mathrm{curl}\ [B_{\mathrm{n}}(\boldsymbol{B}_{\mathrm{h}} - \boldsymbol{B}_{\mathrm{hp}})]|_{z=z_2}\ . \tag{13}$$

CONCLUDING REMARKS

An interesting point concerning the boundary condition for the magnetic field modeling was recently raised by Wang and Sheeley (1992). The conventional source surface model is constructed in such a way that the calculated line-of-sight component of the field matches the observed line-of-sight component. The model does not reproduce the strength of the polar field, and a certain amount of polar magnetic flux is added to the model in rather ad hoc way. Wang and Sheeley (1992) assumed that the magnetic field is radial at the solar surface. Then one can easily convert the observed line-of-sight component of the magnetic field into the radial component, and the calculation is carried out to match this radial field. They found that this procedure gives several favorable aspects, including a correct reproduction of the polar field strength.

The assumption that the magnetic field is radial at the solar surface, as was introduced by Wang and Sheeley (1992), is physically understood in terms of the effect of buoyancy force on magnetic flux tubes. If the radial field assumption turns out to be satisfied, the method described in section 3 can be reformulated by using the Green's function already presented by Sakurai(1982). On the other hand, observations by vector magnetographs show unambiguously the existence of transverse magnetic fields in active regions. Whether the field is close to radial or the field has significant transverse components depends on the field strength and the spatial scales that are under study. In active regions, the field is not necessarily radial. In global scales, the field can be approximately radial at the solar surface. If the spatial resolution in the source surface model calculations increases in the future, one may find an intermediate situation in which neither the radial field approximation nor the transverse field measurements are applicable. Modeling techniques in such a case are not yet established.

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